DALITZ PLOT DENSITY ANALYSIS IN $K^+_{\mu 3}$ DECAY

X2 Collaboration

D. HAIDT, H. SCHEPERS, J. STEIN and H. J. STEINER
III. Physikalisches Institut der Technischen Hochschule, Aachen, Germany

S. NATALI, G. PISCITELLI and F. ROMANO
Istituto di Fisica dell'Università di Bari, Istituto Nazionale di Fisica Nucleare, Sezione di Bari, Italy

E. FETT
Fysisk Institutt, Universitetet i Bergen, Norway

J. LEMONNE, R. MØLLERUD *, T. I. PEDERSEN ** and S. N. TOVEY
CERN, Geneva, Switzerland

V. BRISSON, L. KLUBERG and P. PETIAU
Ecole Polytechnique, Paris, France

C. D. ESVELD, J. J. M. TIMMERMANS and R. T. Van de WALLE
Physics Labaratory, University of Nijmegen, Nijmegen, The Netherlands

B. AUBERT, L. M. CHOUNET and DONG LÊ
Laboratoire de l'Accélérateur linéaire, Orsay, France

F. BOBISUT, E. CALIMANI, S. CIAMPOLILLO and H. HUZITA
Istituto di Fisica dell'Università di Padova, Istituto Nazionale di Fisica, Nucleare, Sezione di Padova, Italy

D. GAMBA, A. MARZARI-CHIESA and A. E. WERBROUCK
Istituto di Fisica dell'Università di Torino, Istituto Nazionale di Fisica Nucleare, Sezione di Torino, Italy

Received 7 July 1969

A sample of K^+ mesons stopped in the CERN 1.1 m^3 heavy-liquid bubble chamber has been used to study the $K^+_{\mu 3}$ Dalitz plot density and hence to examine the four-momentum dependence of the form factors $f_+(q^2)$ and $\xi = f_-(q^2)/f_+(q^2)$. The result of a three-parameter fit is: $\lambda_+ = 0.053^{+0.026}_{-0.021}$, $\xi(0) = -0.4^{+1.6}_{-1.5}$, $\xi(6.8\ m_\pi^2) = -0.80 \pm 0.50$. The variation of the optimum value for ξ with λ_+ is described.

In the "X2" experiment, a complete analysis of the $K^+ \to \pi^0 \mu^+ \nu$ decay has been performed. Results from the $K^+_{\mu 3}$ polarization analysis [1] and from a $K^+_{\mu 3}/K^+_{e3}$ branching ratio measurement [2] have recently been reported. In this letter the $K^+_{\mu 3}$ Dalitz plot density analysis is presented.

Assuming the V-A theory of weak interactions, the matrix element of the hadronic current in $K^+_{\mu 3}$ decay may be expressed in terms of two form factors f_+ and f_-:

$$\langle \pi | J_\mu^{h, \Delta S \neq 0} | K \rangle \propto f_+(p_K + p_\pi)_\mu + f_-(p_K - p_\pi)_\mu \ ,$$

where p_K and p_π are the K and π four-momenta, and f_+ and f_- are complex functions of the four-momentum transfer squared:

$$q^2 = (p_K - p_\pi)^2 = m_K^2 + m_\pi^2 - 2m_K E_\pi \ ,$$

E_π being the total energy of the pion. In $K^+_{\mu 3}$ de-

* Now at Niels Bohr Institute, Copenhagen, Denmark.
** Now at University of Bergen, Norway.

cay, q^2 extends from 0.6 m_π^2 to 7.1 m_π^2.

It has been shown [1,3] that the ratio $\xi(q^2) = f_-(q^2)/f_+(q^2)$ can be considered as real, and this will be assumed in our analysis.

The usual parametrization of $f_+(q^2)$ and $f_-(q^2)$ is a first-order development:

$$f_\pm(q^2) = f_\pm(0)(1 + \lambda_\pm q^2/m_\pi^2) .$$

The parameter ξ can be expanded as

$$\xi(q^2) = \xi(0) + \Lambda q^2/m_\pi^2 ,$$

where, if λ_+ is small:

$$\Lambda = \xi(0)(\lambda_- - \lambda_+) .$$

The Dalitz plot density is proportional to the following function:

$$\rho = f_+^2(q^2)[A(E_\mu, E_\pi) + B(E_\mu, E_\pi)\xi(q^2) + C(E_\pi)\xi^2(q^2)] ,$$

with:

$$A(E_\mu, E_\pi) = M_K(2E_\mu E_\nu - M_K E'_\pi) + \tfrac{1}{4} m_\mu^2 E'_\pi - m_\mu^2 E_\nu$$

$$B(E_\mu, E_\pi) = m_\mu^2 (E_\nu - \tfrac{1}{2} E'_\pi)$$

$$C(E_\pi) = \tfrac{1}{4} m_\mu^2 E'_\pi$$

$$E'_\pi = E_\pi^{\mathrm{Max}} - E_\pi = \frac{1}{2m_K}(m_K^2 + m_\pi^2 - m_\mu^2) - E_\pi .$$

In these expressions E_μ, E_ν and E_π are the muon, neutrino and pion total energies.

Lines of equal density ρ, in an arbitrary scale, are shown in fig. 1 for $\xi = 0$ (full lines) and for $\xi = -2$ (dashed lines). The relative variation of the density $\rho(E_\mu, E_\pi)$ as a function of ξ is greatest for low values of E_π. This effect is so strong that despite its small population the lower part of the plot provides most of the information on ξ.

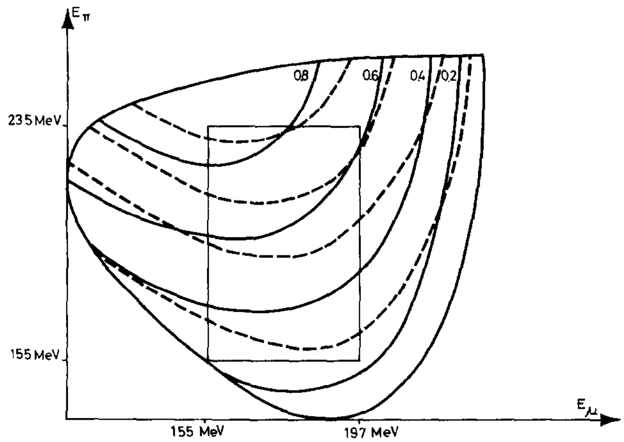

Fig. 1. Curves of equal Dalitz plot density for $\xi = 0$ (full lines) and $\xi = -2$ (dashed lines). The analysis was confined to events within the rectangle shown.

If ξ is independent of q^2, the pion energy spectra, for given values of E_μ, are straight lines, the slopes of which are quadratic functions of ξ. Their determination leads to two solutions for ξ. The value of the false solution depends only weakly on the value of E_μ. This ambiguity can in any case be removed by studying the muon spectra, for given values of E_π, which are parabolae. Two different values of ξ can never lead to the same spectrum. Hence in an analysis of the total Dalitz plot density, the ghost solution for ξ is expected to be buried.

The "X2" run consisted of $5 \times 10^6 K^+$'s stopped in the CERN heavy-liquid bubble chamber, filled with freon C_2F_5Cl. The $K^+ \to \pi^0\mu^+\nu$ events used in the Dalitz plot analysis were extracted from the same sample as the events used for the muon polarization analysis, namely events with a stopped μ^+ and two converted γ-rays. The energy of the muon is well determined from range (3%) and, after fitting, the energy of the pion is known to a similar accuracy. More experimental details have been described previously [1]. The analysis was confined to a region of the Dalitz plot (fig. 1) selected so as to reduce the backgrounds and to ensure well-known detection efficiencies for the μ^+ and π^0. Events were selected provided that $E_\pi < 235$ MeV, and that $155 < E_\mu < 197$ MeV. This ensures that

i) the background due to τ' and $K_{\pi 2}$ decays at rest is eliminated;

ii) the contamination from the mode $K_{\pi 2}$, with K or π decay in flight, is less than 0.1%.

In the accepted part of the Dalitz plot the contamination due to the $K_{\mu 3}$ decays in flight is about 1%. A convenient fiducial volume for the K^+ decay point ensures that muons from the selected energy interval stop in the chamber. Their detection efficiency was found to be independent of length. Moreover, the pion detection efficiency is uncorrelated with the muon energy.

The π^0 detection efficiency has been computed from the variation of the γ-ray detection probability with energy. The latter was computed directly from the $K_{\mu 3}$ sample. The fraction of γ-rays of a given energy detected can be determined from their conversion length distribution if it is assumed that there is no loss at short distances (i.e. from 1 to 10 cm). Even at short distances, some unmeasurable γ-rays are lost and a correction varying from 1% to 6% has been applied.

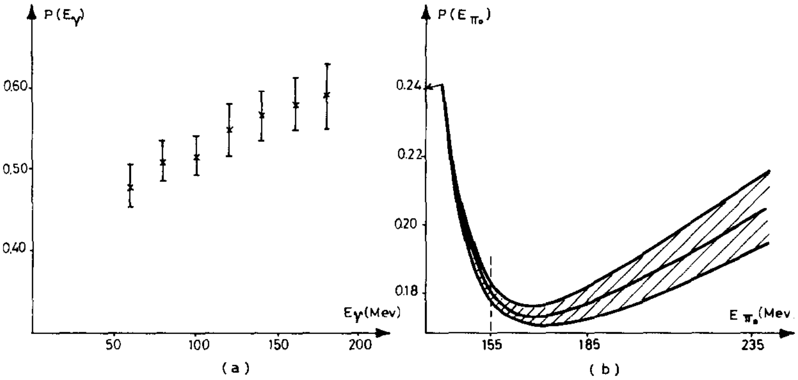

Fig. 2. a) γ-ray detection efficiency; b) π^0 detection efficiency; the curves limiting the hatched region represent the π^0 detection efficiencies computed from the extreme variations of the γ-ray detection probability allowed within one standard deviation.

The detection probability of γ-rays is shown in fig. 2a as a function of energy.

This result has ben checked, using the fact that the angular distribution of the γ-rays in the π^0 rest frame is isotropic at all π^0 energies. Both for our $K_{\mu 3}$ sample and for about 4000 $K_{\pi 2}$ events, the agreement is very satisfactory for γ energies greater than 50 MeV. Thus it has been required that all events entering the Dalitz plot analysis have both γ-ray energies greater than 50 MeV.

The relative π^0 detection efficiency (fig. 2b) has been computed, integrating the results of fig. 2a over the allowed γ energy range for each value of the π^0 energy. The quick fall observed between 10 MeV and 20 MeV is due to the 50 MeV cut on the γ-ray energy. Since the determination of ξ is very sensitive to the shape of the π^0 spectrum for the lower values of the π^0 energy, all events with E_{π^0} smaller than 155 MeV have been removed from the analysed sample. In the accepted zone of the Dalitz plot, the relative variation of the π^0 detection efficiency never exceeds 10%.

A first step in the determination of the form factors is the analysis of the muon energy spectra at fixed π^0 energies, which has the advantage of being independent of the π^0 detection efficiency and avoids the problem of the double solution. A maximum likelihood analysis has been performed on the sample of 4347 events remaining after the application of all the above cuts apart from that on the γ-ray energy spectrum. The likelihood function was defined as

$$\log L_{\mu} = \sum_{\text{all events}} \log \left[\frac{\rho(E^{i}_{\pi}, E^{i}_{\mu}, \xi, \lambda_{+})}{\int \rho(E^{i}_{\pi}, E_{\mu}, \xi, \lambda_{+})\, dE_{\mu}} \right],$$

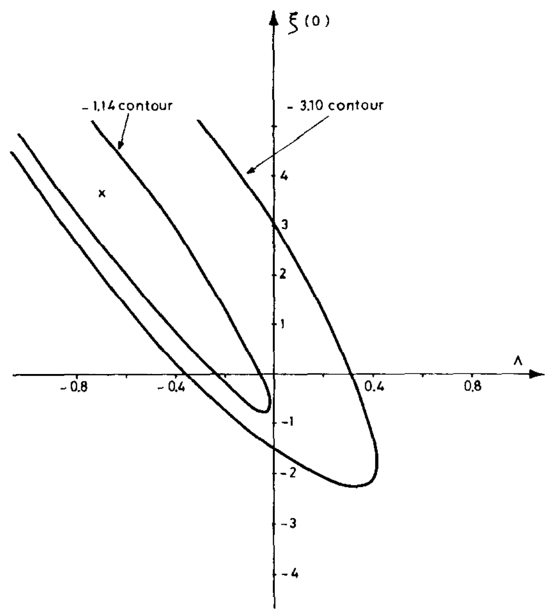

Fig. 3. Muon energy spectra analysis at fixed π^0 energies. The curves represent the -1.14 and -3.10 contours of the likelihood function.

where the normalization integral is taken from 155 MeV to 197 MeV.

From the Dalitz plot density expression it follows that the likelihood function L_{μ} at fixed pion energies is independent of the factor $f_{+}(q^2)$ and hence of λ_{+}. The likelihood plot of the two remaining parameters Λ and $\xi(0)$ is shown in fig. 3. No parasitic solution appears on this plot, but the parameters used are seen to be strongly correlated. From the slope of the regression line of

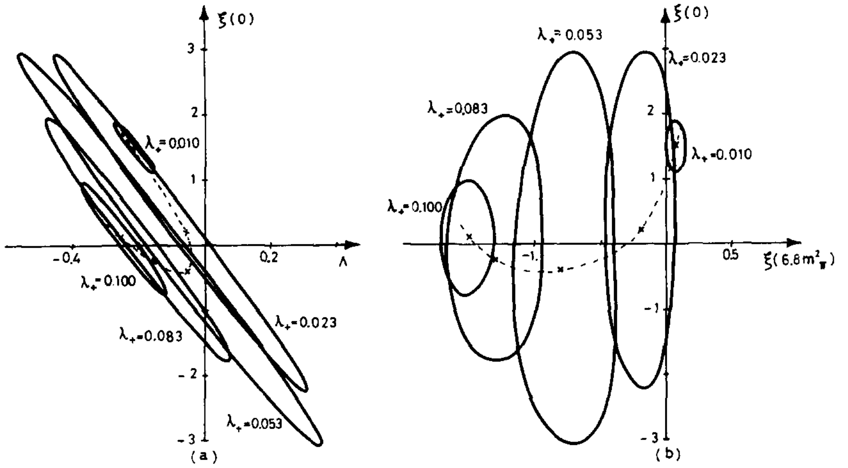

Fig. 4. Three-parameter analysis of the Dalitz plot density: a) λ_+, $\xi(0)$, Λ, b) λ_+, $\xi(0)$, $\xi(6.8\ m_\pi^2)$. The contours are drawn for several values of λ_+ and give the limits of the volume inside which there is a 68% probability of finding the three parameters simultaneously. This volume is defined by $\log L_{max} - \log L \leqslant 1.8$.

$\xi(0)$ into Λ, one finds that $\xi(5.5\ m_\pi^2)$ is uncorrelated to Λ. For this point the result is

$$\xi(5.5\ m_\pi^2) = -0.6^{+1.2}_{-0.4}\ .$$

This method uses only part of the available information. As a second step, the variation of the Dalitz plot density with the energy of both the pion and the muon has been studied. After cuts, 3240 $K^+_{\mu 3}$ events remain in the selected region of the Dalitz plot. The likelihood function

$$\log L = \sum_{\text{all events}} \log \left[\frac{\rho(E^i_\pi, E^i_\mu, \xi, \lambda_+) D(E^i_\pi)}{\iint \rho(E_\pi, E_\mu, \xi, \lambda_+) D(E_\pi)\, \mathrm{d}E_\mu\, \mathrm{d}E_\pi} \right]$$

has been computed. $D(E_\pi)$ is the π^0 detection efficiency, and the normalization integral is made over the accepted energy ranges. The result of a three-parameter fit is shown in fig. 4a. The one standard deviation surface resembles a very thin oblique ellipsoid, showing that the parameters $\xi(0)$ and Λ are strongly correlated. The optimum values are

$$\lambda_+ = 0.053^{+0.026}_{-0.021}\ , \qquad \xi(0) = -0.4^{+1.6}_{-1.5}\ ,$$

$$\Lambda = -0.05^{+0.23}_{-0.25}\ .$$

The quoted errors give the uncertainty on each parameter computed from its marginal distribution, i.e. evaluated independently of the others. They are obtained by subtracting 0.5 from the logarithm of the maximum value of the likelihood function, and including the small contribution of the uncertainty on the π^0 detection efficiency.

For each λ_+ value, $\xi(0)$ and Λ can be replaced by uncorrelated parameters that can be taken as $\xi(0)$ and $\xi(6.8\ m_\pi^2)$ as illustrated in fig. 4b. The result of the analysis is

$$\lambda_+ = 0.053^{+0.026}_{-0.021}\ , \qquad \xi(0) = -0.4^{+1.6}_{-1.5}\ ,$$

$$\xi(6.8\ m_\pi^2) = -0.80 \pm 0.50\ .$$

The parasitic solution was seen in the likelihood plot as a ridge displaced by $\Delta\xi(6.8\ m_\pi^2) \gtrsim -3$ with respect to the solution shown in fig. 4b. This second solution is ruled out by the results of the analysis of the muon energy spectra at fixed pion energies.

The variation of the result for $\xi(6.8\ m_\pi^2)$ with λ_+ can be expressed as

$$\xi(6.8\ m_\pi^2) = -22\ \lambda_+ + 0.25\ .$$

If λ_+ is fixed at 0.029, the value obtained from a compilation of previous K^+ experimental data [4], one gets:

$$\lambda_+ \equiv 0.029\ , \qquad \xi(0) = 0.0 \pm 2.0\ ,$$

$$\xi(6.8\ m_\pi^2) = -0.36 \pm 0.24^{\ddagger}\ .$$

The compatibility between the Dalitz plot analysis result and the polarization and branching ratio data previously published [1,2] is discussed in the next letter of this issue.

‡ As λ_+ has a fixed value, only two parameters are fitted and the error on ξ becomes much smaller.

References

1. J.Bettels et al. (X2 Collaboration), Nuovo Cimento 56A (1968) 1106.
2. T.Eichten et al. (X2 Collaboration), Phys. Letters 27B (1968) 586.
3. K.K.Yound, M.J.Longo and J.A.Helland, Phys. Rev. Letters 18 (1967) 806.
4. C.Rubbia, Proc. of Topical Conf. on Weak interactions, CERN 69-7 (1969) p. 227.

* * * * *