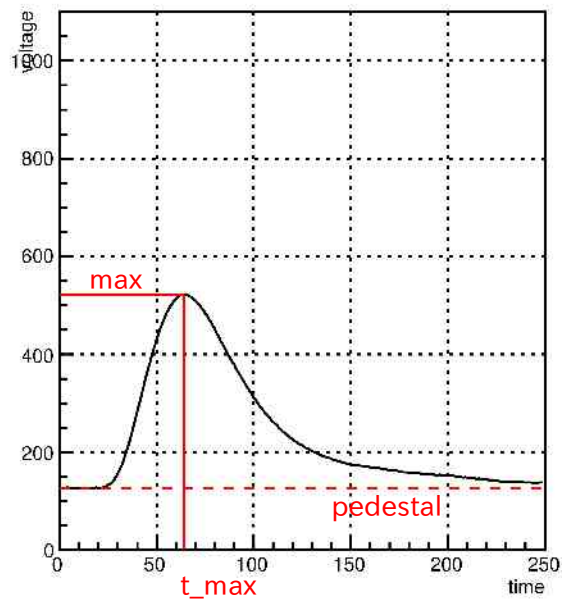


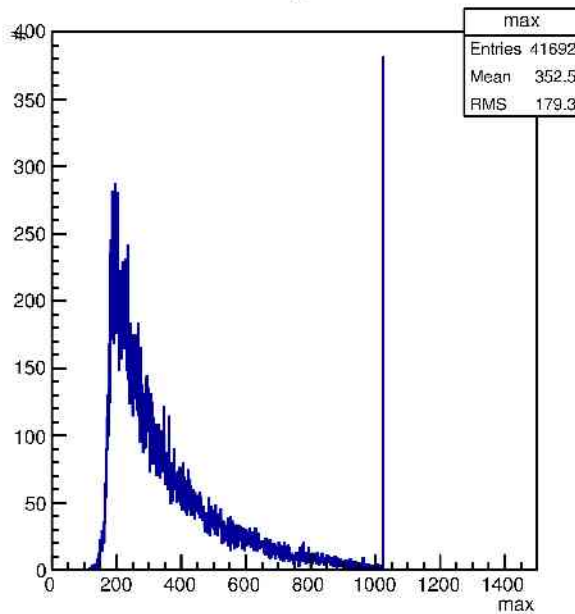
CsI Photon Detector Waveform Analysis

2015/09/08
H. Ito

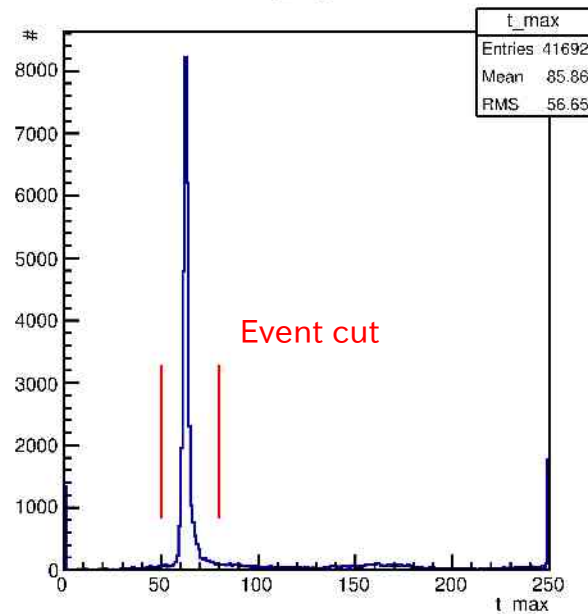
waveform(n,x,y)=(12,7,19)



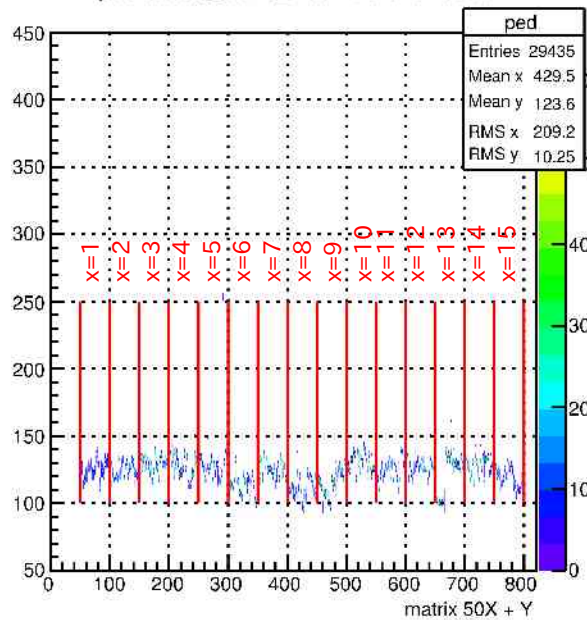
max



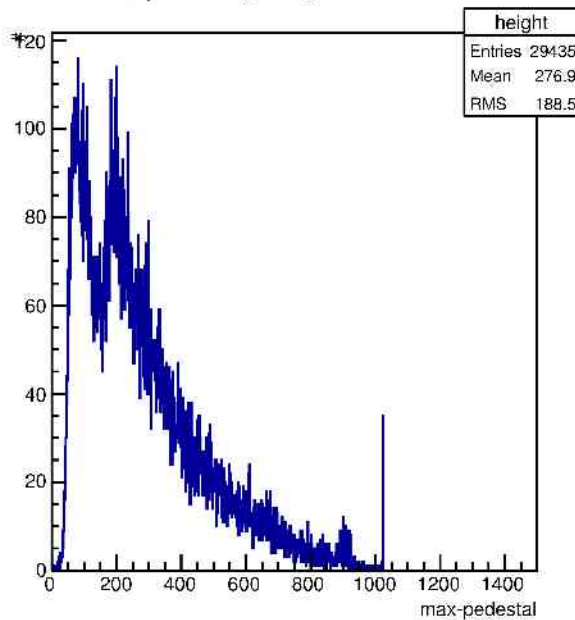
t_max



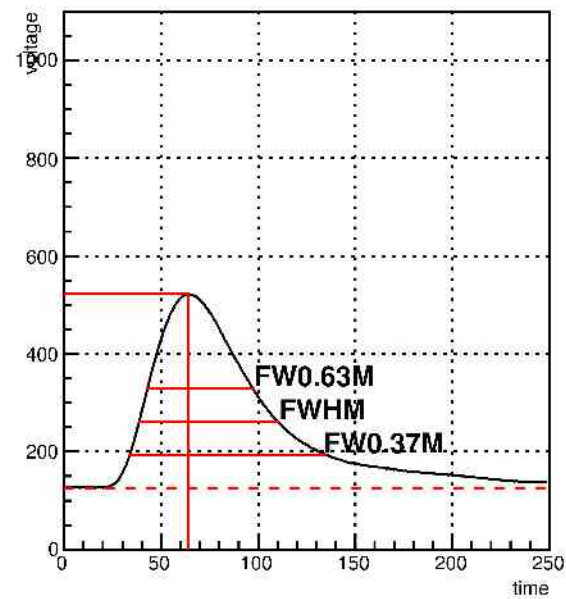
pedestal @50<t_max<80 event cut

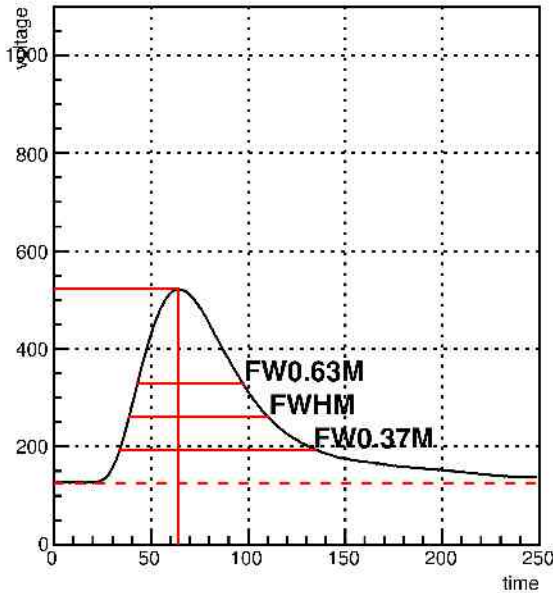


max-pedestal @50<t_max<80 event cut



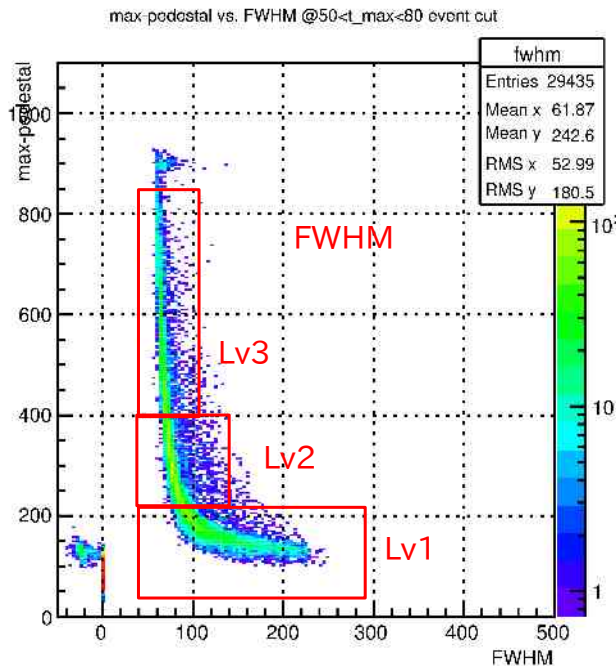
waveform(n,x,y)=(12,7,19)



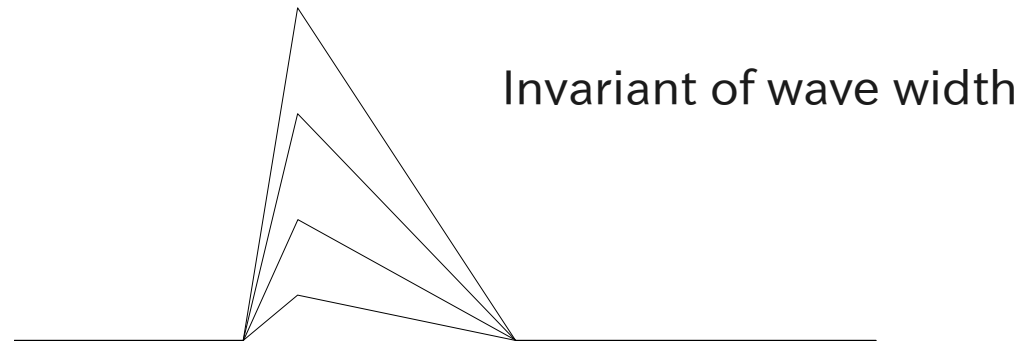


First Motivation: decision of function model

- Step1. Divide 3 level in pulse height
- Step2. Main Model Fix in each the level ←← This time
- Step3. Decide the Model in all level
Research of the property
- Step4. applying Multiple pulse event



Expected waveform

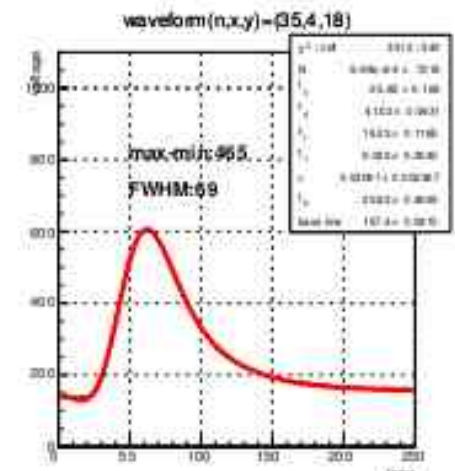
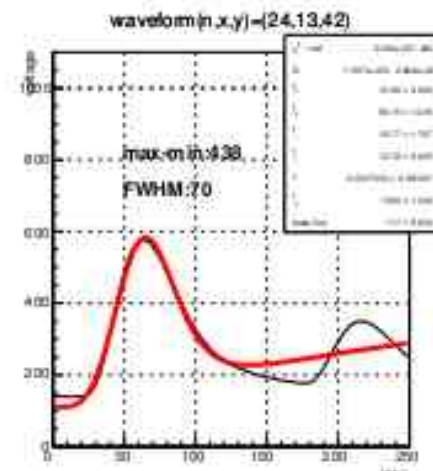
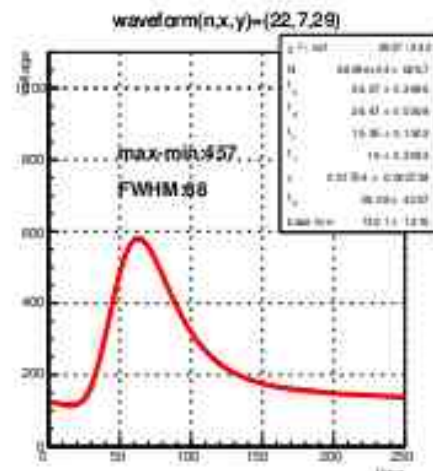
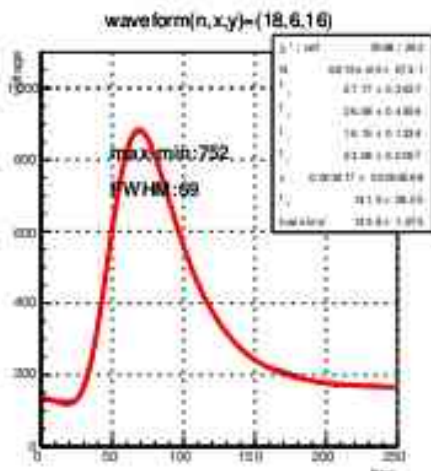
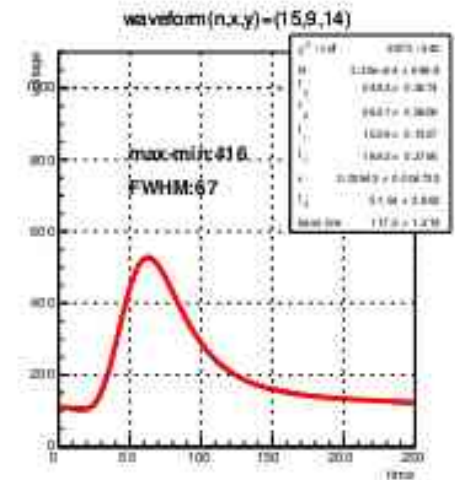
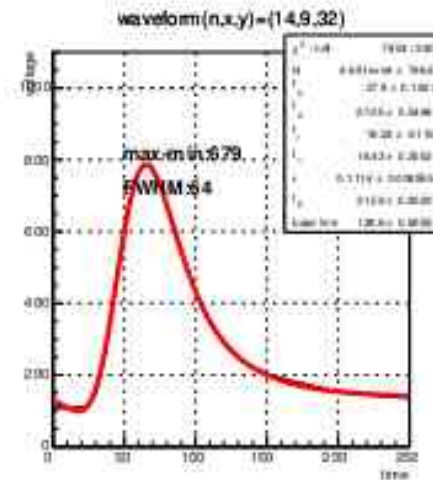
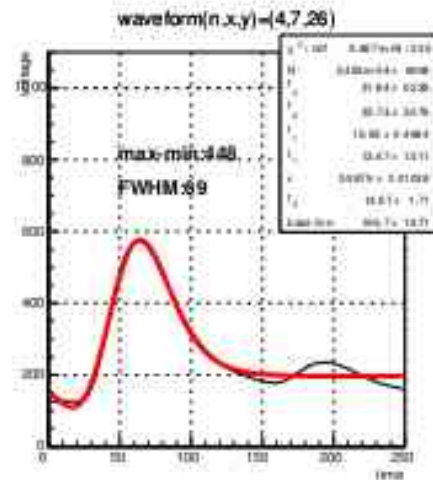
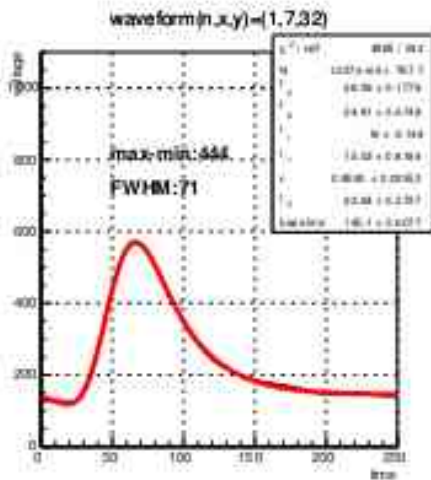


1. Simple Analysis
→ Level 3 waveform fitting

Lv3 waveform

$$V(t) = N \text{ Freq} \left[\frac{t - (t_0 + t_d)}{\tau_r} \right] \frac{t - t_0}{\tau_1^2} \left(\exp \left[\frac{-(t - t_0)}{\tau_1} \right] + \varepsilon \exp \left[\frac{-(t - t_0)}{\tau_2} \right] \right)$$

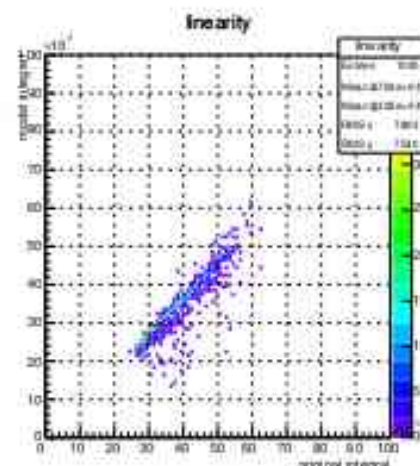
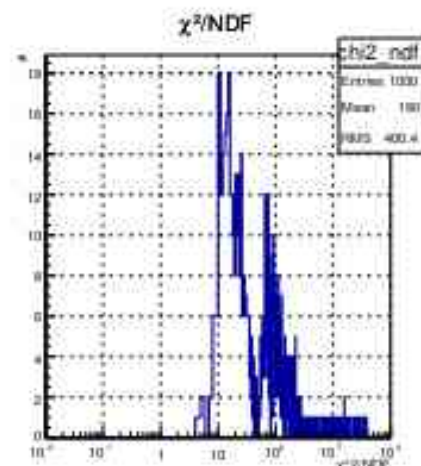
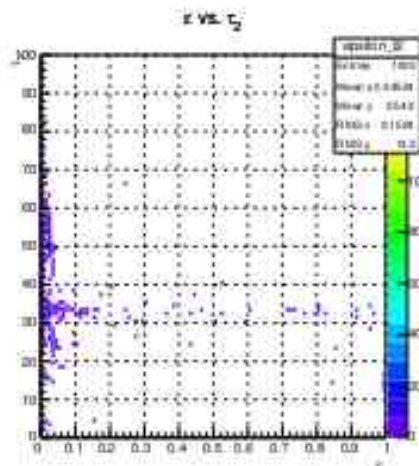
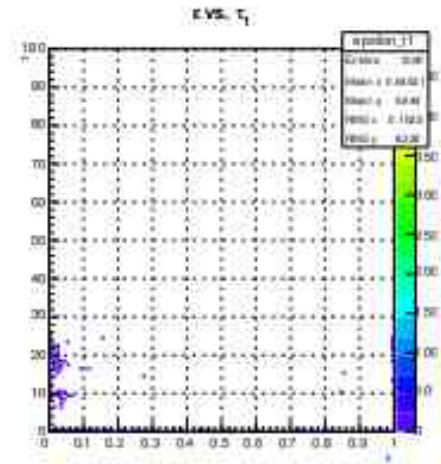
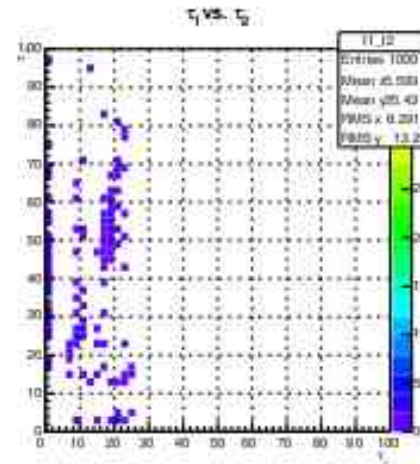
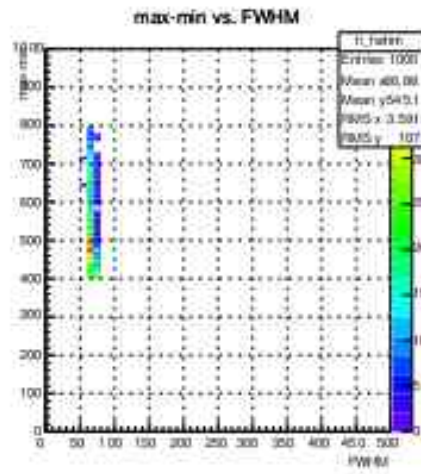
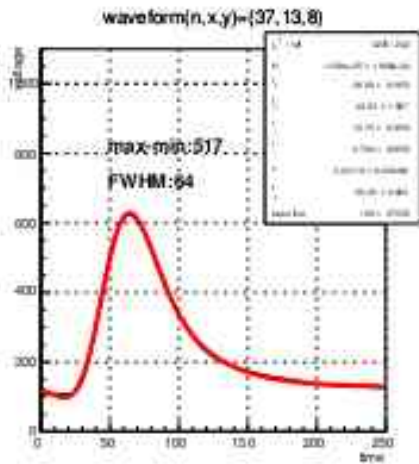
Model: yamazaki



Lv3 waveform

$$V(t) = N \text{ Freq} \left[\frac{t - (t_0 + t_d)}{\tau_r} \right] \frac{t - t_0}{\tau_1^2} \left(\exp \left[\frac{-(t - t_0)}{\tau_1} \right] + \varepsilon \exp \left[\frac{-(t - t_0)}{\tau_2} \right] \right)$$

Model: yamazaki

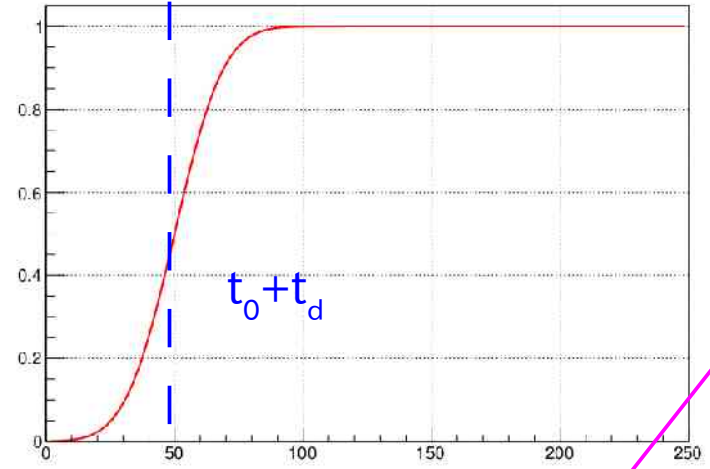


Lv3 waveform

$$V(t) = N \operatorname{Freq} \left[\frac{t - (t_0 + t_d)}{\tau_r} \right] \frac{t - t_0}{\tau_1^2} \left(\exp \left[\frac{-(t - t_0)}{\tau_1} \right] + \varepsilon \exp \left[\frac{-(t - t_0)}{\tau_2} \right] \right)$$

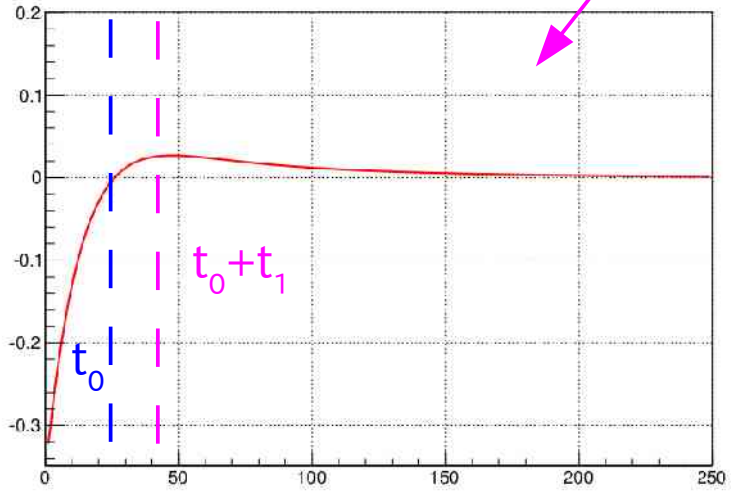
Model: yamazaki

TMath::Freq((x-[1]-[2])/[3])

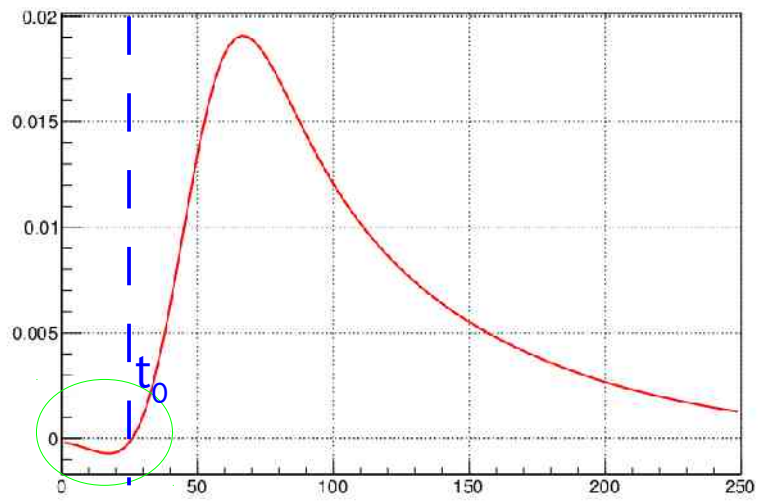


Parameters
 $N = 1$ $\tau_1 = 18$
 $t_0 = 26$ $\tau_2 = 50$
 $t_d = 24$ $\varepsilon = 0.16$
 $t_r = 15$

$(x-[1])/[4]/[4]*(\exp(-(x-[1])/[4])+[5]*\exp(-(x-[1])/[6]))$



TMath::Freq((x-[1]-[2])/[3])*(x-[1])/[4]/[4]*(\exp(-(x-[1])/[4])+[5]*\exp(-(x-[1])/[6]))



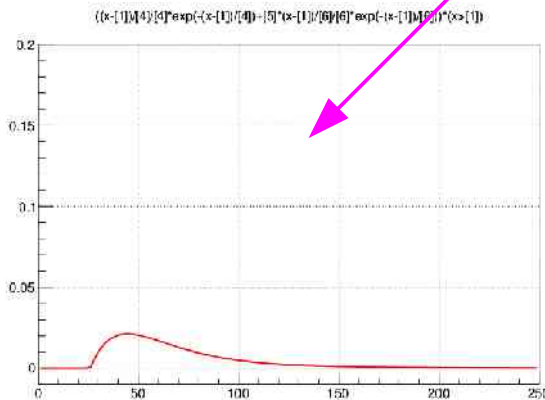
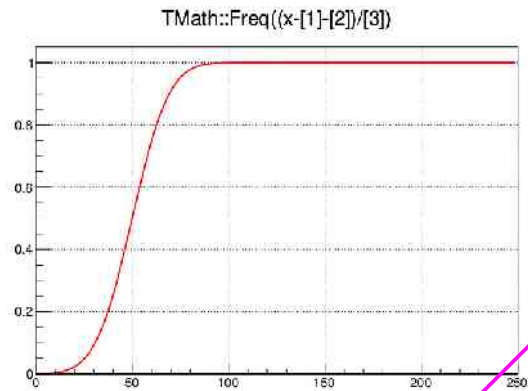
Lv3 waveform

$$V(t) = N \text{ Freq} \left[\frac{t - (t_0 + t_d)}{\tau_r} \right] \frac{t - t_0}{\tau_1^2} \left(\exp \left[\frac{-(t - t_0)}{\tau_1} \right] + \varepsilon \exp \left[\frac{-(t - t_0)}{\tau_2} \right] \right)$$

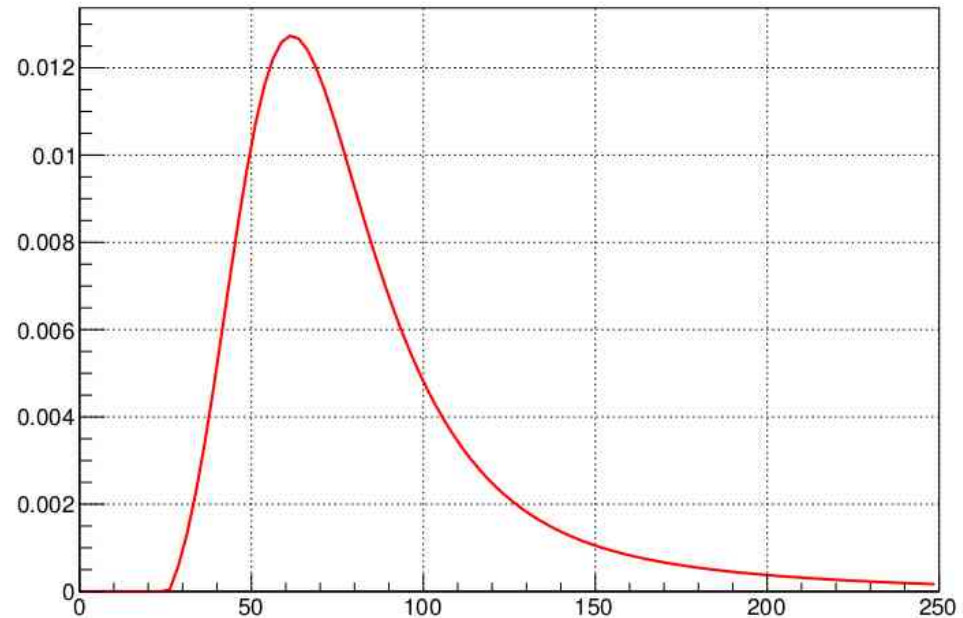
Model: yamazaki

$$V(t) = N \text{ Freq} \left[\frac{t - (t_0 + t_d)}{\tau_r} \right] \left\{ \frac{t - t_0}{\tau_1^2} \exp \left[\frac{-(t - t_0)}{\tau_1} \right] + \varepsilon \frac{t - t_0}{\tau_2^2} \exp \left[\frac{-(t - t_0)}{\tau_2} \right] \right\} \quad (t > t_0)$$

Model: ito



TMath::Freq((x-[1]-[2])/[3]) * ((x-[1])/[4])^4 * exp(-(x-[1])/[4]) + [5]*(x-[1])/[6])^6 * exp(-(x-[1])/[6])) * (x>[1])

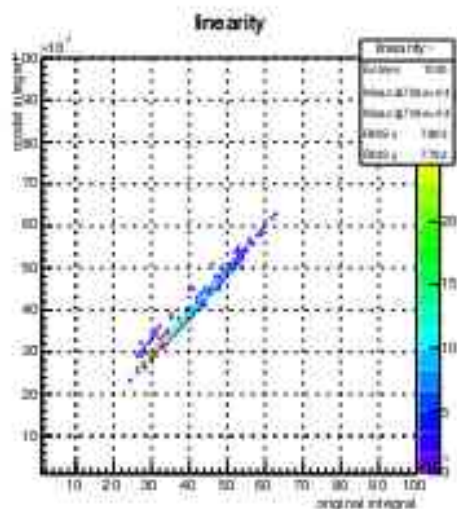
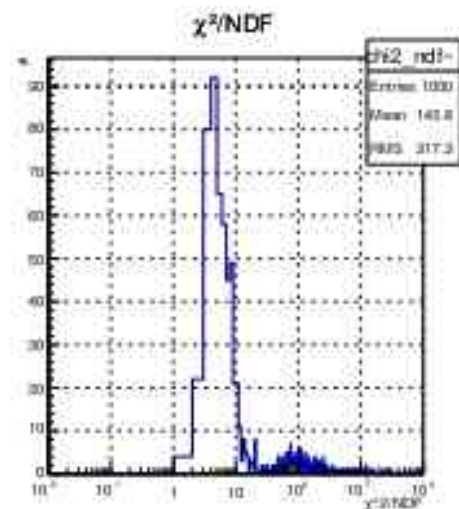
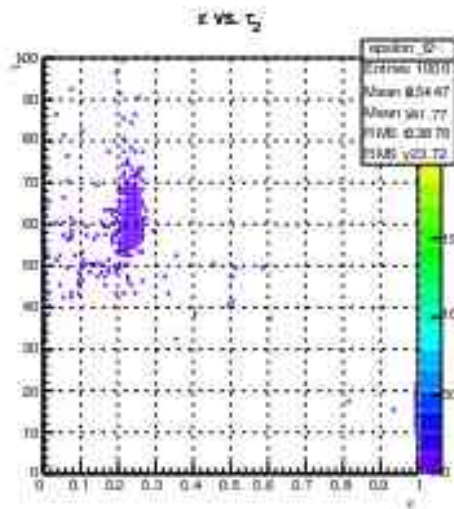
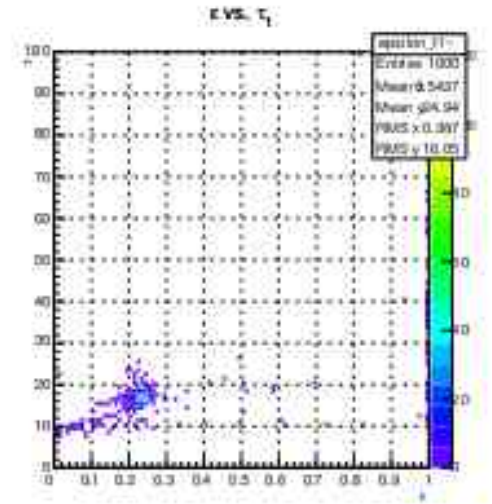
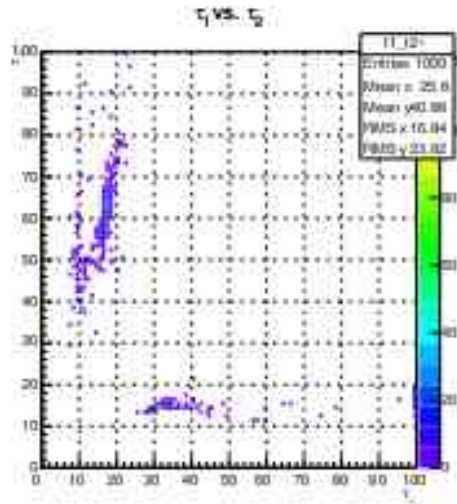
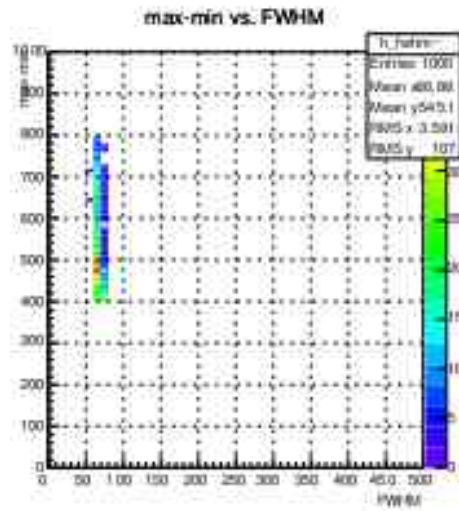
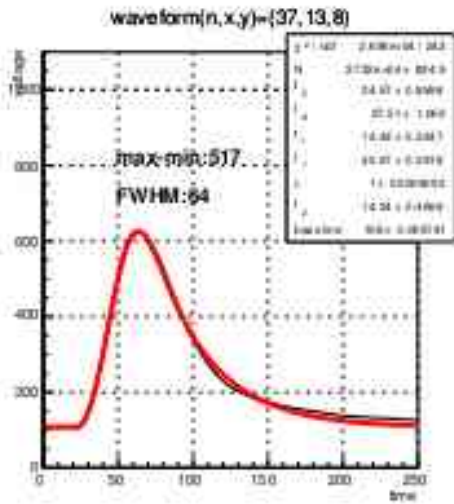


Lv3 waveform

Model: ito

$$V(t) = N \text{ Freq} \left[\frac{t - (t_0 + t_d)}{\tau_r} \right] \left\{ \frac{t - t_0}{\tau_1^2} \exp \left[\frac{-(t - t_0)}{\tau_1} \right] + \varepsilon \frac{t - t_0}{\tau_2^2} \exp \left[\frac{-(t - t_0)}{\tau_2} \right] \right\}$$

($t > t_0$)



model results:

Waveform: $f(t) = N \text{ Freq} \left[\frac{t - (t_0 + t_d)}{\tau_r} \right] \left\{ \frac{t - t_0}{\tau_1^2} \exp \left[\frac{-(t - t_0)}{\tau_1} \right] + \varepsilon \frac{t - t_0}{\tau_2^2} \exp \left[\frac{-(t - t_0)}{\tau_2} \right] \right\}$

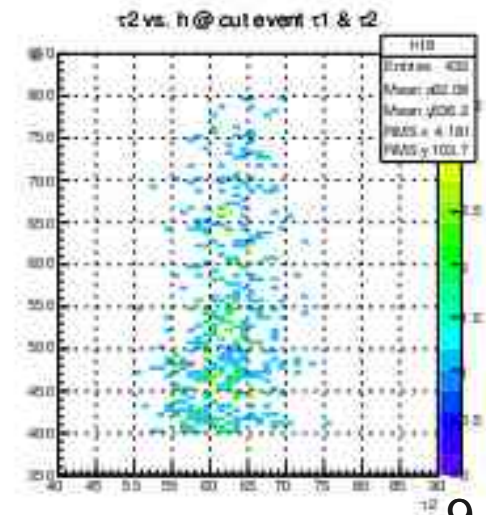
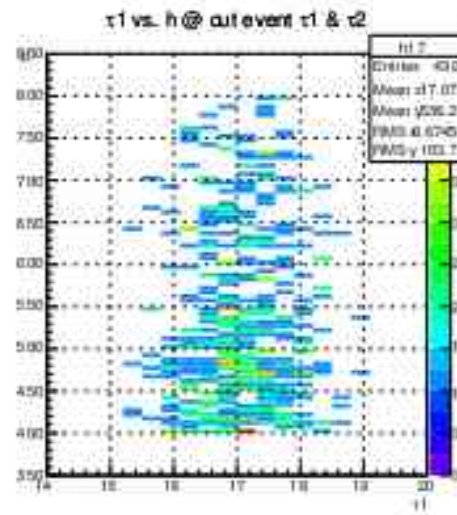
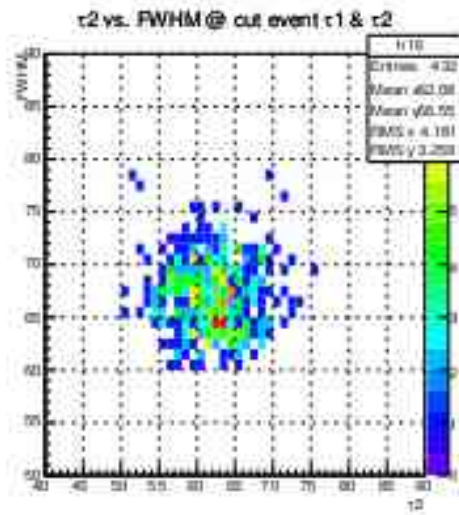
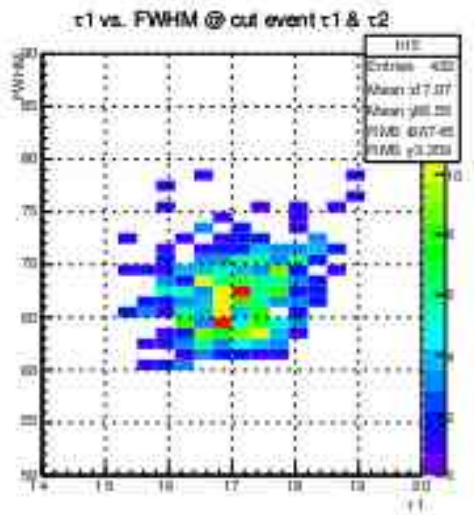
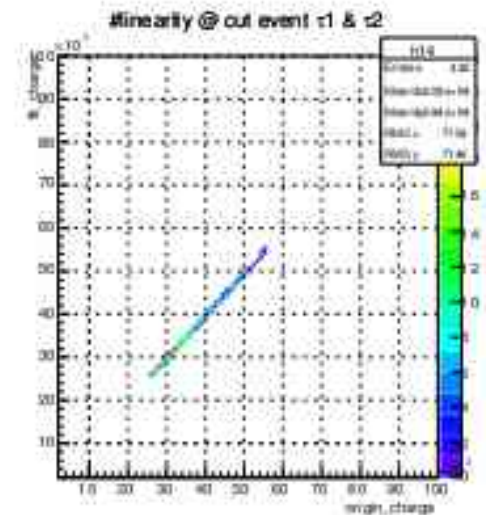
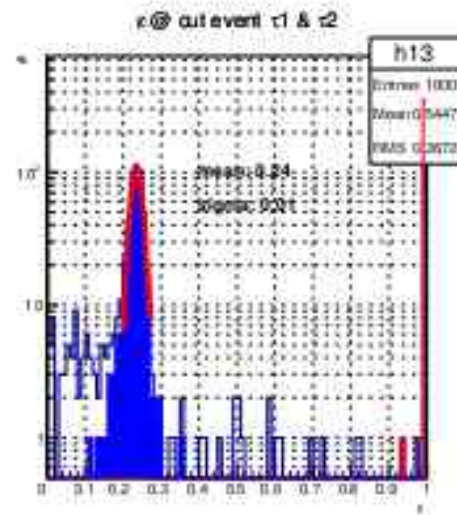
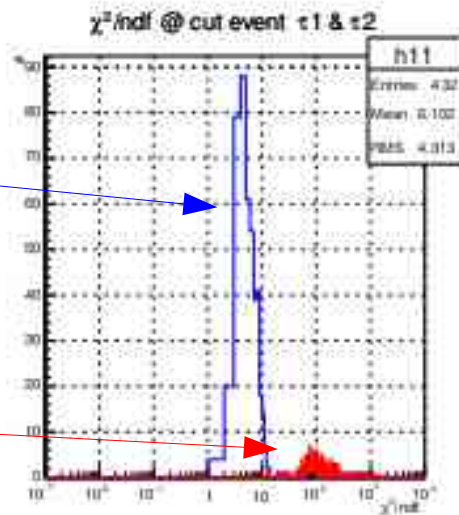
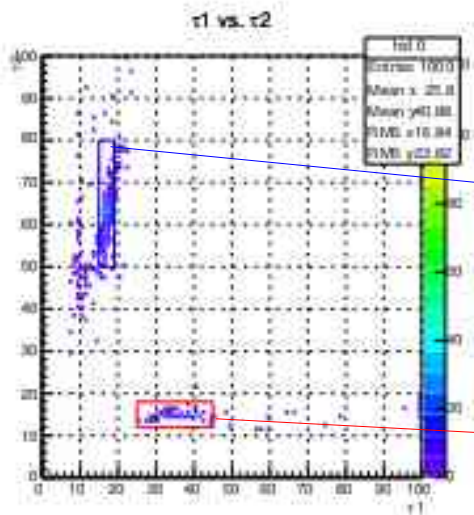
variables (t):

Parameters (t):

Lv3 waveform

Model: ito

$$V(t) = N \text{ Freq} \left[\frac{t - (t_0 + t_d)}{\tau_r} \right] \left\{ \frac{t - t_0}{\tau_1^2} \exp \left[\frac{-(t - t_0)}{\tau_1} \right] + \varepsilon \frac{t - t_0}{\tau_2^2} \exp \left[\frac{-(t - t_0)}{\tau_2} \right] \right\} \quad (t > t_0)$$

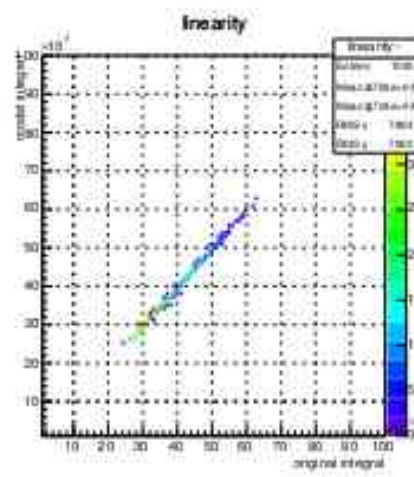
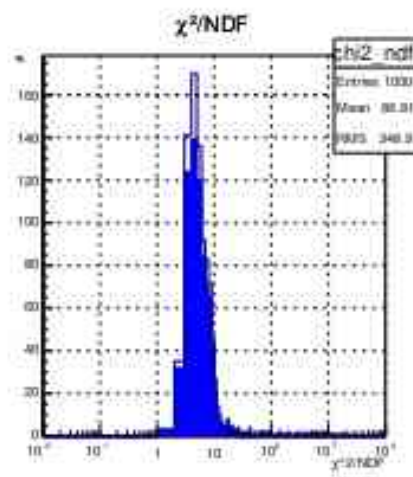
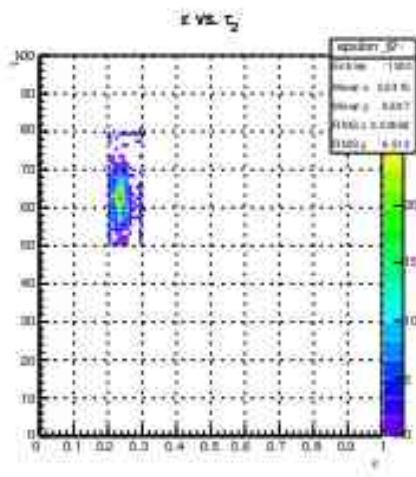
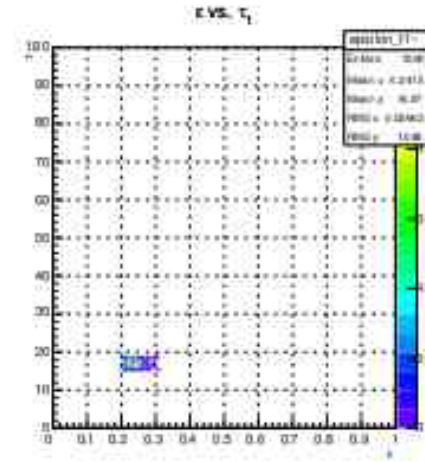
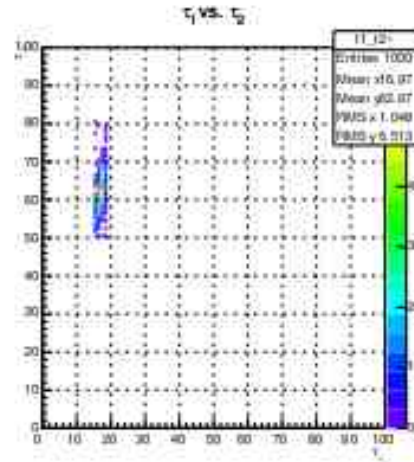
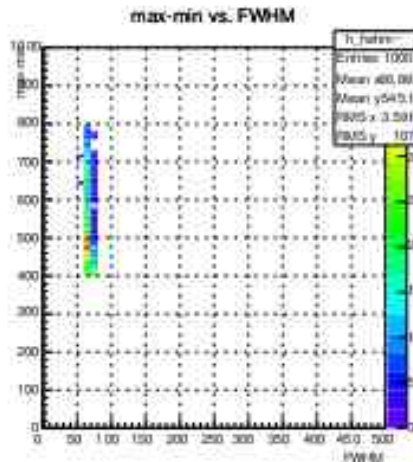
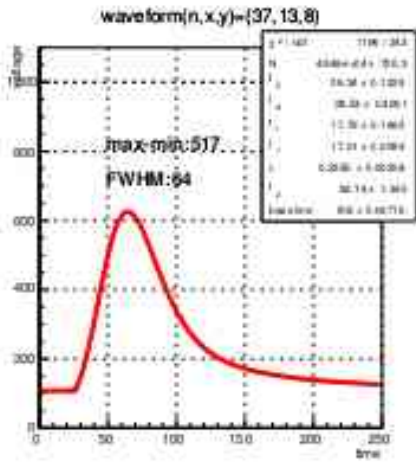


Lv3 waveform

Model: ito

$$V(t) = N \text{ Freq} \left[\frac{t - (t_0 + t_d)}{\tau_r} \right] \left\{ \frac{t - t_0}{\tau_1^2} \exp \left[\frac{-(t - t_0)}{\tau_1} \right] + \varepsilon \frac{t - t_0}{\tau_2^2} \exp \left[\frac{-(t - t_0)}{\tau_2} \right] \right\}$$

$$\begin{aligned} 14 < \tau_1 < 19 \\ 50 < \tau_2 < 80 \\ 0.2 < \varepsilon < 0.3 \end{aligned} \quad (t > t_0)$$



Result

$$\begin{aligned} \tau_1 &= 17.07 \\ \tau_2 &= 62.08 \\ \varepsilon &= 0.24 \end{aligned}$$

$\chi^2/\text{ndf} = 6.1$ @ good model

Conclusion

Pulse height vs. FWHM
 $h \propto 1/\text{FWHM}$?

Level 3 waveform analysis

Yamazaki model \rightarrow Ito Model

$$\tau_1 = 17.07 \pm 0.67$$

$$\tau_2 = 62.08 \pm 4.18$$

$$\varepsilon = 0.24 \pm 0.01$$

$$\chi^2/\text{NDF} = 6.1 \text{ @good event}$$

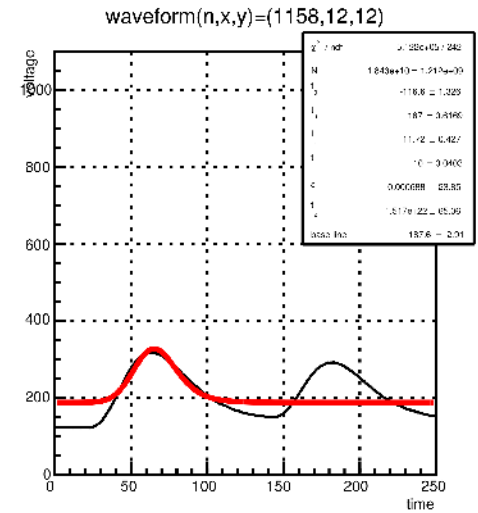
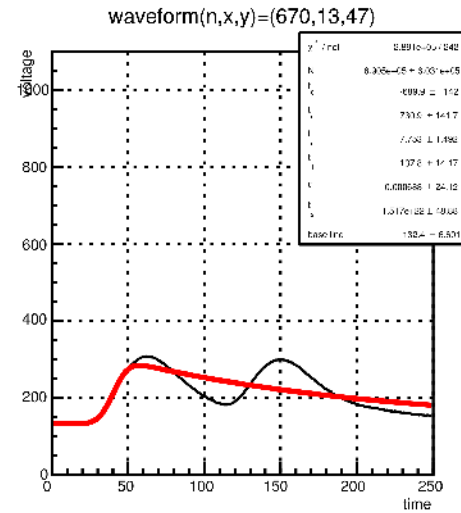
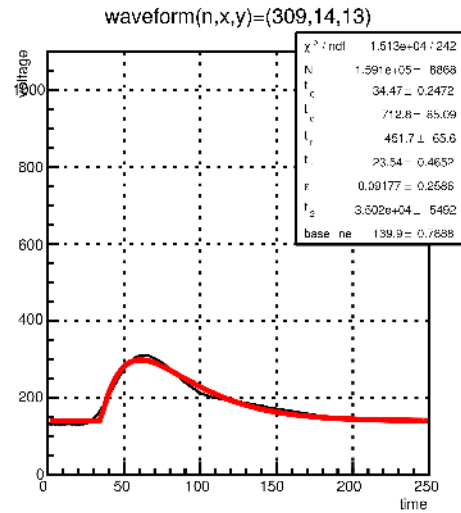
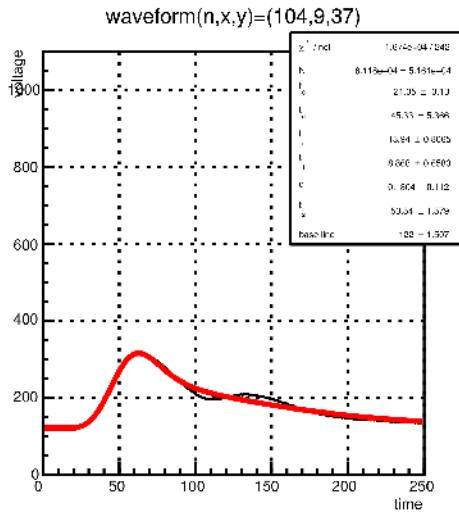
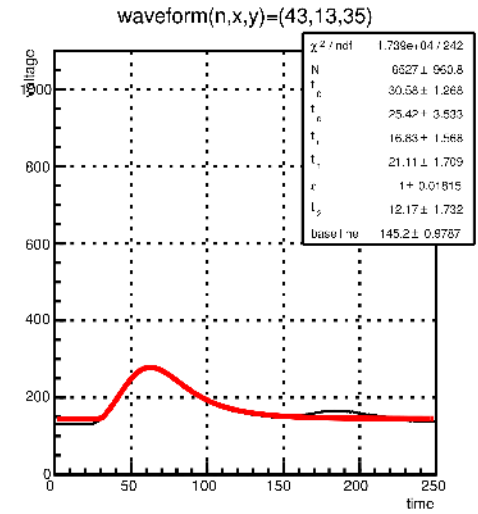
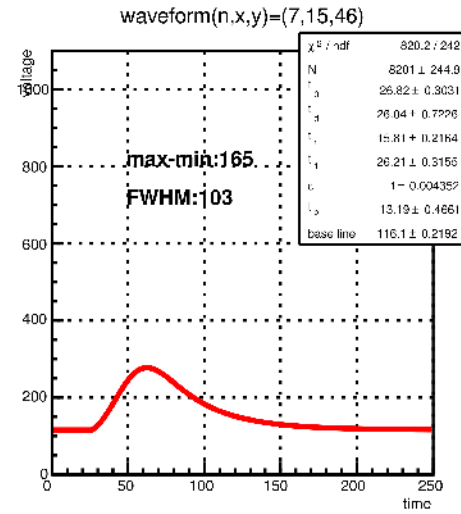
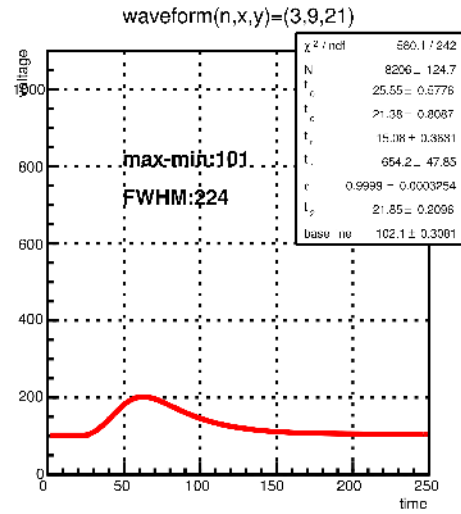
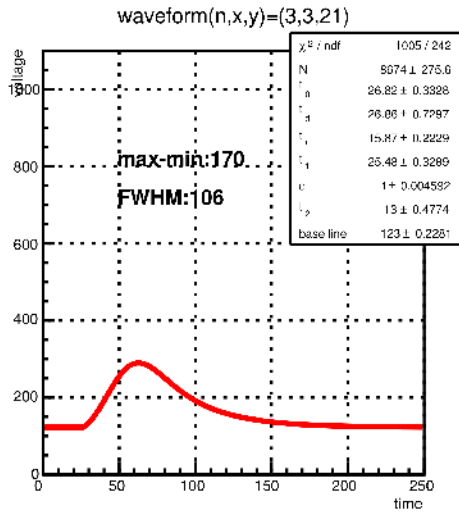
Linearity: good

} Fixed parameter for the model

Next work

1. How much is the linearity concretely?
2. Decision of main waveform function model
3. Level 1 and 2 waveform analysis
Ito Model Fitting \leftarrow multi-pulse fitting
4. Applying Multi pulse event

Buck up



Buck up

ROOTさんの χ^2 検定法について

例

x	0	1	2	3	4	5
y	2	8	17	21	23	31

$$\chi^2 = \sum_x \frac{(f(x) - y(x))^2}{f(x)}$$

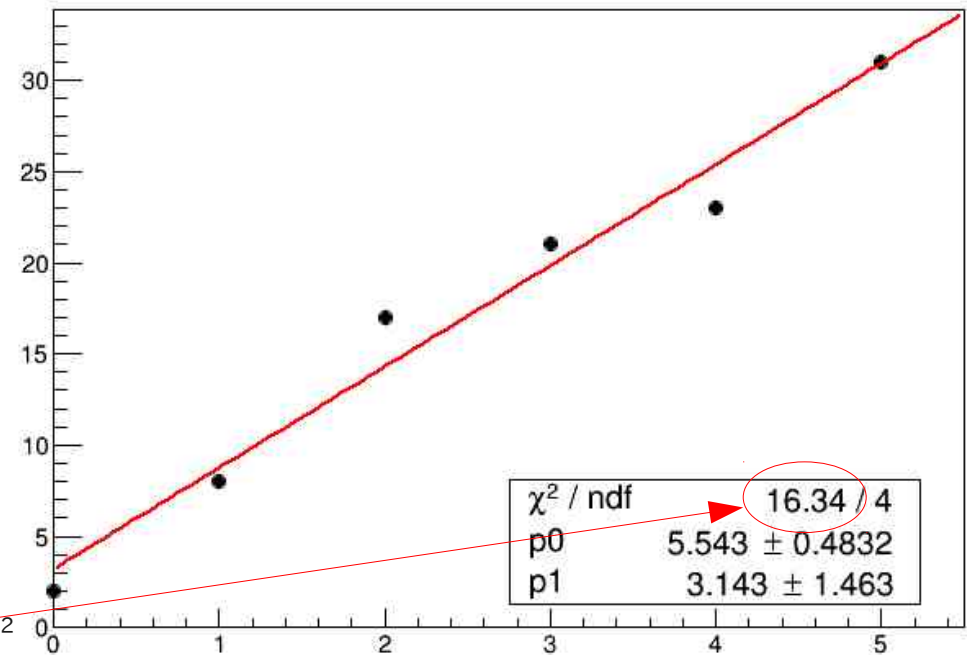
$$= \sum_i \frac{(y(x_i; a_1, a_2, \dots) - y(x_i))^2}{\sigma_i^2}$$

$$= (3.143 - 2)^2 + (5.543 + 3.143 - 8)^2 \\ + (5.543 \times 2 + 3.143 - 17)^2 + (5.543 \times 3 + 3.143 - 21)^2 \\ + (5.543 \times 4 + 3.143 - 23)^2 + (5.543 \times 5 + 3.143 - 31)^2$$

~ 16.34

誤差なしグラフについて、 σ は
 χ^2 の計算に入れない
-> というよりも、 $\sigma=1$ ですべて
計算している。

Graph



Buck up

ROOTさんの χ^2 検定法について

例

x	0	1	2	3	4	5
y	2	8	17	21	23	31
dy	1	2	2	1	3	2

$$\chi^2 = \sum_x \frac{(f(x) - y(x))^2}{f(x)}$$

$$= \sum_i \frac{(y(x_i; a_1, a_2, \dots) - y(x_i))^2}{\sigma_i^2}$$

誤差ありのグラフでは σ を用いる

$$= (2.481 - 2)^2 / 1^2$$

$$+ (5.962 + 2.481 - 8)^2 / 2^2$$

$$+ (5.962 \times 2 + 2.481 - 17)^2 / 2^2$$

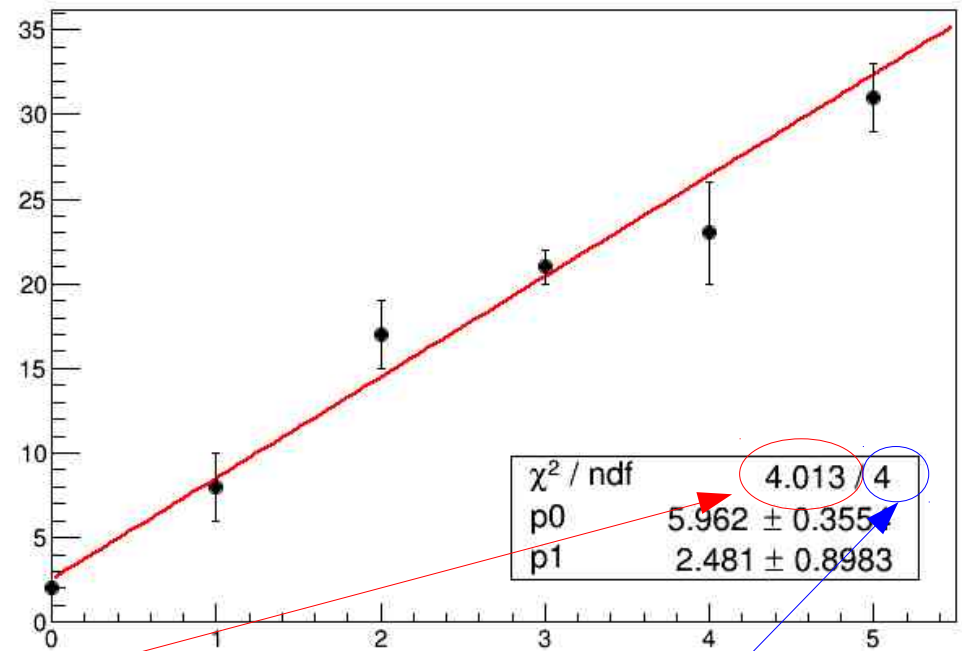
$$+ (5.962 \times 3 + 2.481 - 21)^2 / 1^2$$

$$+ (5.962 \times 4 + 2.481 - 23)^2 / 3^2$$

$$+ (5.962 \times 5 + 2.481 - 31)^2 / 2^2$$

~ 4.012

Graph



自由度 = グラフの点数 - 関数のパラメータ数