Mathematical Note on semi-analytical PDF approach for high energy muon track

Shigeru Yoshida

March 1, 2006

A high energy muon track is associated with multiple cascades mainly by the pair creation reaction but also bremsstrahlung and photo-nuclear interaction. Their cross sections calculated by JULIeT are shown in the left panel of Figure 1. The pair creation cross section dominates over those of the other channels. Its mean free path is

$$\begin{aligned}\lambda_{e+e-}^{-1} &\sim \frac{N_A}{A}\sigma_{e+e-}\rho_{ice} \\ &\simeq 1.6 \times 10^{-25}\ [cm^2] \times 0.9\ [g/cm^3] \times\ 6.02 \times 10^{23}/18 \\ &\sim 3 \times 10^{-3} \qquad [cm^{-1}] \end{aligned} \tag{1}$$

Thus, a muon sees its pair creation cascade almost every $3m$ along its track. This situation is reasonably described by the continuous energy loss (CEL) approximation. In this picture, the **average** energy loss per unit length dE/dX is given by

$$\begin{aligned} -\frac{dE}{dX} &\equiv \beta E \\ &= E\frac{N_A}{A}\int dyy\frac{d\sigma(E)}{dy} \end{aligned} \tag{2}$$

where $y = 1 - E'_{\mu}/E_{\mu}$ is the inelasticity of muons. The β term is shown in the right panel of Figure 1. As you can see, β is almost energy independent in UHE/EHE regime except the photo-nuclear interaction, which simplifies our calculation.

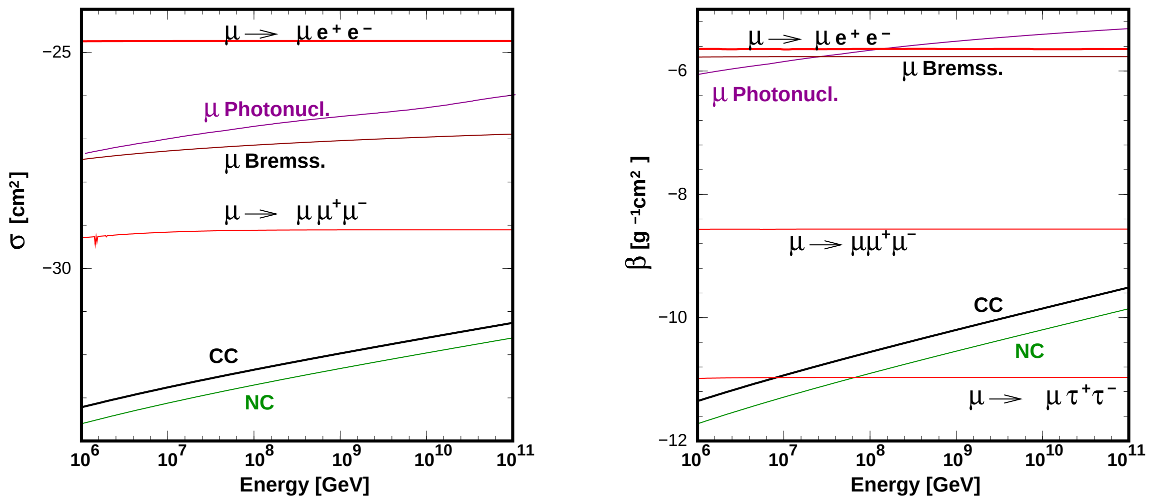

Figure 1: Left panel : Total cross sections of muons for various interaction channels as a function of energy. Right panel : The energy loss coefficient β of muons.

When β is constant, Eq. 2 can be easily resolved and we get the energy profile as

$$E(X) = E_0 \exp\left[-\beta X\right] \tag{3}$$

where X $[g/cm^2]$ is the slant depth of a given track and E_0 is initial energy at $X = 0$. Because the mean free path of the pair creation ($\sim [m]$) is definitely shorter than our detector resolution, we can assume a series of small cascade is generated every $\Delta X \leq \lambda_{e+e-}$ along the track with energy of

$$\Delta E = E(X) - E(X + \Delta X) \simeq E_0 \exp\left[-\beta X\right] \beta \Delta X \tag{4}$$

Therefore, a cascade energy relative to the primary (initial) energy,

$$\frac{\Delta E}{E_0} = \exp^{-\beta X} \beta \Delta X \tag{5}$$

is energy independent, which implies that you can scale this energy of the "small" cascade to any given primary muon energy.

The analysis above allows us to build a PDF of a given muon track as superpose of single cascade PDF at vertex every ΔX along track geometry with energy given by Eq. 5. You can give E_0 as whatever you want because you can scale it to true muon energy anyway. E_0 is just a normalization factor of PDF which can be derived in minimizing the likelihood, in other words. An individual cascade PDF will be built by using the photonics table or an alternative analytical function.

There are some issues. First, the CEL approximation should work perfectly for the pair creation, but not for the Bremsstrahlung and the photo-nuclear reaction, because their cross sections are two orders of magnitude lower than that of the pair creation. Those two reactions are sources of stochastic energy loss. But significant fraction of cascades generated by these reactions deposits energies too small to be resolved. So you have to include their contributions in this COE approximation. Only when the cascade energy happens pretty big, we are hopefully able to resolve it by the Sean's module as a 1st order approximation (Note that the CEL approximation discussed here can be considered as "0th order"). Secondly, β of the photo-nuclear reaction is not energy independent, as shown in Figure 1. The cross section itself is also rather uncertain. So its treatment is not so simple and leaves us significant systematic errors in reconstruction. We should keep this possibility in our mind.