The Empirical Atmospheric Muon Bundle model: Mathematical Representation

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The Elbert formula [1] gives the number of muons with energies greater than E_μ in a bundle as follows:

$$\begin{aligned} N_\mu &= \frac{E_T}{E_0}\frac{A^2}{\cos\theta}\xi^{-\alpha}(1-\xi)^\beta \\ \xi &= \frac{AE_\mu}{E_0} \end{aligned} \tag{1}$$

where A is the mass number of primary cosmic rays with energy of E_0, θ is zenith angle of a muon bundle. The other parameters are $E_T = 14.5$ GeV, α=1.757, β=5.25.

One can differentiate it to obtain the differential spectrum of muons in a bundle dN_μ/dE_μ.

At this stage of the IceCube analysis, we are not resolving individual muons, but measuring its bundle energy resulted from summing all the muon energies contained in the bundle. It is, therefore, useful to define the bundle energy as follows:

$$E_\mu^B \equiv \int\limits_{E_{th}}^{E_0/A} \frac{dN_\mu}{dE_\mu}E_\mu dE_\mu \tag{2}$$

The approximation using the fact that the integration above is mainly determined in the regime of $\xi \ll 1$ leads to the relatively simple formula to represent the bundle energy as

$$E_\mu^B = E_T\frac{A}{\cos\theta}\frac{\alpha}{\alpha-1}\left(\frac{AE_{th}}{E_0}\right)^{-\alpha+1}. \tag{3}$$

We approximate a bundle event as a single muon with energy of E_μ^B. This approximation should work well in the analysis based on N_{pe} which in turn calorimetrically measures the energy deposit that is proportional to E_μ^B to the first approximation. Then we can estimate the rate of muon "bundle" by rewriting the primary cosmic ray flux as a function of E_μ^B, *i.e.*,

$$\frac{dJ_\mu^B}{dE_\mu^B} = \frac{dE_0}{dE_\mu^B}\frac{dJ_{CR}}{dE_0}(E_0(E_\mu^B, \cos\theta, A, E_{th})). \tag{4}$$

Here dJ_{CR}/dE_0 is the relatively well-measured cosmic ray spectrum that can be represented by the power law form $\kappa E_0^{-\gamma}$. The formula described by Eq 3, then gives the bundle rate. We get

$$\begin{aligned} \frac{dJ_\mu^B}{dE_\mu^B} &= \frac{1}{\alpha-1}\frac{E_0}{E_\mu^B}\frac{dJ_{CR}}{dE_0}(E_0) & (5)\\ E_0 &= \left(\frac{\cos\theta}{A}\frac{\alpha-1}{\alpha}\frac{E_\mu^B}{E_T}\right)^{\frac{1}{\alpha-1}} AE_{th} & (6) \end{aligned}$$

Eq. 5 provides the atmospheric muon intensity at the surface. The parameters in Eq. 6 are to be determined by the IceCube high energy data samples. The most uncertain parameter which is a deciding factor in the normalization to give the absolute intensity is E_{th}, the threshold energy of muons in the integral of Eq. 2. Because the IceCube detector is only sensitive to sufficiently energetic muons, it is likely that E_{th} is around 1 TeV. The real data will tell us this value.

Since the IceCube is an underground detector, we see muons after propagating inside the earth. The propagation reduces the muon energy from E_μ to E_μ^{I3}. The bundle energy at the IceCube depth is given by

$$E_\mu^{B,I3} \equiv \int_{E_{th,I3}}^{E_{max}} \frac{dN_\mu}{dE_\mu^{I3}} E_\mu^{I3} dE_\mu^{I3} \tag{7}$$

The numerical calculation as the JULIeT does is able to estimate distribution of E_μ^{I3}, but the simple analytic representation is more useful in the real data fitting to obtain the parameters such as E_{th}. The CEL approximation gives

$$E_\mu^{I3} = (E_\mu + \epsilon)\exp^{-\beta_\mu X} - \epsilon \tag{8}$$

where $\beta_\mu \sim 4 \times 10^{-6}$ cm^2 g^{-1} is the inelasticity parameter due to the muon interactions, and $\epsilon \sim 500$GeV is the critical energy below which the ionization loss dominates over the radiative interactions. X is the slant depth of the muon trajectory. Eqs. 7 and 8 give

$$E_\mu^{B,I3} \simeq e^{-\beta_\mu X} E_\mu^B - \frac{\beta}{\alpha}(1 - e^{-\beta_\mu X}) A \frac{\epsilon}{E_0} E_\mu^B. \tag{9}$$

Here E_μ^B must be calculated for the threshold energy of muons at the IceCube depth, $E_{th,I3}$, which is related to the one at the earth surface by Eq. 8. One can see that the second term represents the contribution from the ionization loss. Because the EHE regime is $E_0 \geq 1$ PeV $\gg \epsilon$, this term is negligible. Note that we used the approximation here that β_μ is energy independent, which is not exactly true because of the photo-nuclear reaction. The systematics by introducing this approximation should be properly accounted in fitting the data.

References

[1] T. K. Gaisser, *Cosmic Rays and Particle Physics*, (Cambridge University Press, Cambridge, England, 1990) Chapter 12.