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Review

Flavor physics and CP violation

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ABSTRACT

We currently live in the age of the CKM paradigm. The 3×3 matrix that links (d, s, b) quarks to (u, c, t) in the charged current weak interaction, being complex and nominally with 18 parameters, can be accounted for by just 3 rotation angles and one *CP* violating (CPV) phase, with unitarity and the CKM phases triumphantly tested at the B factories. But the CKM picture is unsatisfactory and has too many parameters. The main aim of Flavor Physics and *CP* violation (FPCP) studies is the pursuit to uncover New Physics beyond the Standard Model (SM). Two highlights of LHC Run 1 period are the CPV phase ϕ_s of B_s mixing and $B_s \to \mu^+\mu^-$ decay, which were found to be again consistent with SM, though the saga is yet unfinished. We also saw the emergence of the P_5' angular variable anomaly in $B^0 \to K^{*0}\mu^+\mu^-$ decay and $R_{K^{(*)}}$ anomaly in $B \to K^{(*)}\mu^+\mu^-$ to $B \to K^{(*)}e^+e^-$ rate ratios, and the BaBar anomaly in $B \to D^{(*)}\tau\nu$ decays, which suggest possible New Physics in these flavor processes, pointing to extra Z', charged Higgs, or leptoquarks. Charmless hadronic, semileptonic, purely leptonic and radiative B decays continue to offer various further windows on New Physics. Away from B physics, the rare $K \to \pi\nu\nu$ decays and ε'/ε in the kaon sector, $\mu \to e$ transitions, muon $g - 2$ and electric dipole moments of the neutron and electron, $\tau \to \mu\gamma$, $\mu\mu\mu$, eee, and a few charm physics probes, offer broadband frontier windows on New Physics. Lastly, flavor changing neutral transitions involving the top quark t and the 125 GeV Higgs boson h, such as $t \to ch$ and $h \to \mu\tau$, offer a new window into FPCP, while a new Z' related or inspired by the P_5' anomaly, could show up in analogous top quark processes, perhaps even link with low energy phenomena such as muon $g - 2$ or rare kaon processes. In particular, we advocate the potential new SM, the two Higgs doublet model without discrete symmetries to control flavor violation, as SM2. As we are close to the alignment limit with h rather SM-like, flavor changing neutral Higgs couplings (FCNH) are suppressed by a small mixing angle, but the exotic Higgs doublet possesses FCNH couplings, which we are just starting to probe. As LHC Run 2 runs its course, and with Belle II B physics program to start soon, there is much to look forward to in the flavor and CPV sector.

2018.01.02
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Contents

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1. Introduction

In the domain of Flavor Physics and CP Violation (FPCP), we are currently under the Cabibbo–Kobayashi–Maskawa, or CKM, paradigm, one of the pillars of the Standard Model (SM) of particle physics. Formulated roughly half a century ago and emergent since 40 years, we have not seen any clear cracks in the past 20 years of B factory, and now LHC(b), scrutiny. In the current period of LHC Run 2, much more data would be collected, while Belle II would finally start B physics data taking. It is a good time to take stock and project what lies ahead, towards the dawn of the 2020s.

Facing a "strangely" prolonged kaon lifetime, young Cabibbo proposed [1] in 1963 a unitary symmetry approach: in modern language of quarks, it is the linear combination of $d' = d\cos\theta_C + s\sin\theta_C$ that enters the charged current. With a small $\theta_C \simeq 14°$, the Cabibbo angle, one also understood neutron beta decay better. As this was the time when the unified electroweak gauge theory was formulated, Cabibbo's picture led to a new problem: if the neutral current Z^0 boson couples[1] to $\bar{d}'d'$, it would lead to flavor changing neutral current (FCNC) $\bar{d}sZ^0$ coupling, hence $K_L \to \mu^+\mu^-$ at a rate that was simply not observed at the time. The electroweak theory had other more fundamental challenges to face, but in 1970, Glashow, Iliopoulos and Maiani (GIM) proposed [2] an elegant solution to the FCNC problem: if Z^0 couples also to a second combination $s' = -d\sin\theta_C + s\cos\theta_C$ that is orthogonal to d', then a new term $-\sin\theta_C\cos\theta_C$ would cancel the offending $\sin\theta_C\cos\theta_C$ by unitarity. The price, or prediction, for this to happen, is the existence of a 4th quark "charm", c, to pair with s' in the charged current. The existence of the charm quark was discovered [3,4] in the November Revolution of 1974, a watershed that precipitated the establishment of SM. Both the electroweak gauge theory and the newly emerged SU(3) color gauge theory of the strong interactions were by then calculable, allowing one to even estimate the charm mass from rare kaon processes, independent from its direct experimental discovery.

There is another great contribution from kaon physics: the observation [5] of *CP* violation (CPV) in 1964. Though the treatment of quark phases was mentioned, GIM did not [2] pursue CPV in their seminal work. Instead, the issue was picked up in late 1972 by two young physicists in the Far East. Besides exploring several different models, arguing that the two generation case possessed no phase in the charged current coupling, Kobayashi and Maskawa showed [6] that a single CPV phase remains when one has three generations of quarks. A new doublet, $(t,\ b')$, couples to the charged current, and d', s' and

[1] Dropping Dirac matrices for simplicity.

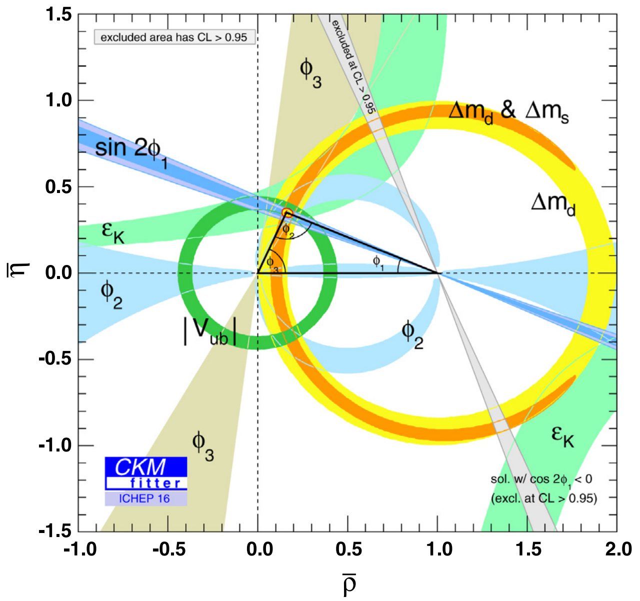

Fig. 1. CKM fit of Eq. (4) as of Summer 2016, with triangle normalized to $|V_{cd}V_{cb}^*|$, or $-V_{cd}V_{cb}^*$ if one takes the parametrization of Eq. (2). *Source:* http://ckmfitter.in2p3.fr/.

b' are related to the mass eigenstates d, s and b by a 3×3 unitary matrix, V. By argument of anomaly cancellation, a third doublet of leptons, (ν_τ, τ) also had to exist, and remarkably, in the same data that gave the ψ and ψ' peaks [4], the τ lepton was discovered [7] in 1975. Repeating the J discovery [3], the b quark was discovered [8] through the Υ resonances in 1977 at hadron facilities. But, although each and every new collider that followed pursued the t quark, it took almost 20 years for the Tevatron to eventually discover [9,10] the top in 1995, which was *unexpectedly heavy*.

The heaviness of the top quark was in fact foretold by the observation [11] of large B^0–$\bar{B}^0$ mixing in 1987, which illustrates the central role that B physics plays in FPCP. This in fact traces back to the discovery [12,13] of prolonged bottom hadron lifetimes, and the equally astounding discovery [14,15] that $b \to u$ transitions are far more suppressed than $b \to c$ transitions. A hierarchical structure

2018.01.04 22:45

$$|V_{ub}| \ll |V_{cb}| \ll |V_{us}| \simeq 0.22, \tag{1}$$

emerged for the CKM matrix V. This led Wolfenstein to write down his namesake parametrization [16]. In the phase convention [17] of keeping V_{us} and V_{cb} real, the unique CPV phase is placed in V_{ub}, hence in V_{td} as well by unitarity of V. The Wolfenstein form [16] of the CKM matrix is

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \simeq \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}, \tag{2}$$

which any FPCP practitioner should memorize, together with[2]

$$\lambda \equiv V_{us} \cong 0.22, \quad A\lambda^2 \equiv V_{cb} \simeq 0.041, \quad A\lambda^3\sqrt{\rho^2+\eta^2} \equiv |V_{ub}| \sim 0.0036, \tag{3}$$

which reflect and quantify the hierarchy pattern of Eq. (1). And the quest began for the CPV phase.

With the long B lifetime, rare B decays became interesting. On top of this, the very heavy top, together with the associated nondecoupled effects of large Yukawa coupling, made the B physics program very attractive. So, when large Δm_{B_d} ($\simeq \Gamma_{B_d}/2!$) was observed, on one hand it implied a heavy top quark, on the other hand, "B factory" planning started at SLAC and KEK, towards the measurement of the CPV phase $\sin\phi_1/\beta$ by the elegant methods developed by Sanda and Bigi [18,19]. By the 1990's, construction started for the corresponding experiments and *asymmetric* e^+e^- colliders, BaBar/PEP-II and Belle/KEKB.

2018.01.08 10:43

[2] We have kept the original Wolfenstein approximate definition for ease of memorising, but as reflected in the $\bar{\rho}$–$\bar{\eta}$ axes in Fig. 1, a more refined definition that holds to all orders in λ and is also rephasing invariant, is typically used: $\lambda = |V_{us}|/\sqrt{|V_{ud}|^2+|V_{us}|^2}$, $A\lambda^2 = |V_{cb}|/\sqrt{|V_{ud}|^2+|V_{us}|^2}$, and $\bar{\rho}+i\bar{\eta} = V_{ud}V_{ub}^*/V_{cd}V_{cb}^*$.

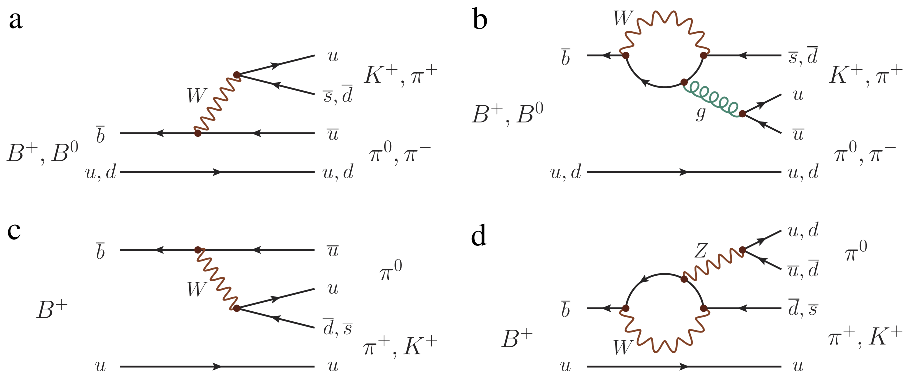

Fig. 2. Feynman diagrams of (a) tree, (b) strong penguin, (c) color-suppressed tree and (d) electroweak penguin processes for $B^+ \to K^+\pi^0$ and $B^0 \to K^+\pi^-$ decays.

The CPV phases $\phi_3/\gamma \equiv \arg V_{ub}^*$ and $\phi_1/\beta \equiv \arg V_{td}$ appear as vertex angles in the triangle or unitarity relation

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0. \tag{4}$$

The Wolfenstein form in Eq. (2) satisfies Eq. (4) to λ^3 order, and can be further extended to λ^5 order. The current status of our CKM paradigm can be summarized in a plot, Fig. 1, using the KM notation of ϕ_1, ϕ_2, ϕ_3 (equivalent to β, α, γ in the notation used by BaBar). This is not only a *beautiful* plot that incorporates Eq. (4), it contains much information and measurements, details of which would be discussed in Section 2. But the main point is, less than 15 years after discovery of large B mixing, the world not only constructed two B factories, but Belle and BaBar both observed [20,21] the CPV phase of B^0–$\bar{B}^0$ mixing in 2001,[3] confirming the CKM paradigm, and the 2008 Nobel prize was awarded to Kobayashi and Maskawa (together with Nambu for spontaneous symmetry breaking, SSB).

A CPV effect requires the interference between a CPV phase and a *CP*-conserving phase [22]. The beauty of the Bigi–Sanda method is that the latter phase, $e^{i\Delta mt}$, the time evolution of the B^0–$\bar{B}^0$ oscillation, is directly measured "*in situ*" by the experiments (thanks to the development of silicon-based vertex detectors), and is thus independent of so-called hadronic uncertainties. The oscillation is a quantum phenomenon of entangled states.

The fact that the sides and angles of the unitarity triangle of Eq. (4) fit nicely together in Fig. 1 and holding up in the past 15 years of intense scrutiny, means that we see no real cracks in the CKM paradigm, even though we know it falls far short of the CPV strength needed for baryogenesis [23]. While such precision measurements and "CKM fits" would certainly continue, in general we seek "anomalies" that suggest deviations from SM predictions, i.e. hints for New Physics (NP). This is the frontier push of FPCP, but here we are oftentimes at the mercies of "hadronic uncertainties". As an example, let us mention the well established [17] measurement of difference of *direct* CPV in $B \to K\pi$ decay,

$$\Delta A_{K\pi} \equiv A_{K^+\pi^0} - A_{K^+\pi^-} = 0.122 \pm 0.022, \quad \text{(PDG)} \tag{5}$$

as first clearly demonstrated by Belle [24].

The well measured direct CPV (DCPV) in $B^0 \to K^+\pi^-$ mode,

$$A_{K^+\pi^-} = -0.082 \pm 0.006, \quad \text{(PDG)} \tag{6}$$

arises from the interference between the subdominant tree amplitude T with the dominant (strong) "penguin" amplitude P, with T carrying the CPV phase in V_{ub} (see Fig. 2). The strength of $A_{K^+\pi^-}$ would depend on the relative strong phase between P and T, which may not be easy to predict. But naively one would expect $A_{K^+\pi^0}$ to be similar in strength to $A_{K^+\pi^-}$, since the two additional amplitudes, electroweak (or Z) penguin $P_{\rm EW}$ and color-suppressed (tree) C, are either suppressed by an extra power of G_F or by color mismatch ($1/N_C$), and are expected to be small. The $P_{\rm EW}$ amplitude, however, is enhanced by nondecoupling of top [25], and could pick up NP phases by analogous enhancements. But, as will be discussed in Section 3, we see no NP effect in the CPV phase measurement of the companion B_s^0–$\bar{B}_s^0$ mixing amplitude, ϕ_s, hence NP effect through $b \to s\ P_{\rm EW}$ amplitude seems unlikely. The large shift in $A_{K^+\pi^0}$ value, even changing sign from $A_{K^+\pi^-}$, would then have to arise from an unsuppressed C amplitude that carries a large strong phase difference with respect to (w.r.t.) T. Such strong rescattering effects [26,27] were originally (i.e. before the start of B factories) thought to be perturbatively suppressed [28,29] by the high m_B scale, but they now turn around to haunt us. The moral,[4] then, is one should avoid hadronic uncertainties if the interest is in NP.

[3] The year after, the two bi-annual "Heavy Flavor" and "B Physics and CP Violation" conferences merged into the annual FPCP conference.

[4] A second example is $\Delta A_{\rm CP} \equiv A_{KK} - A_{\pi\pi}$ in $D^0 \to K^+K^-$ vs $\pi^+\pi^-$ decays, which arose [30] then disappeared [31] again with Run 1 data of the LHCb experiment. Had it stayed, it would still be even more likely than $\Delta A_{K\pi}$ to be due to hadronic effects.

In this review, we deemphasize processes that are susceptible to large hadronic corrections. This includes lattice calculations of well-known hadronic parameters, such as form factors or decay constants, even though these are promising directions in the long run. Also, lattice calculations have not reached the stage of handling multiple dynamical scales, which is often the case in rare B decays. Another very important aspect of flavor physics we deemphasize is spectroscopy. In fact, the discovery of new, unexpected hadrons such as [17] $X(3872)$, $Y(4260)$ and $D_{sJ}(2320)$ are listed among the best cited B factory papers, with a rather large theoretical and experimental following purporting to four-quark or molecular states. The LHCb experiment also claims to have discovered pentaquarks. But while interesting, these have little to do with our quest for NP beyond SM, and we refer to the companion article by Stone [32].

Instead, let us list the leading and recent flavor highlights (some of them "anomalies"), or otherwise probes of NP, at our current frontier:

- ϕ_s from $B_s \to J/\psi\,\phi, J/\psi\, f_0$;
- $B^0_{s,\,d} \to \mu^+\mu^-$;
- P_5' and $R_{K^{(*)}}$ anomalies in $B \to K^{(*)}\ell^+\ell^-$;
- $B \to D^{(*)}\tau\nu$ (BaBar anomaly);
- $B^+ \to \tau^+\nu$ and $b \to s\gamma$ (H^+ probes);
- $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$;
- muon $g-2$ anomaly and $\mu \to e$ processes;
- $\tau \to \mu\gamma$ etc.;
- $h(125) \to \mu\tau$ and $t \to ch$.

As a rule, however, "anomalies" come and go. In fact, they mostly just go (away), as B factory workers could ruefully attest to. Thus, which of the above anomalies are "real"? We do not know. We have kept $g-2$ only because it has been so long-standing, and because of continued experimental (and lattice) interest. The P_5' "anomaly" is less long-standing while also susceptible to hadronic effects. It is kept because of its volatile experimental situation, and because there might be a common trend with other measurements, such as the $R_{K^{(*)}}$ anomaly. Other hints, such as some discrepancy of direct measurements of $\sin 2\phi_1/\beta$ vs CKM fit result, or the long standing "discrepancies" of inclusive vs exclusive measurements of V_{ub} and V_{cb}, would be covered as traditional elements of CKM measurements in the following section. Recent progress in lattice studies have reopened ε'/ε as potentially harboring NP, which we briefly discuss with rare kaon decays, while we do not really discuss ε_K (which plays a supporting role in Fig. 1) as it has well documented dependence on decay constant and "bag" parameter which are being steadily improved by lattice. Note that we did not list any charm process above. Be it rare D decays or D^0–$\bar{D}^0$ mixing, we have no indications for NP yet in charm physics. The reason is opposite to why B physics is prominent: charm hadron lifetimes are unsuppressed by $V_{cs} \simeq 1$, while loop-induced decays are suppressed by the smallness of down type quark masses on the weak scale, hence suffer strong GIM cancellation. New Physics in charm is hard to demonstrate, and we discuss it in a regular subsection. In a similar vein, rare top decay is also a "harder" subject, because the top quark decays faster than even the strong interaction time scale. But its association with the Higgs particle, $h(125)$, throws in some new light.

The Higgs boson h was discovered at LHC Run 1, but no New Physics appeared at 7 and 8 TeV. For LHC Run 2 at 13 TeV, there is unfortunately a repeat: again no New Physics has appeared so far, with the 750 GeV "diphoton" disappearing with more data. Although discovery is still possible with new particles or effects with weaker coupling, there is no new high energy facility that is firmly in sight and that would run after completion of the high luminosity LHC (HL-LHC) running around 2035. This only enhances the importance of FPCP studies. Although in general it probes NP only indirectly, it provides us with hope that some indication or evidence for NP might appear in the not so distant future. One of course has New Physics (beyond SM) in the neutrino sector, the fact that neutrinos mix hence have mass, but that is covered by another review [33].

Bottom, charm, τ and top are traditionally viewed as heavy flavors, but kaon and μ physics may still offer surprises. In the following sections, we first discuss CKM measurements in Section 2, then the highlight B anomalies (first 4 items above) in Section 3. Some further discussion of B decays are given in Section 4, including connection to Dark sector. Rare K decays, mainly $K \to \pi\nu\nu$ (plus Dark sector connections), are discussed in Section 5, with brief comments on ε'/ε. This is followed by rare muon processes and $g-2$, with a brief touch upon electric dipole moments. Tau and charm physics are covered only briefly in Section 6, but we introduce the new subject of flavor changing top and Higgs couplings in Section 7, followed by our conclusion. Our emphasis is on the physics and the experiment–theory interplay, rather than on technical details, with a view towards improvements and updates in the LHC Run 2 period.

2. CKM paradigm

The CKM paradigm stands as the pillar of flavor physics and CP violation of SM, but the CKM matrix elements are free parameters that have to be measured experimentally. Our mission is to verify if the matrix is unitary, and whether the matrix elements measured with various methods are consistent with each other. If one can demonstrate violation of unitarity, or if convincing discrepancies emerge somewhere, it may indicate New Physics.

The nine complex CKM matrix elements of V can be expressed in just four independent real parameters if V is unitary, as given in Eq. (2) in a convenient parametrization. In this section, we first describe the methods to measure each matrix element, then report the current experimental results. The most recent fit results to extract the Wolfenstein parameters from a global fit to available measurements will be discussed. Possible discrepancies will be highlighted together with the future prospect.

2.1. Magnitudes

The magnitudes of CKM matrix elements are often extracted from measurements of transition or decay rates and comparing with theoretical calculations, where the magnitudes of the elements are the unknowns. Since theoretical calculations typically involve nonperturbative parameters such as decay constants and form factors, the extracted magnitudes have both experimental and theoretical uncertainties.

2.1.1. The 2 × 2 submatrix

The four elements of the 2 × 2 submatrix in the upper left corner of the CKM matrix are related to the Wolfenstein parameter λ only, with which the suggestion of unitarity first emerged [1,2].

$|V_{ud}|$ *and* $|V_{us}|$

The element V_{ud} is the first to be determined, based on $u \leftrightarrow d\,(p \leftrightarrow n)$ transitions, and is the most precisely known. The magnitude of V_{ud} is determined from the study of $0^+ \to 0^+$ nuclear beta decays, which is governed purely by the weak current. The measured half lifetimes, t, and Q values, which give the decay rate factor $\mathcal{F}$, are used to determine $|V_{ud}|$ using the formula [34],

$$|V_{ud}|^2 = \frac{2984.45\ \text{s}}{\mathcal{F}t(1+\Delta)}, \tag{7}$$

where Δ denotes the combined effect of electroweak radiative corrections, nuclear structure and isospin violating nuclear effects. The average over 14 precise results [35] gives,

$$|V_{ud}| = 0.97417 \pm 0.00021, \tag{8}$$

where the error is dominated by theoretical uncertainty.

Extractions of $|V_{ud}|$ are also performed with measurements of neutron lifetime and the $\pi^+ \to \pi^0 e^+ \nu_e$ decay rate. Both give results consistent with Eq. (8), but with uncertainties larger by more than an order of magnitude. Determination from neutron lifetime is limited by knowledge of the ratio of axial–vector to vector couplings, while semileptonic π^+ decay suffers from statistics. The details can be found in the review by Blucher and Marciano in PDG [17].

The magnitude of V_{us} may be extracted from the decays of kaons, hyperons and τ leptons. Two different approaches are employed: extracting $|V_{us}|$ directly, and measuring $|V_{us}/V_{ud}|$. The aforementioned review in PDG documents the efforts.

In direct extraction, $|V_{us}|$ is often determined from semileptonic $K \to \pi\ell\nu$, or $K\ell 3$, decays. The partial decay width is related to the product of $|V_{us}|$ and the form factor at zero momentum transfer, $f_+(0)$, of the lepton–neutrino system. Experimental inputs are decay branching fractions and kaon lifetimes, and form factor measurements that enable theoretical computation of phase space integrals. Theory is needed to compute short and long distance radiative corrections, isospin breaking strength, and $f_+(0)$. Recent high statistics and high quality measurements have facilitated good constraints to justify the comparison between different decay modes. PDG therefore follows the prescription of Ref. [36] to average results from $K_L^0 \to \pi e\nu, \pi\mu\nu$, $K^\pm \to \pi^0 e^\pm \nu, \pi^0\mu^\pm\nu$, and $K_S^0 \to \pi e\nu$, giving the product $|V_{us}|f_+(0) = 0.2165 \pm 0.0004$. The form factor average[5] $f_+(0) = 0.9677 \pm 0.0037$ from the three-flavor lattice QCD calculations [37] gives $|V_{us}| = 0.2237 \pm 0.0009$.

The second method aims at determining the ratio $|V_{us}/V_{ud}|$, and $|V_{us}|$ can be obtained by use of the well determined $|V_{ud}|$. This ratio can be extracted from the ratio of decay rates of $K \to \mu\nu(\gamma)$ and $\pi \to \mu\nu(\gamma)$, along with the ratio of decay constants from lattice QCD calculations. The KLOE measurement of $K \to \mu\nu(\gamma)$ [38] leads to $|V_{us}| = 0.2254 \pm 0.0008$, where the uncertainty is dominated by the ratio of decay constants. Combining the two approaches to $|V_{us}|$ measurements, one gets

$$|V_{us}| = 0.2248 \pm 0.0006. \tag{9}$$

Other determinations of $|V_{us}|$ include extraction from hyperon decays [39], and from semihadronic τ decays. The former approach gives results consistent with Eq. (9) but with larger errors. As for semihadronic τ decays, $\tau \to K\nu(+X), \pi\nu(+X)$, the average of inclusive and exclusive rates yields[6] $|V_{us}| = 0.2204 \pm 0.0014$ [40]. The average $|V_{us}|$ value determined from τ decays deviates from the kaon average by almost 3σ, and the discrepancy mainly comes from inclusive τ measurements, which gives 0.2186 ± 0.0021. Further investigation is needed.

We can check CKM unitarity using $|V_{ud}|$, $|V_{us}|$ and $|V_{ub}|$, where the latter is known to be too small for any impact. Taking $|V_{ud}|$ and $|V_{us}|$ from Eqs. (8) and (9), respectively, the squared sum gives,

$$|V_{ud}|^2 + |V_{us}|^2\,(+|V_{ub}|^2) = 0.9995 \pm 0.0005. \tag{10}$$

The good agreement with unitarity already confirms the radiative corrections from Standard Model at better than 50σ level [41]. Moreover, it also sets good constraints on New Physics effect [42] that may contribute in nuclear beta decay, kaon decay and muon decay.

5 See the review by Rosner, Stone, and Van de Water in PDG [17].

6 We skip detail discussions here, and refer to Ref. [40], which is the HFAG update for the PDG 2015 update. Thus, PDG 2016 often uses this reference, while in Section 3 we would more often quote the latest from HFAG.

$|V_{cd}|$ and $|V_{cs}|$

Early determinations of $|V_{cd}|$ were performed using neutrino scattering data. The strategy is to compare the production rates of opposite sign dimuons with single muons, from both neutrino and anti-neutrino beams on nucleus. The nuclear scattering can be expressed as $\nu_\mu + N \to \mu^- + c + X$, where the c quark hadronizes into a charm hadron and subsequently decays semileptonically into a muon plus other particles. The underlying process is a neutrino interacting with a d or s quark in a nucleus, and similarly $\bar{d}$ or $\bar{s}$ quark for anti-neutrino beam. The difference of the double-muon to single-muon production ratio by neutrino and antineutrino beams is proportional to the charm cross section of valence d quarks, and therefore to $|V_{cd}|^2$ times the average semimuonic branching fraction of charm hadrons, $\mathcal{B}_\mu$. PDG 2016 uses the average over CDHS, CCFR and CHARM, $\mathcal{B}_\mu |V_{cd}|^2 = 0.463 \pm 0.034$, and the average $\mathcal{B}_\mu = 0.087 \pm 0.005$ of two estimates [43,44], to obtain $|V_{cd}| = 0.230 \pm 0.011$.

Similar to the $|V_{us}|$ case, the magnitude of V_{cd} can be extracted from semileptonic and leptonic D decays, where the form factor $f_+^{D\pi}(0)$ is needed for the former, and the decay constant f_D for the latter. Using the form factor $f_+^{D\pi}(0) = 0.666 \pm 0.029$ [37] from three-flavor lattice QCD calculations, and the $D \to \pi \ell \nu$ branching fractions measured from BaBar, Belle, BESIII and CLEO-c, PDG obtains $|V_{cd}| = 0.214 \pm 0.003 \pm 0.009$, where the first uncertainty is experimental and the second is theoretical (form factor). Averaging the $D^+ \to \mu^+ \nu$ branching fraction measurements from BESIII and CLEO-c and using $f_D = 209.2 \pm 3.3$ MeV [37], PDG gives $|V_{cd}| = 0.219 \pm 0.005 \pm 0.003$. The three different approaches give consistent values, and the average by PDG yields

$$|V_{cd}| = 0.220 \pm 0.005. \tag{11}$$

Analogous to $|V_{us}|$ and $|V_{cd}|$ extraction, the magnitude of V_{cs} can be extracted from semileptonic D decays and leptonic D_s decays, using the form factor and D_s decay constant calculated from lattice QCD, respectively. Using the form factor $f_+^{DK}(0) = 0.749 \pm 0.019$ [37] and the average branching fraction of $D \to K \ell \nu$ decays, one obtains $V_{cs} = 0.975 \pm 0.007 \pm 0.025$, where the first uncertainty is experimental and the second is mainly due to the form factor. For leptonic decays, the average branching fractions $(5.56 \pm 0.24) \times 10^{-3}$ and $(5.56 \pm 0.22) \times 10^{-2}$ for $D_s^+ \to \mu^+ \nu$ and $\tau^+ \nu$, respectively, are documented in the same review in footnote 5. The magnitude of V_{cs} can be determined for each decay branching fraction using the decay constant $f_{D_s} = (248.6 \pm 2.7)$ MeV [37], D_s^+ lifetime and mass, and the corresponding lepton masses. The average of the two gives $|V_{cs}| = 1.008 \pm 0.021$, where the uncertainty is dominated by the decay constant. Combining with $D \to K \ell \nu$ decays, the average gives

$$|V_{cs}| = 0.995 \pm 0.016. \tag{12}$$

Another information on $|V_{cs}|$ can come from W decay. The W decay branching fraction to each lepton is inversely proportional to $3[1 + \Sigma_{u,c,d,s,b} |V_{ij}|^2 (1 + \alpha_s(m_W)/\pi) + \cdots]$. Assuming lepton universality and using the branching fraction of W leptonic decay measured by LEP-2, one obtains $\Sigma_{u,c,d,s,b} |V_{ij}|^2 = 2.002 \pm 0.027$ [45], which is a good test of CKM unitarity. The magnitude of $|V_{cs}|$ can also be determined from flavor tagged $W^+ \to c\bar{s}$ decays. However, The precision suffers from the low statistics and purity of flavor tagging.

Comments

As pointed out in the first section, the four magnitudes discussed here are much larger than the others except $|V_{tb}|$. Numerous experimental measurements together with progressively improved theoretical calculations of form factors and decay constants yield precise determination of the four magnitudes. The obtained results show that $|V_{ud}|$ is indeed close to $|V_{cs}|$, and $|V_{us}|$ close to $|V_{cd}|$. The quadratic sum of $|V_{ud}|$ and $|V_{cd}|$ is very close to unity. It is quite impressive that various determinations using different decays or different methods for the same amplitudes are consistent with each other, demonstrating the accuracy of the Standard Model. The uncertainties of the magnitudes except for $|V_{cd}|$ are dominated by theory uncertainties, while experimental and theory uncertainties are comparable for $|V_{cd}|$.

So far the only notable discrepancy at $\sim 3\sigma$ level comes from $|V_{us}|$ determination using exclusive and inclusive τ decays. Owing to large data samples collected by a multitude of experiments that enable the extraction in a systematic and coherent way, PDG adopts the $|V_{us}|$ value determined from kaon decays as default. The $|V_{us}|$ extraction using τ decays is facilitated by measuring the decay rate with kaon (strange quark) in the final state. The deviation in $|V_{us}|$ value between the kaon and τ methods mainly arises from the inclusive method for τ, which is more challenging experimentally. The result using the exclusive method is closer to the default $|V_{us}|$. Therefore, a $|V_{us}|$ anomaly seems unlikely, not to mention New Physics effects. It is the responsibility of the experiments to understand the deviation in the inclusive mode and provide more precise measurements.

2.1.2. $|V_{ib}|$ and $|V_{tj}|$

$|V_{cb}|$ and $|V_{ub}|$

Semileptonic decays of B^+ and B^0 mesons proceed via leading order weak interactions. Modes involving light leptons ($\ell = e$ or μ) are expected to be free from non-SM contributions. Therefore, these decays serve the crucial role for determining $|V_{cb}|$ and $|V_{ub}|$, which are extracted from measurements of exclusive and inclusive branching fractions, and theoretical formulae based on the operator product expansion (OPE). A dedicated PDG review documents the details of the extraction method and current results.

The differential branching fractions for exclusive $B \to D^{(*)}\ell\nu$ can be expressed as the product of $|V_{cb}|^2$, form factor squared $\mathcal{F}^2(\omega)$, where ω is the inner product of B and $D^{(*)}$ four velocities, and other factors that need to be computed. For example, the differential decay rate of $B \to D^*\ell\nu$ is given by

$$\frac{d\Gamma}{d\omega}(B \to D^*\ell\nu) = \frac{G_F^2 m_B^5}{48\pi^3}|V_{cb}|^2(\omega^2-1)^{1/2}P(\omega)(\eta_{\mathrm{ew}}\mathcal{F}(\omega))^2, \tag{13}$$

where $P(\omega)$ is a phase space factor, η_{ew} accounts for leading electroweak corrections that include an estimate of uncertainty from (still) missing long-distance QED radiative corrections. Two unknowns in the equation are $|V_{cb}|$ and $\mathcal{F}(\omega)$. The idea is to obtain $|V_{cb}|\mathcal{F}(\omega)$ from the measured differential rates and extrapolate it to the end point of $\omega = 1$, which corresponds to maximum momentum transfer for the leptons. In the infinite mass limit, the form factor at the end point is unity. For finite quark masses, $\mathcal{F}(1)$ needs to be computed by lattice QCD [46–48]. The same strategy can be applied to the $B \to D\ell\nu$ case. Using all available data from LEP, CLEO, Belle and BaBar, PDG 2016 gives

$$|V_{cb}| = (39.2 \pm 0.7) \times 10^{-3}, \quad \text{(exclusive)} \tag{14}$$

where the dominant theoretical uncertainty comes from the form factor, and the experimental uncertainty is due to the decay rate near $\omega = 1$. Owing to possible New Physics in the data, the branching fraction measurements of $B \to D^{(*)}\tau\nu$ were not included in the extraction of $|V_{cb}|$. The discussion of $B \to D^{(*)}\tau\nu$ can be found in the B highlights given in Section 3.

Extracting $|V_{cb}|$ from inclusive decays requires the measurement of total semileptonic decay branching fractions, moments of lepton energy, and hadronic invariant mass spectra in $b \to c$ transitions. Theoretical calculations for the decay rate can be performed reliably in terms of nonperturbative parameters based on $1/m_b$ expansion, where there are subtleties at $O(1/m_b^2)$ [49,50] The nonperturbative parameters can be extracted from information of moments from experiments. Measurements of inclusive processes have been performed using B mesons from Z^0 decays at LEP and $\Upsilon(4S)$ decays at B factories. Experimental information on the leptonic and hadronic invariant masses in $B \to X_c\ell\nu$, and the photon moments from $B \to X_s\gamma$ all help in determining the nonperturbative parameters. An average of experimental measurements leads to [17]

$$|V_{cb}| = (42.2 \pm 0.8) \times 10^{-3}. \quad \text{(inclusive)} \tag{15}$$

There is a slight tension (2.9σ) between the two extractions of $|V_{cb}|$. Belle II will have two orders of magnitude more data in the near future, which facilitates more precise measurements of the semileptonic decay rates. However, one needs to reduce theoretical uncertainties to verify the discrepancy between the two $|V_{cb}|$ extractions. The PDG combination of the two, after scaling the error by $\sqrt{\chi^2} = 2.9$, is

$$|V_{cb}| = (40.5 \pm 1.5) \times 10^{-3}. \tag{16}$$

Similar to the inclusive $|V_{cb}|$ extraction, the extraction of $|V_{ub}|$ from inclusive $B \to X_u\ell\nu$ decays can also be pursued using the same effective theory. However, the large inclusive $B \to X_c\ell\nu$ background poses a challenge for measuring the branching fractions. To calculate partial decay rates in regions of phase space where the $B \to X_c\ell\nu$ decays are suppressed requires a nonperturbative distribution function called the shape function [51,52]. Note that the nonperturbative physics for the $|V_{ub}|$ case is modelled by a few parameters. At leading order, the shape function is universal, therefore it can be extracted from the photon energy spectrum of the $B \to X_s\gamma$ decays. An alternative approach is to measure the semileptonic decay rate more inclusively, covering some parts of the $B \to X_c\ell\nu$ region. Large data samples collected at the B factories enable the tagging of accompanying B meson as decaying hadronically or semileptonically, hence a wider kinematic region for the signal can be allowed. Averaging over the two approaches, PDG gives the inclusive average,

$$|V_{ub}| = (4.49 \pm 0.16^{+0.16}_{-0.18}) \times 10^{-3}, \quad \text{(inclusive)} \tag{17}$$

where the first uncertainty is experimental and the second theoretical.

Exclusive charmless semileptonic B decays provide another way to determine $|V_{ub}|$. Although exclusive decays result in better signal-to-background ratios experimentally, their branching fractions are rather small, leading to large statistical uncertainties. Among all the $b \to u$ semileptonic decay modes, the decay $B \to \pi\ell\nu$ is the most promising. To extract $|V_{ub}|$, the form factor from theoretical calculations, lattice QCD or light-cone sum rule, is needed. With the available $B \to \pi\ell\nu$ form factor for high leptonic momentum squared (q^2) region calculated by lattice QCD [53,54], a fit to the experimental partial rates and lattice results versus q^2 gives $|V_{ub}| = (3.72 \pm 0.16) \times 10^{-3}$ [54]. PDG rescales the uncertainty by $\sqrt{\chi^2/\text{d.o.f.}} = 1.2$ and gives

$$|V_{ub}| = (3.72 \pm 0.19) \times 10^{-3}. \quad \text{(exclusive)} \tag{18}$$

The inclusive and exclusive $|V_{ub}|$ results again show a 2.6σ tension. Belle II will be able to reduce the experimental uncertainties in the near future, but we need to pin down the theoretical uncertainties to verify the discrepancy. The average over inclusive and exclusive results after scaling the uncertainty by 2.6 is

$$|V_{ub}| = (4.09 \pm 0.39) \times 10^{-3}. \tag{19}$$

Information from other B decays can be used to probe $|V_{ub}|$. For instance, the decay rate of $B^+ \to \tau^+\nu$ is directly proportional to $|V_{ub}|^2$. Using the branching fraction of $B^+ \to \tau^+\nu$, the decay constant f_B from lattice QCD calculations and B^+ lifetime, PDG obtains $|V_{ub}| = (4.04 \pm 0.38) \times 10^{-3}$, consistent with Eq. (19). Since the decay $B^+ \to \tau^+\nu$ is sensitive to the charged Higgs contribution at tree level, this mode is often used to examine possible New Physics (see Section 3). A large Λ_b sample collected by LHCb facilitates the measurement of $|V_{ub}/V_{cb}|$. Based on the lattice calculation of Ref. [55], LHCb reported $|V_{ub}/V_{cb}| = 0.083 \pm 0.006$ [56] by measuring the ratio of branching fractions of $\Lambda_b^0 \to p^+\mu^-\nu$ and $\Lambda_b^0 \to \Lambda_c^+\mu^-\nu$ with $q^2 > 15$ GeV$^2/c^2$ and $q^2 > 7$ GeV$^2/c^2$, respectively. The ratio of the two magnitudes can also be included along with other determination methods, and therefore test the CKM paradigm.

$|V_{td}|$ and $|V_{ts}|$

Due to the difficulty of tagging d or s jets, it is unlikely that $|V_{td}|$ and $|V_{ts}|$ can be determined from top quark decays. Instead, these two CKM elements can be extracted from B–$\overline{B}$ oscillations mediated by box diagrams, or from rare K and B decays via penguin loop diagrams. In general, the ratio $|V_{td}/V_{ts}|$ is determined with better precision, since the hadronic uncertainties in $B_d^0 - \overline{B}_d^0$ and $B_s^0 - \overline{B}_s^0$ mixings are equal in the flavor SU(3) limit, and therefore cancel in the ratio.

Experimental measurements of the mass difference for both B_d^0 and B_s^0 meson systems have reached rather good accuracy: $\Delta m_d = (0.5064 \pm 0.0019)$ ps^{-1} and $\Delta m_s = (17.757 \pm 0.021)$ ps^{-1} [17]. Using the lattice QCD results $f_{B_d}\sqrt{\hat{B}_{B_d}} = (216 \pm 15)$ MeV and $f_{B_s}\sqrt{\hat{B}_{B_s}} = (266 \pm 18)$ MeV [37], where $\hat{B}_{B_d}$ is the bag parameter, PDG obtains

$$|V_{td}| = (8.2 \pm 0.6) \times 10^{-3}, \quad |V_{ts}| = (40.0 \pm 2.7) \times 10^{-3}, \tag{20}$$

where the uncertainties are dominated by lattice QCD calculations. As stated, some of the uncertainties for the decay constants and bag parameters can be reduced by taking ratio. Using $f_{B_d}\sqrt{\hat{B}_{B_d}}/f_{B_s}\sqrt{\hat{B}_{B_s}} = 1.268 \pm 0.064$ [37], the ratio of the two magnitudes is extracted to be

$$|V_{td}/V_{ts}| = 0.215 \pm 0.001 \pm 0.011. \tag{21}$$

The second uncertainty is from lattice QCD calculation, which is substantially reduced.

Other information related to $|V_{td}|$ or $|V_{ts}|$ come from rare B decays. However, the statistics of rare decays is typically limited, hence the extracted numbers may not be precise. For instance, $|V_{td}/V_{ts}|$ can also be determined from the ratio of $B \to \rho\gamma$ and $B \to K^*\gamma$ decay branching fractions. To increase statistics, PDG combines both charged and neutral B meson decay rates. With the assumption of isospin symmetry (e.g. $\Gamma(B^0 \to \omega^0\gamma) = \Gamma(B^0 \to \rho^0\gamma)$) and taking the heavy quark limit, one can write $|V_{td}/V_{ts}|^2/\xi_\gamma^2 = [\Gamma(B^+ \to \rho^+\gamma) + 2\Gamma(B^0 \to \rho^0\gamma)]/[\Gamma(B^+ \to K^{*+}\gamma) + \Gamma(B^0 \to K^{*0}\gamma)] = (3.19 \pm 0.46)\%$ [40]. The factor ξ_γ contains information of hadronic physics. Taking ξ_γ from various theoretical computations gives $|V_{td}/V_{ts}| = 0.214 \pm 0.016 \pm 0.036$, consistent with the value from the mixing analysis but with larger uncertainties.

The decay rates of two very rare decays, $B_s \to \mu^+\mu^-$ and $K^+ \to \pi^+\nu\overline{\nu}$, are proportional to $|V_{ts}V_{tb}|^2$ and $|V_{td}V_{ts}|^2$, respectively. The current measured $B_s \to \mu^+\mu^-$ branching fraction is consistent with the SM prediction, while a few $K^+ \to \pi^+\nu\overline{\nu}$ events are observed, suggesting a branching fraction that is slightly higher than SM expectations at 10^{-10} level. Since possible New Physics may appear in these rare decays, it is better to determine the corresponding CKM matrix elements precisely from other channels, such that the SM branching fractions can be predicted with good accuracy. More discussions for these two very rare decays are given in Sections 3 and 5, respectively.

$|V_{tb}|$

$|V_{tb}|$ can be extracted directly by measurement of the single top production cross section at hadron colliders. The combined CDF and DØ measurements give $|V_{tb}| = 1.02^{+0.06}_{-0.05}$ [57]. ATLAS and CMS have measured single top production cross sections for t-channel, Wt and s-channel processes at 7, 8 and 13 TeV [58]. The average of the LHC measurements gives $|V_{tb}| = 1.005 \pm 0.036$, assuming that systematic and theoretical uncertainties are correlated. The average of the Tevatron and LHC measurements from PDG is

$$|V_{tb}| = 1.009 \pm 0.031, \tag{22}$$

where error is dominated by experimental systematic uncertainties.

The second method to determine $|V_{tb}|$ uses the ratio of branching fractions,

$$R = \mathcal{B}(t \to Wb)/\Sigma_q\mathcal{B}(t \to Wq) = |V_{tb}|^2, \tag{23}$$

where $q = d$, s or b. Assuming unitarity, the ratio R is $|V_{tb}|^2$. Experiments at Tevatron and LHC have followed this method and performed the extractions by selecting $t\bar{t}$ events with various number of b-tagged jets. Since the measurements are not precise, experiments reported either an allowed range [59] or lower limits [60,61] at 95% C.L. All experimental results are consistent with each other, and with Eq. (22).

Comments

The current PDG values of $|V_{ib}|$ and $|V_{tj}|$ show, as expected, that $|V_{tb}|$ is close to unity and the rest of $|V_{ib}|$ and $|V_{tj}|$ are small. Based on single top production at LHC, the precision of $|V_{tb}|$ can be improved with more data, while the theory uncertainties need to be reduced to extract $|V_{td}|$ and $|V_{ts}|$ precisely from B^0–$\overline{B}^0$ and B_s^0–$\overline{B_s}^0$ mixing analyses, respectively. As for $|V_{cb}|$ and $|V_{ub}|$, the uncertainties of both magnitudes will be reduced by 10 to 50 times more data at Belle II. However, the discrepancies between the exclusive and inclusive methods need to be understood. See Section 4.2 for more discussion.

2.2. Phases

We turn to discuss the determination of CKM phases. In SM, *CP* violation involves the phases of CKM elements. Therefore, CPV measurements in a host of *B* decays provide good constraints on the angles of the unitarity triangle shown in the $\bar{\rho} - \bar{\eta}$ plane of Fig. 1.

We have already given a brief account of the pursuit of direct CPV, and lamented that the DCPV difference in $B \to K^+\pi$ decay, Eq. (5), seems to be due to an "unsuppressed color-suppressed tree diagram *C*", that carries also a large strong phase difference w.r.t. the tree amplitude *T*, that makes the extraction of short distance information, such as CKM elements, impossible, even at the *B* decay scale. We give a little more discussion in Section 4.1. The precursor of realization of the "hadronic menace" was in fact in the pursuit of DCPV in kaon decay.

The measurement of the direct *CP* violation parameter, $\mathrm{Re}(\varepsilon'/\varepsilon)$, was viewed as a way to test CKM mechanism and constrain many New Physics scenarios. Theoretically, $\mathrm{Re}(\varepsilon'/\varepsilon)$ is proportional to $\mathrm{Im}(V_{td}V_{ts}^*)$. But since the electroweak penguin contribution tends to cancel the gluonic penguin contributions for large m_t [62], this greatly amplifies the hadronic menace. Therefore, it is rather difficult to use $\mathrm{Re}(\varepsilon'/\varepsilon)$ to extract CKM parameters. Unlike ε_K, indirect CPV in neutral kaon mixing, it does not appear in Fig. 1 as a constraint. Inspection of Fig. 1, however, shows that ε_K as a constraint provides consistency support for the determination of the ϕ_1/β, ϕ_2/α and ϕ_3/γ program at the *B* factories, therefore we do not go into any details. Suffice to say that ε_K, the original discovery of CPV [5], depends on the decay constant f_K and bag parameter $\hat{B}_K$, and carries its own hadronic uncertainties. We note, however, that recent lattice developments suggest a major deficit for SM to account for the measured $\mathrm{Re}(\varepsilon'/\varepsilon)$ result, suggesting it could possibly be a renewed probe of New Physics. More discussion is given in Section 5.3.

2.2.1. ϕ_1/β (and ϕ_2/α)

ϕ_1/β

The triumph of the B factories has been the clean measurement of the CPV phase angle ϕ_1/β. Thanks to the method developed by Bigi and Sanda [18,19], unlike our discussion so far about measurement of magnitudes of the CKM matrix, the measurement of ϕ_1/β is especially free of hadronic uncertainties. As the subject is relatively well known, let us be brief.

Precise determination of ϕ_1 is done experimentally by studying neutral *B* meson decays into a *CP* eigenstate *f* via $b \to c\bar{c}s$ transitions, which has no CPV phase in the decay amplitude, but makes these excellent probes of CPV phase of the B_d mixing amplitude. The time-dependent CPV asymmetry can be written as

$$\mathcal{A}_f(t) = \frac{\Gamma(\overline{B}^0 \to f) - \Gamma(B^0 \to f)}{\Gamma(\overline{B}^0 \to f) + \Gamma(B^0 \to f)} = S_f \sin(\Delta m_d\, t) - C_f \cos(\Delta m_d\, t), \tag{24}$$

where

$$S_f = \frac{2\,\mathrm{Im}\lambda_f}{1+|\lambda_f|^2}, \quad C_f = \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}, \quad \lambda_f = \frac{q}{p}\frac{\overline{A}_f}{A_f}. \tag{25}$$

The mixing parameter q/p can be approximated as $V_{tb}^* V_{td}/V_{tb}V_{td}^* = e^{-2i\phi_1 + O(\lambda^4)}$, where λ is defined in Eq. (3). A_f ($\overline{A}_f$) is the amplitude of the decay $B^0 \to f$ ($\overline{B}^0 \to f$). Minus C_f is the parameter of direct *CP* violation. If the amplitude is dominated by a single diagram such that the strong phase factorize, $|\lambda_f| = 1$ and $S_f = \sin(\arg\lambda_f) = \eta_f \sin 2\phi_1$, where η_f is the *CP* eigenvalue. Such is the case, e.g. for the golden $B \to J/\psi K_S$ mode. If there are at least two diagrams with two CKM phases contributing to the amplitude, S_f may be sensitive to the difference of strong phases between the two diagrams, and C_f may not be zero. The *t*-depend study using Eq. (24) can therefore measure DCPV directly.

The extraction of ϕ_1 with $B \to J/\psi K_S$ and other golden modes was the main target for the *B* factory experiments at the dawn of the new millennium. Large mixing induced *CP* violation of $O(1)$ was observed in such golden modes. The most recent average performed by HFAG [40] using the precise measurements from BaBar and Belle, early LEP and CDF results and recent LHCb measurements with 3 fb^{-1}, gives

$$\sin 2\phi_1 = 0.691 \pm 0.017. \tag{26}$$

The measured value of $\sin\phi_1$ has a four-fold ambiguity, which can be resolved by a global fit to all available measurements of CKM parameters, such as the one displayed in Fig. 1 from CKMfitter. The four-fold ambiguity can be reduced to two, ϕ_1 and $\pi + \phi_1$, by a time dependent angular analysis of $B^0 \to J/\psi K^{*0}$, or a time dependent Dalitz plot analysis of $B^0 \to \overline{D}^0 h^0$

($h^0 = \pi^0,\ \eta,\ \omega$). Results by BaBar and Belle on both type of modes show that negative $\cos 2\phi_1$ is unlikely, as can be seen in Fig. 1.

The same time dependent analysis has been applied to B^0 decays via $b \to c\bar{c}d$ transitions. However, $b \to d$ penguin pollution may cause S_f and C_f to deviate from the values of $\sin 2\phi_1$ and $\cos 2\phi_1$, respectively. Therefore, the experimental measurements of S_f and C_f for $B^0 \to J/\psi\pi^0$, $B^0 \to J/\psi\rho^0$ and $B^0 \to D^{(*)+}D^{(*)-}$ are not included in the global fit by the CKMfitter group, although these results are consistent with that from the golden modes. The $b \to s\bar{q}q$ penguin-dominated decays, on the other hand, have the same CKM phase as the $b \to c\bar{c}s$ modes, and should yield the same S_f and C_f. But since New Physics phases may contribute in the penguin loop, resulting in deviations for S_f and C_f from those measured in the golden modes (which were indicated by early data), these time-dependent analyses provide a clean probe for New Physics, which is discussed in Section 4.

ϕ_2/α

The second phase angle, ϕ_2, of the triangle of Eq. (4) is the angle between $V_{td}V_{tb}^*$ and $V_{ud}V_{ub}^*$. It can be extracted from time dependent analysis in $b \to u\bar{u}d$ transitions, analogous to the ϕ_1 case. But the $b \to d$ penguin may have significant strength compared to the $b \to u\bar{u}d$ tree amplitude, resulting in a phase shift from the CKM phase in S_f and making the ϕ_2 determination difficult. Experimentally the angle ϕ_2 has been extracted with three decay channels: $B \to \pi\pi$, $\rho\rho$ and $\rho\pi$.

From the observation of direct CP violation in $B \to \pi^+\pi^-$ we know that the d penguin contribution is not small. Therefore, $S_{\pi^+\pi^-}$ is not $\sin(2\phi_2)$, but can be expressed as,

$$S_{\pi^+\pi^-} = \sqrt{1 - C^2_{\pi^+\pi^-}}\,\sin(2\phi_2 + \Delta), \tag{27}$$

where Δ is the phase difference between the two amplitudes, $e^{-2\pi\phi_3}\bar{A}_{\pi^+\pi^-}$ and $A_{\pi^+\pi^-}$. This phase angle shift can be extracted with isospin analysis using the branching fractions and direct CP asymmetries for the three $B \to \pi\pi$ decays [63]. PDG uses the world average of BaBar, Belle and LHCb results [40], $S_{\pi^+\pi^-} = -0.66 \pm 0.06$, $C_{\pi^+\pi^-} = -0.31 \pm 0.05$, the branching fractions of all three modes, and $C_{\pi^0\pi^0} = -0.43^{+0.25}_{-0.24}$. Since the analysis with the data used in 2015 leads to 16 mirror solutions, while the branching fraction and CP asymmetry of the $\pi^0\pi^0$ mode have sizeable uncertainties, the constraint on ϕ_2 is rather loose, $0° < \phi_2 < 3.8°$, $86.2° < \phi_2 < 102.9°$, $122.1° < \phi_2 < 147.9°$, and $167.1° < \phi_2 < 180°$ at 68% confidence level.

Compared with $B \to \pi\pi$, the ρ is a vector meson, hence the $\rho^+\rho^-$ final state from B decays could be a mixture of CP even or CP odd components. This mixture state complicates the analysis, and the phase angle cannot be extracted if the mixture is half and half. Fortunately, experimental data show that the longitudinal polarization fractions in $B^+ \to \rho^+\rho^0$ and $B^0 \to \rho^+\rho^-$ decays are close to unity, indicating that final states are almost CP even. Moreover, the small branching fraction of $B^0 \to \rho^0\rho^0$ is a factor of twenty less than that of $B^0 \to \rho^+\rho^-$ and $B^+ \to \rho^+\rho^0$, indicating the penguin contribution is small. Isospin analysis is applied to the $\rho\rho$ modes using the world average [40] of $S_{\rho^+\rho^-}$, $C_{\rho^+\rho^-}$, the branching fractions of the three $B \to \rho\rho$ modes, together with the $S_{\rho^0\rho^0}$ and $C_{\rho^0\rho^0}$ values from Ref. [64]. The ϕ_2 phase angle is obtained in six regions at 68% confidence level: $0° < \phi_2 < 5.6°$, $84.4° < \phi_2 < 95.3°$, $174.7° < \phi_2 < 180°$, and the other three regions in mirror solutions $3\pi/2 - \phi_2$.

Although the $B^0 \to \rho^+\pi^-$ final state is not a CP eigenstate, both B^0 and $\bar{B}^0$ can decay to $\rho^\pm\pi^\mp$. Furthermore, the decay $B^0 \to \rho^\pm\pi^\mp$ proceeds via the same diagrams as $B^0 \to \pi^+\pi^-$, making interference of four diagrams possible. The time-dependent Dalitz plot analysis of $B^0 \to \pi^+\pi^-\pi^0$ enables the extraction of ϕ_2 with a $\phi_2 \to \pi + \phi_2$ ambiguity since the variation of strong phase in the interference regions of $\rho^+\pi^-$, $\rho^-\pi^+$ and $\rho^0\pi^0$ can be known [65]. PDG averages the Belle [66] and BaBar [67] measurements to give $\phi_2 = (54.1^{+7.7}_{-10.3})°$ and $(141.8^{+4.7}_{-5.4})°$.

Combining all the three ϕ_2 measurements, ϕ_2 is constrained to be rather close to 90°,

$$\phi_2 = (88.6^{+3.5}_{-3.3})°. \tag{28}$$

2.2.2. ϕ_3/γ

The third CKM angle ϕ_3 is between $V_{ud}V_{ub}^*$ and $V_{cd}V_{cb}^*$, and is practically the phase angle of V_{ub}^* in the standard phase convention. Since the top quark is not involved in the CKM elements for ϕ_3, it cannot be extracted using time-dependent analysis. Three different methods are used to extract ϕ_3 experimentally. All employ the principle that the interference of $B^- \to D^{(*)0}K^{(*)-}(b \to c\bar{u}s)$ and $B^- \to \bar{D}^{(*)0}K^{(*)-}$ $(b \to u\bar{c}s$, which carries the CPV phase) can be studied by comparing some $D^{(*)0}$ and $\bar{D}^{(*)0}$ decays. In principle, the interference permits the extraction of the B and D amplitudes, the weak phase and the relative strong phases. The crucial parameter is r_B, defined as the amplitude ratio,

$$r_B = \frac{A(B^- \to \bar{D}^{(*)0}K^{(*)-})}{A(B^- \to D^{(*)0}K^{(*)-})}, \tag{29}$$

which ranges from 0.1 to 0.2. Note that a smaller r_B causes the interference to be less effective.

The GLW method [68,69] considers D decays to CP eigenstates. Therefore, both the Cabibbo-suppressed and favored decays go into the same final state, facilitating the interference. To enhance r_B and make the amplitudes of B and D mesons compatible for both decays, the ADS method [70] considers Cabibbo-allowed $\bar{D}$ decay and doubly-Cabibbo-suppressed D

Table 1
Summary of measured CKM matrix element and phase values.

	$\|V_{ud}\|$	$\|V_{us}\|$	$\|V_{ub}\|$
	0.97417 ± 0.00021	0.2248 ± 0.0006	0.00409 ± 0.00039
	$\|V_{cd}\|$	$\|V_{cs}\|$	$\|V_{cb}\|$
	0.220 ± 0.005	0.995 ± 0.016	0.0405 ± 0.0015
$\|V_{td}/V_{ts}\|$	$\|V_{td}\|$	$\|V_{ts}\|$	$\|V_{tb}\|$
$0.215 \pm 0.001 \pm 0.011$	0.0082 ± 0.0006	0.0400 ± 0.0027	1.009 ± 0.031
	$\sin 2\phi_1$	ϕ_2	ϕ_3
	0.691 ± 0.017	$(88.6^{+3.5}_{-3.3})^\circ$	$(73.2^{+6.3}_{-7.0})^\circ$

decay. Extensive measurements of ϕ_3 are performed at the B factories, CDF and LHCb, which can be found in the HFAG web pages.

The most promising and effective method, if not limited by statistics, comes from the Dalitz plot method. The point is that some three-body D^0 decays, such as $D^0 \to K^0_S\pi^+\pi^-$, have large branching fractions, and the interference can be studied across the Dalitz plot for such D decays [71,72]. Belle and BaBar have measured ϕ_3 with good accuracy using the Dalitz plot method. The obtained results are $\phi_3 = (78^{+11}_{-12} \pm 4 \pm 9)^\circ$ [73] for Belle, and $\phi_3 = (68 \pm 14 \pm 4 \pm 3)^\circ$ [74] for BaBar, where the last uncertainty of these measurements is from D decay modeling. But the beauty of the Dalitz analysis is that, by a bin-by-bin fit, one can fit for the variation of the strong phase across the Dalitz plot, and the eventual measurement of ϕ_3 would be model independent. The LHCb result with this model independent approach yields $\phi_3 = (62^{+15}_{-14})^\circ$ [75].

Combining the results from GLW, ADS and Dalitz analyses, PDG gives

$$\phi_3/\gamma = (73.2^{+6.3}_{-7.0})^\circ. \tag{30}$$

But with the large data sets expected by Belle II and LHCb in the future, the error stands major improvement. Combining Eqs. (26), (28) and (30), one can easily check that the three angles sum to 180°within errors, which can be further tested in the future.

Comments

When the B factories were proposed, only the first phase angle ϕ_1 was assured to be measured with good accuracy. The angle ϕ_3 was known to be difficult at the beginning of the B factory era. With large accumulated data, however, the data analysis is gradually improved, and several new methods were brought out. Now the uncertainty for ϕ_3 is at the 7°level.[7] When high quality data become available in the future, it is likely that we can achieve better accuracy than originally conceived. With Belle II data taking to start soon, and with the LHCb detector upgrade, the angle ϕ_3 is expected to reach an accuracy of 1°for both Belle II and LHCb with full data samples (with LHCb aiming to push below with a Phase 2 Upgrade). It is worth reemphasizing that ϕ_3 measurement is based on *tree level* processes, and accesses the phase of V^*_{ub} directly.

2.3. CKM summary and global fit

We have given a short review on the determination of CKM elements based on values given by PDG 2016. The magnitudes of the matrix elements are extracted from various semileptonic and leptonic decays of mesons, as well as muon and tau decays. As for the phase angles, time-dependent CP analysis is used for ϕ_1 and ϕ_2, while ϕ_3 can be extracted by using $B^- \to D^{(*)0}K^{(*)-}$ decays with various methods. Footnote 7 below makes clear the impact of LHCb, especially on γ/ϕ_3.

The PDG 2016 averages of the measured magnitudes of CKM matrix elements, as well as the three phase angles, are summarized in Table 1. In most cases, extractions using different channels for the same elements or phase angles show consistent results, strongly supporting the CKM paradigm. Among these, many already reach good accuracy with errors dominated by theory uncertainties. However, there are also noticeable tensions in some of the measurements. The discrepancies between the inclusive and exclusive determinations of $|V_{cb}|$ and $|V_{ub}|$ need to be resolved with efforts from both experiment and theory. An almost 3σ tension on $|V_{us}|$ coming from inclusive τ decays also need to be understood.

The CKM paradigm stands tall, but there is still work to be done.

CKM Global Fit

With the nine magnitudes of the CKM matrix elements and three angles summarized in Table 1, the unitarity of the matrix and sum of the three angles can be readily checked. The squared sum of the first row of the CKM matrix gives $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9996 \pm 0.0005$, which can be compared with Eq. (10), where adding $|V_{ub}|^2$ has moved the fourth decimal place. The squared sums for 1st column, 2nd row and 2nd column are, $|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 0.9975 \pm 0.0022$, $|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1.040 \pm 0.032$ and $|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 1.042 \pm 0.032$, respectively, where uncertainties of

[7] To show the volatility of the subject, as reported at EPS-HEP 2017, the new combined LHCb result [76], $\gamma/\phi_3 = (76.8^{+5.1}_{-5.7})^\circ$ by a single experiment. This leads to the HFLAV combined result of $\gamma/\phi_3 = (76.2^{+4.7}_{-5.0})^\circ$ for EPS 2017, which is at the 5°level! Belle II data is eagerly awaited.

the latter two are dominated by $|V_{cs}|$. These squared sums are all consistent with unity. The angle sum of ϕ_1, ϕ_2 and ϕ_3 is $\left(183^{+7}_{-8}\right)^\circ$, consistent with a triangle.

Since the matrix elements can be determined more precisely using a global fit to all available measurements under the constraint of three generation unitarity, PDG conducts a fit using two approaches: the frequentist statistics [77] used by CKMfitter [78], as well as a Bayesian approach from UTfit [79,80]. Fit results from both approaches are similar, where the Wolfenstein parameters obtained from the frequentist approach are

$$\lambda = 0.22506 \pm 0.00050, \quad A = 0.811 \pm 0.026, \quad \bar{\rho} = 0.124^{+0.019}_{-0.018}, \quad \bar{\eta} = 0.356 \pm 0.011, \tag{31}$$

which can be compared with the Bayesian approach: $\lambda = 0.22496 \pm 0.00048$, $A = 0.823 \pm 0.013$, $\bar{\rho} = 0.141 \pm 0.019$ and $\bar{\eta} = 0.349 \pm 0.012$. The fitted magnitudes of the CKM matrix elements are

$$(|V_{ij}|) = \begin{pmatrix} 0.97434^{+0.00011}_{-0.00012} & 0.22506 \pm 0.00050 & 0.00357 \pm 0.00015 \\ 0.22492 \pm 0.00050 & 0.97351 \pm 0.00013 & 0.0411 \pm 0.0013 \\ 0.00875^{+0.00032}_{-0.00033} & 0.0403 \pm 0.0013 & 0.99915 \pm 0.00005 \end{pmatrix}, \tag{32}$$

and the Jarlskog invariant [81], defined as $\mathrm{Im}\,(V_{us}V_{cb}V^*_{ub}V^*_{cs})$, is

$$J = \left(3.04^{+0.21}_{-0.20}\right) \times 10^{-5}. \tag{33}$$

Eq. (32) is consistent with Table 1, but with higher precision.

Comments

So far we have briefly described the determination of the magnitudes and phase angles of the CKM matrix elements. A total of 12 parameters are extracted from hundreds of experimental measurements, from the decays of light and heavy mesons, baryons, tau leptons, W bosons and top quarks, together with perturbative and nonperturbative theoretical calculations based on SM. Data analyses were performed to measure the decay rates, mixing parameters, time-dependent or direct *CP* violating asymmetries, etc. No apparent anomaly is found in the CKM sector, and all the obtained parameters give a consistent picture. This is rather remarkable, indicating that the Higgs mechanism with Yukawa coupling and the quark mixing matrix constitute the fundamental theory that governs the particle physics realm. Consistent experimental measurements also imply that New Physics contribution, if any, is rather small. Therefore, it maybe more fruitful to search for New Physics in the channels where the SM contribution is forbidden or highly suppressed.

3. Recent B highlights

The B factories were very successful, measuring $\sin 2\phi_1/\beta$ precisely and free of hadronic uncertainties. They confirmed the Kobayashi–Maskawa picture, and further sealed the CKM paradigm with a plethora of measurements. It is quite amazing that the tiny CPV phase in B_s mixing, $\phi_s = -2\beta_s$, is now also measured to be consistent with SM expectation at -0.04. It is not "anomalous", but there were excitements along the path of measurement, which is still partially reflected in the latest HFAG [82] combination plot, Fig. 3, as one moved from the Tevatron to the LHC. The very rare decay $B_s \to \mu^+\mu^-$ was toted as a great vehicle for discovering the effect of SUSY in the flavor sector, but it again turned out to be consistent with SM. These two measurements are triumphs of experimental physics, and epitomize the success of the LHC(b).

While the B factories saw many anomalies, they have mostly gone away. There are, however, two anomalies that emerged during the LHC Run 2 period, and not just from the LHC. One is the so-called "P_5' anomaly" uncovered by LHCb in the angular analysis of $B \to K^*\mu^+\mu^-$ decay. It is reminiscent of the A_{FB} anomaly (which got eliminated by LHCb) from the B factories, but not the same. It may call for an extra Z' boson. Another is called the "BaBar anomaly" in the $B \to D\tau\nu$ and $D^*\tau\nu$ final states, which cannot be understood in the usual (SUSY) type of two Higgs doublet models (called 2HDM-II). In this section, we would go through these recent B highlights, along with a few lesser effects. As an anticlimax, we mention the spectacular DCPV asymmetries observed over the Dalitz plot of three body B decays, to reemphasize that hadronic effects remain a menace.

3.1. CPV phase ϕ_s

Recall that it was the discovery [11] of large B_d mixing, i.e. Δm_d is comparable to Γ_d, that stimulated the construction of the B factories. From that perspective, it was the magnificent observation [83] of B_s mixing at the Tevatron that was the precursor to the measurement of ϕ_s at the LHC, which we now discuss. The new "B factory", called LHCb, a hybrid of a fixed-target and collider detector, was already under construction at that time. A new era of B physics at hadronic colliders was dawning.

Inspection of Fig. 3 shows that ϕ_s is measured together with $\Delta\Gamma_s$, which is different from the B_d case. As discussed already with $\sin 2\phi_1/\beta$ measurement, for neutral B_q meson decaying into a *CP* eigenstate that is common to both B_q and $\overline{B}_q$ mesons, the interference between the amplitudes of the direct decay and decay after mixing generates a time-dependent CPV asymmetry. For the B_s and $\overline{B}_s$ meson system, however, $b \to c\bar{c}s$ decay is not Cabibbo suppressed like $b \to c\bar{c}d$ decay,

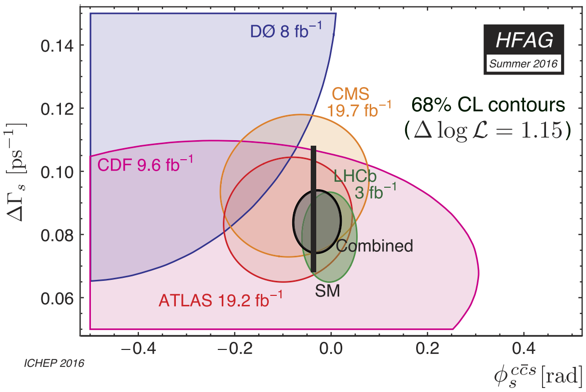

Fig. 3. Measured 68% C.L. contours in the ϕ_s–$\Delta\Gamma_s$ plane from all relevant experiments. The combined contour (black ellipse) is consistent with SM expectation (thick black line).
Source: http://www.slac.stanford.edu/xorg/hfag/.

which can link between $\bar{b}s$ and $b\bar{s}$ mesonic states. That is, the mixing of B_s and $\bar{B}_s$ mesons has a non-negligible absorptive part, or a width difference $\Delta\Gamma_s$, in addition to Δm_s. Thus, the CPV asymmetry study fits for both the weak phase ϕ_s and the decay width difference $\Delta\Gamma_s$. The expected decay width difference in SM is determined [84] to be

$$\Delta\Gamma_s^{(\mathrm{SM})} = 0.088 \pm 0.020 \text{ ps}^{-1}, \tag{34}$$

where $\Delta\Gamma_s$ is defined as the difference between decay widths Γ_L and Γ_H of the light and heavy B_s states. The weak phase ϕ_s is predicted to be rather small but with high precision (β_s is used by CDF, while ϕ_s is used by DØ and LHCb),

$$\phi_s^{(\mathrm{SM})} = -2\beta_s^{(\mathrm{SM})} = -2\arg(-V_{ts}V_{tb}^*/V_{cs}V_{cb}^*) = -0.0376^{+0.0008}_{-0.0007}, \tag{35}$$

where the numerical value is from indirect determination by a global fit to experimental data [85]. Since the prediction is rather precise, any deviation between the measured value and Eq. (35) would be strong hint for New Physics, and it was stressed that many New Physics scenarios could greatly enhance ϕ_s [86].

From the relation to the CKM matrix elements as displayed in Eq. (35), one can see that the unitarity triangle[8]

$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0 \tag{36}$$

is very squashed in SM compared with Eq. (4), where the measurement of the latter is displayed in Fig. 1. This enhances one's appreciation of the current world average, largely based on LHC Run 1 data,

$$\phi_s = -0.030 \pm 0.033, \tag{37}$$

which is quite consistent with the SM prediction, as one can see pictorially from Fig. 3. But the game is far from over, and one needs much improvement in data size, as well as analysis methods, to scrutinize whether there is New Physics in ϕ_s, or it is fully consistent with SM. However, at the dawn of the LHC era, there was much anticipation for a potentially large, New Physics CPV phase in ϕ_s. Though this wish or hope was dashed by the measured value of Eq. (37), it was truly a recent highlight of B physics. So let us give some account.

The process $B_s \to J/\psi\,\phi$ is the main decay used to measure ϕ_s as well as $\Delta\Gamma_s$. The $B_s \to J/\psi\,\phi$ decay final state is a mixture of CP eigenstates, and an angular analysis is required to disentangle the CP-odd and CP-even components. A time-dependent angular analysis has to be carried out by measuring the decay angles of the final state particles and the proper decay length of the B_s candidate. The differential decay rate can be expressed as,

$$\frac{d^4\Gamma[B_s(t)]}{d\Theta\, dt} = \sum_{i=1}^{10} \mathcal{O}_i(\alpha, t)\, g_i(\Theta), \tag{38}$$

where $\mathcal{O}_i$ are time-dependent functions, g_i the angular functions, α represents the physics parameters of interest ($\Delta\Gamma_s$ and ϕ_s), and Θ and t represent the measured angles and proper decay time of B_s decay. Because of the finite width difference, the functions $\mathcal{O}_i$ can be expanded as,

$$\mathcal{O}_i(\alpha, t) \propto e^{-\Gamma_s t}\Big[a_i \cosh(\Delta\Gamma_s t/2) + b_i \sinh(\Delta\Gamma_s t/2) + c_i \cos(\Delta m_s t) + d_i \sin(\Delta m_s t)\Big], \tag{39}$$

[8] One can verify easily by use of Eq. (2), but expanding to $O(\lambda^5)$ terms.

where the non-oscillatory $\Delta\Gamma_s$ effects also enter. The extraction of ϕ_s mostly comes through the parameters b_i and d_i. The angular parameters symbolized by Θ contain the polar and azimuthal angles θ_T and ϕ_T of the μ^+ in the J/ψ rest frame, and the helicity angle ψ_T of the ϕ meson w.r.t. the direction of K^+. The details of the description of the time-dependent decay rate can be found in Ref. [87] (see also Ref. [84] for a review).

From the angular distribution alluded to above, one can already tell the complexity of the analysis. The required ingredients include a well modeled detection efficiency along the decay angles, decay proper time, and the flavor of the B_s meson. In particular, flavor tagging is a critical element in the analysis, where the tagging power in recent studies is still at the order of 1%–3%. Possible interference between the $J/\psi\,\phi$ and $J/\psi\,K^+K^-$ components is one of the dominant systematic issues, and a detailed study of the K^+K^- invariant mass distribution is needed to disentangle the interference pattern. There are several similar decays that can be further used to determine ϕ_s, such as $B_s \to \psi(2S)\,\phi$ and $J/\psi f_0$ (i.e. $J/\psi\,\pi^+\pi^-$), which require similar or simpler modeling of the differential decay rate. The $B_s \to J/\psi\,\phi$ mode dominated early studies, but for the measurement by LHCb, the $J/\psi f_0$ ($\to \pi^+\pi^-$) mode was critical as well (see e.g. Ref [84]).

To aim at measuring $|\phi_s| < 0.1$ appeared rather daunting at the Tevatron. However, with the discovery [24] of large DCPV difference in $B \to K\pi$, it was argued [88] that a New Physics phase in the electroweak penguin that could potentially explain the effect of Eq. (5), could also show up as large CPV in B_s mixing, and that CDF and DØ had the chance to get the first glimpse of an $O(1)$ value for $|\sin 2\beta_s|$. Intriguingly, the Tevatron results indeed suggested [89] ϕ_s that is away from the predicted value of -0.04. The situation is somewhat still preserved in the latest HFAG combination plot shown in Fig. 3. The final CDF and DØ results tend towards rather sizable negative values but with large error bars (while one can only tell that $\Delta\Gamma_s > 0$). There was thus much anticipation at the turn-on of the LHC, where the earliest LHCb result was not inconsistent [90] with that from the Tevatron. Alas, the hope was dashed by the LHCb announcement at Lepton–Photon 2011 held in Mumbai, that ϕ_s also turned out to be consistent with SM. Further results from ATLAS and CMS, and in particular the LHCb result with full Run 1 data using both $B_s \to J/\psi\,\phi$ and $J/\psi\,f_0$ ($\to \pi^+\pi^-$) modes are much closer to the SM prediction, with the current world average given in Eq. (37). The uncertainty is still dominated by statistics, so further improvement is expected with more LHC data and improvement of analysis techniques. More refined algorithms (as well as statistics) should also improve systematic errors. But the game has definitely switched from large deviation search to precision test of SM.

The reverse argument of Ref. [88], that no sign of New Physics in ϕ_s makes a Z-penguin explanation of Eq. (5) implausible, points to the color-suppressed amplitude C as culprit, causing us to lament in the Introduction that hadronic effects are still a menace in B decays. Theorists never predicted $|C| \sim |T|$.

As seen from Fig. 3, the measurement of $\Delta\Gamma_s$ is now also in agreement with SM. To a lesser extent compared with ϕ_s measurement, due to its connection with the like-sign dimuon charge asymmetry A_{sl}^b, $\Delta\Gamma_s$ had its own saga. Based on the final data set from DØ [91], the measured A_{sl}^b value differs from the SM prediction by 3.6σ, consistent with what was reported at the dawn of the LHC. Although it has been difficult for other experiments to confirm or rule out this asymmetry, it can be expressed in terms of the wrong-sign asymmetries for B_d and B_s mesons, a_{sl}^d and a_{sl}^s, respectively. The wrong-sign asymmetry a_{sl}^d for B_d is measured by B factory experiments and shows no deviation from SM. Attributing the "DØ anomaly" to the wrong-sign B_s asymmetry implies a value for a_{sl}^s that is much larger than in SM, which could probe the product of $\Delta\Gamma_s$ and $\tan\phi_s$. This was quite interesting before summer 2011, when ϕ_s might turn out sizable. Since the prediction of $\Delta\Gamma_s$ based on OPE is quite robust, and long distance effects have been demonstrated to have little impact [92], the issue has gone away with ϕ_s being consistent with the small value expected in SM (but "DØ anomaly" remains).

The current $\Delta\Gamma_s$ measurement has comparable precision as SM expectation. With the upcoming improved measurement of $B_s \to J/\psi\phi$ and similar decays, together with the improvements of SM-based calculations, the situation might be different in the future. This also echoes the recent experimental development of the B_s effective lifetime measurement using $B_s \to \mu^+\mu^-$ events. As discussed in the next subsection, only the heavier state of B_s is allowed in this super-rare decay process.

3.2. $B_s \to \mu^+\mu^-$ and $B^0 \to \mu^+\mu^-$

Fig. 4 is a highlight achievement of B physics at LHC Run 1: the joint measurement [93] of $B_s \to \mu^+\mu^-$ by CMS and LHCb that is consistent with the SM expectation, and a mild hint for the $B^0 \to \mu^+\mu^-$ process that is further suppressed by $|V_{td}/V_{ts}|^2$. But this statement does not really capture how much a highlight it actually is.

Just like the absence of $K_L \to \mu^+\mu^-$ and other FCNC effects led to the invention of the GIM mechanism [2], the $B_s \to \mu^+\mu^-$ and $B^0 \to \mu^+\mu^-$ processes are forbidden at tree level and suppressed by GIM cancellation in SM. It suffers from further suppression from quark annihilation within the B_q meson, hence suppressed by $f_{B_q}^2/m_{B_q}^2$, and by helicity suppression, hence another factor of $m_\mu^2/m_{B_q}^2$. The branching fractions of these two decays are predicted [94] reliably in SM as

$$\mathcal{B}^{\rm SM}(B_s \to \mu^+\mu^-) = (3.66 \pm 0.23) \times 10^{-9}, \tag{40}$$

$$\mathcal{B}^{\rm SM}(B^0 \to \mu^+\mu^-) = (1.06 \pm 0.09) \times 10^{-10}. \tag{41}$$

These rates are indeed tiny, but not totally out of reach experimentally, considering the huge b-hadron production cross section at hadron colliders and the simple dimuon tracks from a well-defined vertex, and as evidenced in the measurement displayed in Fig. 4. Because of their extreme rarity, these modes are great probes of New Physics — any deviation from the

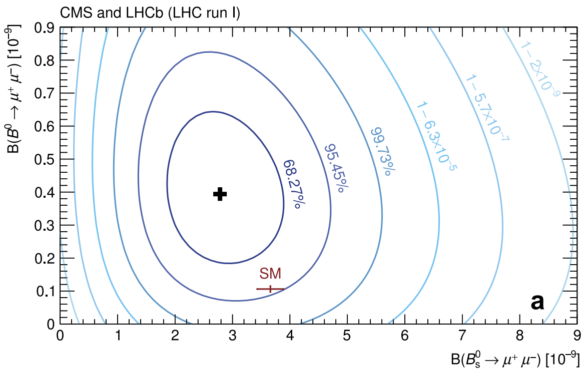

Fig. 4. Likelihood contours in the $\mathcal{B}(B_s \to \mu^+\mu^-)$ vs $\mathcal{B}(B^0 \to \mu^+\mu^-)$ plane. The black cross indicates the best-fit value, while the SM prediction (and its uncertainty) is also marked.
Source: Courtesy CMS and LHCb collaborations, from Ref. [93].

SM branching fractions could provide a smoking gun signal for New Physics. Alternatively, measurements that confirm the SM predictions would be a triumph of experimental physics, and provide stringent constraint of New Physics.

Introducing new particles or non-SM couplings in the loop processes can modify these two decays significantly. In SUSY models, $\mathcal{B}(B_s \to \mu^+\mu^-)$ can be proportional to $\tan^6\beta$, which can easily increase the branching fractions by orders of magnitude [95–99], an effect largely arising from additional Higgs bosons, and because of the aforementioned multiple suppression of $B_q \to \mu^+\mu^-$ rates. The potential for enhancement by three orders of magnitude over SM expectation galvanized the great pursuit at the Tevatron, with the hope to discover SUSY indirectly through the flavor sector. This nicely illustrates the complementarity between direct search at colliders, compared with indirect search inferred by rare decay studies.

As the Tevatron pursuit pushed down the orders of magnitude and reaching the endpoint of sensitivities, but just before early LHC Run-1 data was fully analyzed, the CDF experiment reported some excess in $B_s \to \mu^+\mu^-$, eventually publishing the large decay rate of $\left(1.3^{+0.9}_{-0.7}\right) \times 10^{-8}$ with the full CDF data set [100]. This is still close to four times larger than Eq. (40), and would constitute good evidence for New Physics, such as SUSY, if the effect was true.

However, the CDF excess was refuted already by CMS and LHCb right at the time of announcement, with CDF data already exhausted and unable to extend further.[9] As LHC data accumulated, and after LHCb and CMS each announced their evidence for $B_s \to \mu^+\mu^-$ with full Run 1 data [17], it took the combination of CMS and LHCb data to announce the 6.2σ discovery [93] of $B_s \to \mu^+\mu^-$ decay, and a hint of $B^0 \to \mu^+\mu^-$ decay was also found. The best fit branching fractions are

$$\mathcal{B}(B_s \to \mu^+\mu^-) = (2.8^{+0.7}_{-0.6}) \times 10^{-9}, \tag{42}$$

$$\mathcal{B}(B^0 \to \mu^+\mu^-) = (3.9^{+1.6}_{-1.4}) \times 10^{-10}, \tag{43}$$

where the two dimensional likelihood contours have already been shown above in Fig. 4. The branching fraction of $B_s \to \mu^+\mu^-$ is in agreement with (if not slightly lower than) the prediction of Eq. (40), while the $B^0 \to \mu^+\mu^-$ rate is higher than the expectation, but still within 2σ of Eq. (41). Note that the $\pm 2\sigma$ (or 95.45%) confidence interval for $\mathcal{B}(B^0 \to \mu^+\mu^-)$ is evaluated to be $[1.4, 7.4] \times 10^{-10}$, which is a two-sided bound, and both CMS and LHCb have some hint of events. But because of possibility of peaking backgrounds and issues of muon quality (see below), the experiments have not emphasized this so far.[10] A new study more optimized towards $B^0 \to \mu^+\mu^-$ is also called for.

Note that the ratio of branching fractions between the two decays also provides a different test to the New Physics [102]. However, theories with the property of minimal flavor violation (MFV), i.e. flavor violation occurring only in CKM sector, preserve the same value as in SM. With the measured branching fractions consistent with SM, these decays can be used as a powerful constraint on New Physics scenarios, e.g. the three orders of magnitude [95–99] "lever arm" is now fully explored. The large parameter space of interest can go beyond the reach of LHC collision energy. This is nicely illustrated by Fig. 2 from Ref. [103]. The "Straub plot", which we do not show here, exhibits the correlations between $B_s \to \mu^+\mu^-$ and $B^0 \to \mu^+\mu^-$ branching fractions in several different SM extensions, mostly different SUSY-based flavor models (which are practically outdated), and the progress of LHC measurements compared with the Tevatron era. Much parameter space has now been eliminated by direct searches at the LHC experiments, but the measurements of $B_s \to \mu^+\mu^-$ and $B^0 \to \mu^+\mu^-$ also contribute as one of the strongest constraints to date, and concurs with direct search. For example: no sign of SUSY.

Although the experiments are not yet paying much attention, and in part because of empirical evidence in support of MFV,[11] it is not easy for an enhancement of $B_d \to \mu^+\mu^-$ as indicated by Eq. (43). A fourth generation [104] of quarks can

9 Recall that the Tevatron collider was shut down in 2011.

10 ATLAS has released a corresponding measurement [101], but with no real excess found. The negative central value of fit has caused concern for some.

11 Or, MFV is based on such evidence: absence of flavor violation other than from CKM paradigm.

achieve it, but it is disfavored by the observation of the 125 GeV Higgs boson. Or one can tune parameters in exotic Higgs representations in SUSY-GUT framework [105], but we have no evidence for SUSY. Very few models can comfortably enhance $B_d \to \mu^-\mu^-$, where the B_d vs B_s ratio test [102] provides another avenue. We note that a fourth generation violates MFV3 (MFV with 3 generations, i.e. SM-like) but not the principle of MFV.

Let us turn to the actual experimental measurement, which is still a work in progress. We note that the situation is different from the Tevatron era, where one was shooting for sizable enhancement above SM, and the situation for signal versus background is quite different. Just to illustrate: had the CDF central value of Ref. [100] been true in Nature, CMS and LHCb would have easily verified it in 2011 already. The challenge now is of course the tiny signal from SM rate, and background reduction is the essential piece of the analysis. The $B_{s,d} \to \mu^+\mu^-$ signal consists of a pair of opposite sign muons from a single displaced vertex, with invariant mass in agreement with the nominal $B_{s,d}$ mass within resolution. The combinatorial background, which arises mostly from muons originating from two different semi-leptonic B meson decays, can be reduced by requiring a good reconstructed vertex, plus low hadronic activity associated with the reconstructed candidate. When one or both of the two muon candidates arise from misidentified hadrons, the situation can be more challenging. In particular, consider B meson decay to two charged hadrons ($B_{s,d} \to K^+K^-, \pi^+\pi^-$ or $K^+\pi^-$), if both hadrons are misidentified as muons, a bump very close to the signal can be produced (peaking background), which is difficult to suppress except by requiring more stringent muon reconstruction quality.

Another important aspect of the analysis is how to normalize the resulting decay rate. A typical choice is the $B^+ \to J/\psi(\to \mu^+\mu^-)K^+$ decay, which has a large, well-measured, branching fraction,

$$\mathcal{B}(B_{s,d} \to \mu^+\mu^-) = \frac{N_{\rm sig}}{N(B^+ \to J/\psi K^+)} \cdot \mathcal{B}(B^+ \to J/\psi K^+) \cdot \frac{\epsilon(B^+)}{\epsilon(B_{s,d})} \cdot \frac{f_u}{f_{s,d}}, \tag{44}$$

where $N_{\rm sig}$ is the signal yield for $B_{s,d} \to \mu^+\mu^-$, and f_u and $f_{s,d}$ are respective hadronization fractions. The detection acceptance times reconstruction efficiency (ϵ) for B^+ and $B_{s,d}$ should be corrected accordingly. The two muons from J/ψ decays can also be used to reduce the systematic uncertainties related to muons. The ratio of B_s and B^+ hadronization composition needs to be determined and additional systematic uncertainty must be introduced for $B_s \to \mu^+\mu^-$ decay. If the $B_s \to J/\psi K^+K^-$ absolute branching fraction can be measured better in the near future (in particular by Belle-II), it can be a good alternative choice of normalization.

A recent update by LHCb [106] that includes both Run-1 and the newly collected 2016 data confirms the observation of $B_s \to \mu^+\mu^-$ decay,

$$\mathcal{B}(B_s \to \mu^+\mu^-) = (3.0 \pm 0.6^{+0.3}_{-0.2}) \times 10^{-9}, \quad \mathcal{B}(B^0 \to \mu^+\mu^-) < 3.4 \times 10^{-10}, \quad \text{(LHCb, up to 2016)} \tag{45}$$

while the excess of $B^0 \to \mu^+\mu^-$ is reduced with respect to the Run 1 only analysis. In the same presentation, LHCb also announced the first effective lifetime measurement for $B_s \to \mu^+\mu^-$, with initial state consisting of only the heavy B_s in SM.

Further investigations are definitely required. The upcoming LHC data would clearly provide better sensitivity, but collisions at higher pile-up (number of pp collisions per beam crossing) rate also degrade the performance, especially for the general purpose CMS experiment. Furthermore, as the measurement of $B_s \to \mu^+\mu^-$ decay is already well established, the new focus would be to have the $B^0 \to \mu^+\mu^-$ rate measured for the first time. The $B^0 \to \mu^+\mu^-$ decay suffers more background contamination, where even $B_s \to \mu^+\mu^-$ constitutes a critical background. An improved dimuon mass resolution is an essential requirement for separating B^0 from B_s events at the upcoming analysis updates. Both CMS and LHCb experiments expect to discover $B^0 \to \mu^+\mu^-$ with HL-LHC operations, if the SM branching fraction is assumed. However, if the actual rate is larger than SM expectation, the first evidence for $B^0 \to \mu^+\mu^-$ decay could be found within the next few years, which makes this program very interesting. Given the historic interest in $B_s \to \mu^+\mu^-$, theorists are encouraged to come up with models that could considerably enhance $B^0 \to \mu^+\mu^-$, as the experimental situation is not settled yet.

3.3. P_5' and $R_{K^{(*)}}$ anomalies in $B \to K^{(*)}\ell^+\ell^-$

FCNC decays such as $B \to K^*\ell^+\ell^-$ are of particular interest since New Physics phenomena can be observed indirectly through their influence in the loop diagrams. Many observables can be probed as a function of $q^2 = m^2_{\ell^+\ell^-}$ with a single decay channel, for example, the differential branching fraction ($d\mathcal{B}/dq^2$), forward–backward asymmetry of the leptons (A_{FB}), and longitudinal polarization fraction of the K^{*0} (F_L). Theoretical calculations for these observables are suitably robust, and any difference between experimental measurements and SM predictions can be interpreted as a hint of New Physics beyond SM (See, e.g. Refs. [107–109]).

Take, for example, the forward–backward asymmetry, A_{FB}. With the observation [25] that the Z-penguin would actually dominate the $b \to s\ell^+\ell^-$ processes with large m_t, A_{FB} was subsequently pointed out [110] as an interesting observable. While one is familiar with A_{FB} measurements at the Z peak, A_{FB} in $B \to K^*\ell^+\ell^-$ is in fact closer to the classic measurement of Ref. [111] that confirmed electroweak theory, only better: the enhanced virtual Z effect is brought down to below m_B^2 to interfere with the virtual photon effect, also loop-induced, and the resulting A_{FB} can be measured within $0 < q^2 < m_B^2$.

Measurement of A_{FB} was first done at the B factories [17], but when Belle announced [112] a 2.7σ deviation from SM expectation in 2009, it caused some sensation. The deviation was mostly in the low q^2 region, where the SM prediction of A_{FB} crosses from positive to negative values around $q^2 = 4\,\text{GeV}^2$. The A_{FB} measured by Belle and BaBar tend to be positive for

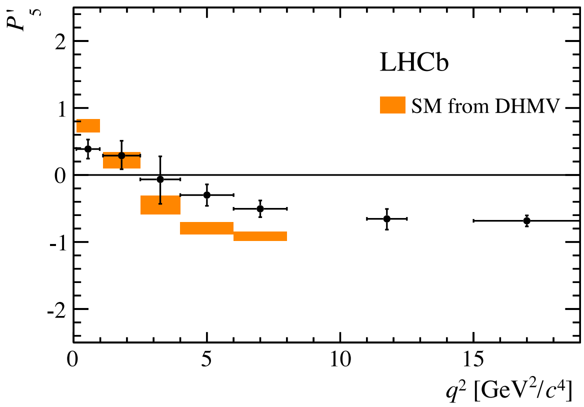

Fig. 5. The P'_5 angular observable in bins of q^2 from LHCb Run 1 data, where some deviation is seen from SM-based predictions (shaded boxes) for 4 GeV$^2 < q^2 <$ 8 GeV^2. See text for discussion.
Source: Courtesy LHCb experiment, from Ref. [114].

all q^2. This "anomaly" was very difficult to accommodate within SM, since the predicted q^2 region of zero-crossing is quite robust. However, the anomaly was *not* confirmed by new LHCb measurement with much larger statistics, and neither by subsequent CMS analysis. Current measurements of A_{FB} are consistent with SM expectations.

But this is definitely not the end of the story, as the $B \to K^*\ell^+\ell^-$ process is rich with observables. To reduce uncertainties from form factors, several new observables were proposed, such as those denoted by P'_n [113] which are claimed to be largely free from form factor uncertainties in the low-q^2 region. Using these observables in the analysis, LHCb found [114] some new discrepancies between measurements and expectations in some q^2 bins in the P'_5 observable, as can be seen from Fig. 5.

The decay has four charged particles in the final state, $B^0 \to K^{*0}\mu^+\mu^- \to K^+\pi^-\mu^+\mu^-$. A full description of the decay requires three angular observables: the helicity angle θ_K of the K^{*0} candidate, the helicity angle θ_ℓ of the dimuon system, and the angle ϕ between the $K^{*0} \to K^+\pi^-$ and dimuon planes. The CP-averaged differential angular distribution can be expressed as,

$$
\begin{aligned}
\frac{1}{d\Gamma/dq^2}\frac{d^4\Gamma}{dq^2 d\Theta} = \frac{9}{32\pi}\Bigg[&\frac{3}{4}(1-F_L)\sin^2\theta_K + F_L\cos^2\theta_K + \frac{1}{4}(1-F_L)\sin^2\theta_K\ \cos 2\theta_\ell \\
&- F_L\cos^2\theta_K\ \cos\ 2\theta_\ell + S_3\sin^2\theta_K\sin^2\theta_\ell\ \cos\ 2\phi + S_4\ \sin\ 2\theta_K\ \sin\ 2\phi_\ell\ \cos\ \phi \\
&+ S_5\ \sin\ 2\theta_K\ \sin\ \theta_\ell\ \cos\ \phi + \frac{4}{3}A_{FB}\sin^2\theta_K\cos\theta_\ell + S_7\sin 2\theta_K\sin\theta_\ell\sin\phi \\
&+ S_8\sin 2\theta_K\sin 2\theta_\ell\sin\phi + S_9\sin^2\theta_K\sin^2\theta_\ell\sin 2\phi\Bigg],
\end{aligned} \tag{46}
$$

where S_n ($n = 3, 4, 5, 7, 8, 9$) are angular observables from the $B^0 \to K^{*0}\mu^+\mu^-$ decay amplitude, which are generally functions of the Wilson coefficients and form factors, hence functions of q^2 as well. The P'_n observables are defined by combining F_L and S_n, with reduced form-factor uncertainties:

$$
P'_n = \frac{S_n}{\sqrt{F_L(1-F_L)}}. \tag{47}
$$

A full angular analysis of $B^0 \to K^{*0}\mu^+\mu^-$ decay is required to determine these observables of interest. Contamination from a possible S-wave component and the associated interference should be taken into account. LHCb carried out the measurements with two different statistical methods, likelihood fit and principle moment analysis, and reported [114] the q^2-dependent angular observables for full Run 1 data, where the measured P'_5 as a function of q^2 is already shown in Fig. 5. There is some deviation between measured values and the SM prediction, denoted as DHMV [115], in the region of $4 < q^2 < 8$ GeV^2. The significance is at 3.4σ level, although the significance of deviation was actually slightly higher in an earlier LHCb publication with 1 fb^{-1} data, which is a cautionary note. A further caution is that, although this deviation could well be due to New Physics, it could still plausibly be explained by introducing large hadronic effects. For example, subsequent to the LHCb result, it was pointed out [116–118] that hadronic effects are very difficult to estimate around the charm threshold, $q^2 \cong 4m_c^2 \simeq 6.8$ GeV^2, precisely the region of deviation indicated by LHCb.

While we prefer to wait for Run 2 data to pan out, the P'_5 anomaly is, however, well known and popular. A representative global fit [119] to New Physics, taking into account many measurements is given in Fig. 6. An order -1 shift in the C_9 Wilson coefficient of one of the Z-penguin operators

$$
O_9 = (\bar{s}\gamma_\mu Lb)(\bar{\ell}\gamma_\mu\ell), \tag{48}
$$

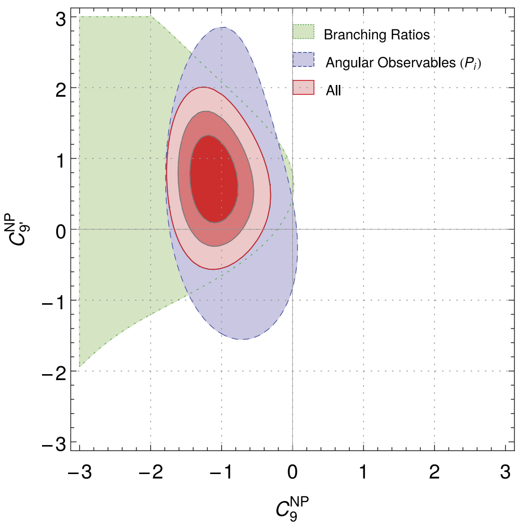

Fig. 6. Combined fit to $B^0 \to K^{*0}\mu^+\mu^-$ angular and rate observables illustrates a possible New Physics shift in the C_9 coefficient by -1, and some shift in other Wilson coefficients.
Source: Courtesy S. Descotes-Genon and J. Matias, from Ref. [119].

seems commonly needed, suggesting the presence of an extra Z' boson. The point was first made in Refs. [120] and [121], and subsequently by many authors. A more recent fit can be found in Ref. [122].

Belle has updated [123] the analysis for both $B^0 \to K^{*0}\mu^+\mu^-$ and $K^{*0}e^+e^-$ modes with full data, using the same approach as LHCb. The result is consistent with LHCb, and a mild discrepancy of 2.6σ is found in a similar q^2 region for the muon channel. However, the electron channel shows better agreement with SM expectations. Very recently, ATLAS and CMS have also measured the P_5' observable. Interestingly, CMS finds [124] better agreement with DHMV predictions with no real deviation in the target q^2 region, hence is at some variance with LHCb (and Belle). The ATLAS result [125] is consistent with LHCb in the $4 < q^2 < 6$ GeV^2 bin and away from DHMV, but does not give results for higher q^2. Currently, LHCb has the best statistical uncertainty, but given the scatter of measured values, and that statistical errors are still quite large, together with the issue of hadronic uncertainties, it seems premature to make a conclusion at this moment. The upcoming LHC and Belle-II data, together with improved SM predictions, should shed further light on this intriguing search.

On a slightly different front, as a test of lepton universality, LHCb measured the ratio of branching fractions of $B^+ \to K^+\mu^+\mu^-$ vs $K^+e^+e^-$ decays [126], in the range of $1 < q^2 < 6$ GeV^2. The ratio, denoted by R_K, is determined to be

$$R_K = 0.745^{+0.090}_{-0.074} \pm 0.036. \quad (\text{LHCb Run 1},\ 1 < q^2 < 6\ \text{GeV}^2) \tag{49}$$

which is slightly below the SM expectation of unity, but still compatible within 2.6σ. This is echoed by the mild deviation of angular distributions between $B^0 \to K^{*0}\mu^+\mu^-$ and $K^{*0}e^+e^-$ modes found by Belle [123]. What has excited many theorists is the recent announcement by LHCb the measurement of $B^+ \to K^{*0}\mu^+\mu^-$ vs $K^{*0}e^+e^-$ decays [127],

$$R_{K^{*0}} = 0.69^{+0.11}_{-0.07} \pm 0.05, \quad (\text{LHCb Run 1},\ 1.1 < q^2 < 6\ \text{GeV}^2) \tag{50}$$

which is in good agreement with Eq. (49), and is by itself of order 2.5σ from SM. Together they offer strong support for the aforementioned shift in C_9. LHCb, however, also gave the number

$$R_{K^{*0}} = 0.66^{+0.11}_{-0.07} \pm 0.03, \quad (\text{LHCb Run 1},\ 0.045 < q^2 < 1.1\ \text{GeV}^2) \tag{51}$$

which is slightly lower but rather similar to Eq. (49), and lower than SM by $\sim 2.2\sigma$. But this poses some puzzle [128] because the lower bin is dominated by $B \to K^{*0}\gamma^*$ that peaks strongly towards small q^2, with a well measured and much larger $B \to K^{*0}\gamma$ parent that should not (QED) distinguish between e^+e^- and $\mu^+\mu^-$. LHCb probably aimed at using this lower bin[12] as "validation", but instead itself showed some disagreement with SM.

Further investigations are clearly needed, and improved measurements from LHCb and Belle II are expected in the near future. Given that several anomalies are dominated by LHCb, a second experiment like Belle II is especially called for to provide cross check.

12 The electromagnetic penguin does not contribute at low q^2 to $B \to K\ell^+\ell^-$.

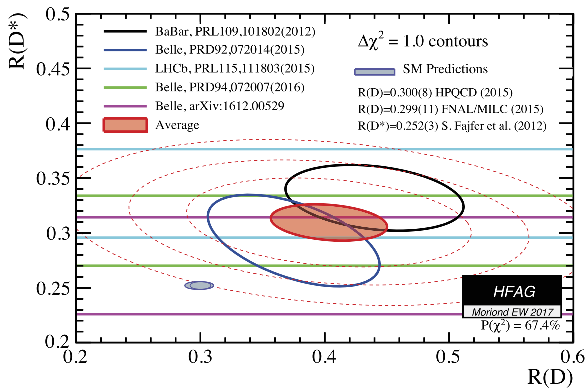

Fig. 7. Current status of R_D and R_{D^*} measurements, illustrating the "BaBar anomaly" in $B \to D^{(*)}\tau\nu$ decays. The average value is shown as a filled ellipse, together with associated uncertainty contours (dashed), which is apparently 4σ away from SM expectations.
Source: HFAG Winter 2017.

3.4. BaBar anomaly: $B \to D^{(*)}\tau\nu$

The measurement of $B \to \tau\nu$ decay is a highlight of the B factories, but by the time of LHC running, it had quieted down, and we defer the discussion to the next section. However, application of the techniques developed for studying $B \to \tau\nu$ lead to a new anomaly [129] uncovered by BaBar in 2012, that the rates and differential distributions in $B \to D^{(*)}\tau\nu$ decays seem to deviate not only from SM, but cannot be understood by charged Higgs boson (H^+) effects from the usual 2HDM-II that arises under SUSY. This has caused a lot of interest, both experimental and theoretical, and the current measurement status is summarized in Fig. 7.

The τ leptons appearing in B decay final states were seldom addressed before the B factory era because of complexity of detection and reconstruction, since there are multiple invisible neutrinos[13] in the final state. But as described in Section 5, $B \to \tau\nu$ emerged as an excellent probe of H^+, and with the large statistics expected at the B factories, Belle and BaBar developed tools to probe such decays. The B factories rely on the coherent production of B meson pairs from the mother $\Upsilon(4S)$ state. "Tagging" one of the B mesons by full reconstruction with either hadronic or semileptonic decays, the kinematics of the other B meson becomes known, which makes it possible to search for signals such as $B \to \tau\nu$. Due to the good coverage of B factory detectors, few missing particles are expected, and the signal can be selected and identified with little extra measured energy, after subtracting off the tagged B meson candidate and the signal τ lepton. A hadronic tag measures full final state four momentum of the tagged B meson and gives better identification, but suffers from low reconstruction/tagging efficiency. On the other hand, the semileptonic decay tag requires a $D^{(*)}$ meson and another charged lepton within an event hence has much higher efficiency, but due to the presence of an additional neutrino on tag side, the kinematic constraints are slightly weaker, resulting in a higher background level. The measurement of $B \to \tau\nu$ (as well as $B \to \mu\nu$) would be discussed in the next section.

The semileptonic $B \to D\tau\nu, D^*\tau\nu$ decays proceed though $b \to c$ transitions, and with branching fractions expected at the couple percent level, they are technically not really rare decays. But the difficulty of detection and the lack of anticipation for any New Physics delayed the study of these decays until late in the B factory era. One can measure the differential partial width in terms of q^2, the squared invariant mass of the τ lepton and neutrino. The lower bound of q^2 depends on the τ mass, while the upper bound is decided by the mass difference between the B and $D^{(*)}$ mesons. By use of ratios of branching fractions, the hadronic uncertainties as well as experimental errors are reduced. The dependence on the CKM element V_{cb} and other common factors also cancel out. The most up-to-date predictions [130,131] of the ratios, R_D and R_{D^*}, are quite accurate,[14]

$$R_D^{\rm SM} = \frac{\mathcal{B}(B \to D\tau\nu)}{\mathcal{B}(B \to D\ell\nu)} = 0.299 \pm 0.003, \tag{52}$$

$$R_{D^*}^{\rm SM} = \frac{\mathcal{B}(B \to D^*\tau\nu)}{\mathcal{B}(B \to D^*\ell\nu)} = 0.252 \pm 0.003, \tag{53}$$

where $\ell = e, \mu$. The experimental measurements of R_D and R_{D^*} rely on studies of the signal $B \to D^{(*)}\tau\nu$ and the normalization channels $B \to D^{(*)}\ell\nu$.

The analyses carried out by BaBar [129,133] and Belle [134,135] are similar to the studies for $B \to \tau\nu$ described above, with one B meson tagged with hadronic or semileptonic decays, but the signal B meson is reconstructed with a charged

13 In the ARGUS and CLEO era, techniques were developed for detecting a single missing neutrino: missing energy/momentum but vanishing missing mass. These techniques were used to measure semileptonic B decays such as $B \to D^{(*)}e\nu$ and $D^{(*)}\mu\nu$.

14 The more recent Ref. [132] gives $R_{D^*} = 0.257 \pm 0.003$, which is slightly higher.

electron or muon, plus a $D^{(*)}$ meson. The signal τ decay $\tau \to \ell\overline{\nu}_\ell\nu_\tau$ gives exactly the same visible final state particles as the normalization channel. The two additional neutrinos from τ decays lead to a wider missing mass distribution calculated by all of the visible particles seen in the event, while the normalization channel has nearly zero missing mass since there is only one neutrino. A very recent Belle measurement [136] of R_{D^*} also looks into hadronic τ decays. Interestingly, LHCb is able to measure $\overline{B} \to D^{*+}\tau\nu$ as well. Although the forward hadron collider environment is not as clean as at the B factories, LHCb reconstructs the B meson with a D^{*+} ($\to D^0\pi^+$) meson plus a muon candidate. The feature of a highly boosted B meson is utilized by requiring a significant distance between the B candidate decay vertex and the proton–proton collision point. Given the missing neutrinos, the momentum of B candidate is in fact unknown, but can be calculated in an approximate way [137].

The HFAG average of all available experimental measurements of R_D and R_{D^*} is already given in Fig. 7. Open ellipses indicate BaBar and Belle analyses with hadronic tag, while other measurements are given by horizontal lines. The average value is shown as a filled ellipse, together with the associated uncertainty contours, which clearly exceeds the SM prediction. The numerical values together with their uncertainties are given by

$$R_D = 0.403 \pm 0.040 \pm 0.024, \tag{54}$$

$$R_{D^*} = 0.310 \pm 0.015 \pm 0.008. \tag{55}$$

Deviations of 2.2σ and 3.4σ from SM predictions (Eqs. (52) and (53)) are found for R_D and R_{D^*}, respectively. Considered together, however, a deviation of 3.9σ is estimated with correlations taken into account.

Could the deviation be due to a statistical fluctuation? More data is needed. But the deviations of R_D and R_{D^*} are enthusiastically embraced by theorists, and the leading candidate New Physics are two Higgs doublet models beyond [138,139] 2HDM-II, and "flavored" leptoquarks (see e.g. Ref. [140]). Note that leptoquarks are also popular for explaining the P_5' and R_K anomalies (see e.g. Refs. [141] and [142]). Both leptoquark and charged Higgs explanations are challenged [143] by direct search for τ pairs at high p_T, while charged Higgs effects to explain the BaBar anomaly tend to cause problems [144,145] with B_c lifetime, or the $B_c \to \tau\nu$ rate. We cannot do justice to the theory activity, but our view is that more data, especially from Belle II and LHCb,[15] is needed to clarify the situation, and that flavored leptoquarks might be more exotic than H^+ beyond 2HDM-II (we will come back to this in Section 8.3).

If the enhancement on the $B \to D^{(*)}\tau\nu$ decay rate is really due to some new particle (or effective operators or operator coefficients that differ from SM), it should modify also the kinematics of the decay itself. Hence a further detailed study of the momentum distribution for the final state particles may provide additional discriminant power. A q^2 dependent measurement of the ratios can indicate how the model should be modified. These extended studies will require much more statistics, which fortunately would become available in the not so distant future. We note in passing that Z and τ decays also lead to strong constraints on models that try to explain R_D and R_{D^*}. See e.g. Ref. [147].

3.5. CPV in three-body B decays

We have emphasized in Section 2 the importance of 3-body Dalitz analysis as part of the "DK" program for extracting the CPV phase angle ϕ_3/γ. In this Highlight section, as an anticlimax, let us discuss CPV in the 3-body phase space of charmless B decays as a highlight of sorts, but to illustrate the depressing nature of the "hadronic menace" over our goal of uncovering New Physics.

CPV rate difference arises when two interfering amplitudes have difference in both CP conserving and CP violating phase. The former usually arise from hadronic or strong interactions, while in SM the latter arise from the CKM matrix. We have mentioned in the Introduction that the large DCPV difference (more than 10%) between $B^+ \to K^+\pi^0$ and $B^0 \to K^+\pi^-$, Eq. (5), could arise from a New Physics CPV phase in the Z penguin. But this hope was dashed by no indication of such phase in the accompanying B_s mixing CPV phase, ϕ_s (Eq. (37), compared with Eq. (35)), and thus one would have to accept that the $B \to K\pi$ DCPV difference is due to the interference of a hadronically enhanced (hence hadronic phase) color-suppressed amplitude C with the dominant strong penguin amplitude P.

LHCb has performed precision analysis of three body charmless $B^+ \to K^+\pi^-\pi^+$, $K^+K^-K^+$, $\pi^+\pi^-\pi^+$ and $\pi^+K^+K^-$ decays with Run 1 data and found significant direct CP asymmetries [148]. For example, the integrated asymmetry is about 6% for $B^+ \to \pi^+\pi^-\pi^+$ with 4.3σ significance. But the CPV asymmetries can be measured in the Dalitz plot, and huge local asymmetries are uncovered. As shown in Fig. 8, the local asymmetries in $B^+ \to \pi^+\pi^-\pi^+$ decays can be over 80%, and change rapidly across the Dalitz plot. As data accumulate with time, these colorful (when seen offline) plots could turn into an artist's palette, but unfortunately such large direct CPV asymmetries can be readily accommodated within SM. The tree level $b \to u\bar{u}d$ process carries the CPV phase, but the strong penguin can rescatter the CKM favored $b \to c\bar{c}d$ process to charmless final states and bring in an amplitude of comparable strength across Dalitz space, and carry varying strong phase. Furthermore, presumably there are resonance interference and final state rescattering effects between all kinds of charmless final states. Thus, even if there is an effect from some New Physics, it will be fully enveloped by these hadronic effects, and there is no way to disentangle them. Beauty may not tell the truth.

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15 A result from LHCb [146] for FPCP 2017 using $\tau^\pm \to \pi^\pm\pi^+\pi^-(\pi^0)\nu$ gives $R_{D^*} = 0.285 \pm 0.019 \pm 0.029$, which is lower in value than using the muonic decay of τ, and the HFLAV (previously HFAG) combination is now $R_{D^*} = 0.304 \pm 0.013 \pm 0.007$, compared with Eq. (55). The "BaBar anomaly" is still quite volatile on experimental side.

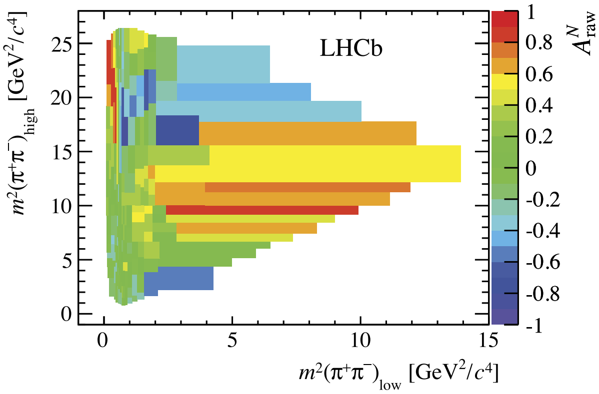

Fig. 8. The measured raw asymmetry in $B^+ \to \pi^+\pi^-\pi^+$ decay Dalitz space, where DCPV asymmetries can reach $O(1)$.
Source: Courtesy LHCb collaboration, from Ref. [148].

4. Beauty and dark

In this section we start with some charmless hadronic rare B decays that remain of interest, then turn to various processes that provide probes to New Physics beyond the SM, some of which provide stringent constraints. We also briefly cover the search for dark particles at the B factories.

4.1. Charmless hadronic

One curiosity before the B factory era was the absence of B decays to charmless baryonic final states, compared with e.g. $B \to K\pi, \pi\pi$ decays. This has to do with the effective four-fermion interaction structure of weak decay, while the composition of baryons is more complicated than mesons. It was suggested [149] that, because of extra suppression of two body modes, perhaps charmless baryonic decays would first emerge in 3-body final states. Sure enough, the first observation of B meson decay to charmless baryonic final states was the $p\bar{p}K^+$ mode discovered by Belle [150] in 2002, which opened a new subfield of B physics with its interesting topics of study. Despite search efforts at the B factories [17], it was not until the LHC era and with the huge statistics of LHCb, that first evidence for two-body $B \to p\bar{p}$ [151] and $p\bar{\Lambda}$ [152] decays start to emerge at the 10^{-8} and 10^{-7} level, respectively. The B meson is the only meson that can have baryons in its decay final state. This notion led to the suggestion [153] to search for baryon number violation (BNV) in heavy meson or fermion decays, but it was realized that these were already highly constrained by the extremely strong bounds on proton decay.[16]

Hadronic uncertainties make it difficult to examine hadronic B decays for physics beyond SM, as we have seen for the $B \to K\pi$ direct CPV difference. There are two CPV variables that meet the purpose better. The first is S_f in time dependent CP violation analysis as discussed in Section 2.2.1. The S_f values in the tree-dominated $b \to c\bar{c}s$ processes and the penguin dominated $b \to sq\bar{q}$ processes are expected to be very similar. Any significant deviation (ΔS) from the mean of $b \to c\bar{c}s$ processes may indicate New Physics phase in the loop.

Among the various $b \to s$ penguin decay modes, two channels are the most promising: $B^0 \to \phi K^0$ and $B^0 \to \eta' K^0$. The former is from pure $b \to s\bar{s}s$ penguin transition, while the latter receives tree contributions but provides the largest rate. The current experimental averages from BaBar and Belle are $S_{\phi K^0} = 0.74^{+11}_{-13}$ and $S_{\eta' K^0} = 0.65 \pm 0.06$ [40], respectively, consistent with $\sin 2\phi_1 = 0.691 \pm 0.017$ determined in charmonium plus K^0 modes. The C_f values in the three cases are all consistent with zero, showing no direct CP violation. The study of such ΔS values became one of the major analysis subjects at the B factories since 2003, and especially when Belle experienced a sign flip of $S_{\phi K^0}$ in the summer update of 2004. Fig. 9 shows the S values measured from various decays that receive contributions from $b \to s$ penguins. The naive average from all $b \to sq\bar{q}$ decay modes gives [40]

$$S(b \to sq\bar{q}) = 0.68^{+0.09}_{-0.10}, \tag{56}$$

which is in good agreement with Eq. (26). It remains one of the clean channels to probe for New Physics in the future. One would need at least an order of magnitude more data to tell if ΔS really deviates from zero. The verdict will be reached by Belle II, with the help of LHCb, in the 2020s.

The second possible approach to probe New physics in charmless hadronic B decays is still through the DCPV asymmetries such as in $B \to K\pi$ modes. As we have seen in Eq. (5), large DCPV difference between the neutral and charged B decays is experimentally well established, which is the difference between $A_{CP}(K^+\pi^-) = -0.082 \pm 0.006$ for $B^0 \to K^+\pi^-$ (Eq. (6))

[16] This did not stop, for example, the search for BNV in top decays by CMS [154].

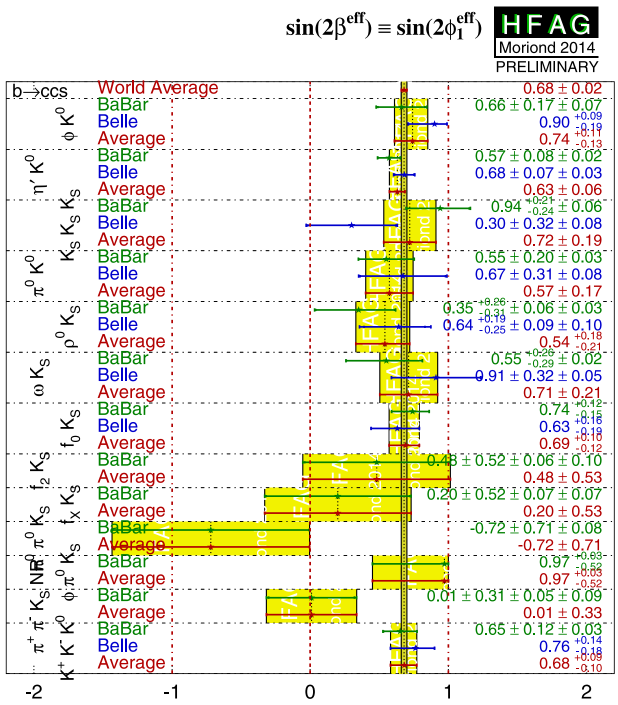

Fig. 9. The S values for decay modes related to $b \to s$ penguin, superimposed with the well measured $\sin 2\phi_1$ from $b \to c\bar{c}s$ modes (narrow vertical band). *Source:* HFAG Winter 2014.

and $A_{CP}(K^+\pi^0) = 0.040 \pm 0.021$ for $B^+ \to K^+\pi^0$. The decay $B^+ \to K^+\pi^-$ is expected to proceed through external W-emission tree diagram T and gluon mediated penguin diagram P, as illustrated in Fig. 2(a) and (b). These diagrams also govern $B^+ \to K^+\pi^0$ decay, but extra diagrams called color-suppressed tree C and electroweak penguin $P_{\rm EW}$, Fig. 2(c) and (d), contribute in addition. The electroweak penguin effect is rather small in SM, and furthermore, it does not bring in any CPV phase. At the start of the millennium, it was generally thought that the magnitude of C is small compared to T (recall the $1/N_C$ expansion of QCD), and carry the same CPV phase, so it was expected that the CP asymmetries for $B^+ \to K^+\pi^0$ and $B^0 \to K^+\pi^-$ decays should be rather similar, i.e. the naive expectation was $\Delta A_{K\pi} \simeq 0$. With the observation of a distinctively large difference, Eq. (5), the explanation can come from either large color-suppressed tree through hadronic effects, or from large electroweak penguin effect with New Physics CPV phase, which would be more exciting. although the electroweak, or Z, penguin and the box diagrams for B_s–$\bar{B}_s$ mixing should probe similar physics. But this reasoning is not foolproof.

One way to test which explanation is correct is to check a CPV asymmetry sum rule [155,156],

$$A_{K^+\pi^-} + A_{K^0\pi^+}\frac{\mathcal{B}(K^0\pi^+)}{\mathcal{B}(K^+\pi^-)}\frac{\tau_0}{\tau_+} = A_{K^+\pi^0}\frac{2\mathcal{B}(K^+\pi^0)}{\mathcal{B}(K^+\pi^-)}\frac{\tau_0}{\tau_+} + A_{K^0\pi^0}\frac{2\mathcal{B}(K^0\pi^0)}{\mathcal{B}(K^+\pi^-)}, \tag{57}$$

where τ_0 and τ_+ are B^0 and B^+ lifetimes, and $\mathcal{B}(K\pi)$ is the branching fraction of $B \to K\pi$. What is missing is a precision measurement of $A_{K^0\pi^0}$.

The branching fractions of all $K\pi$ decays will become precision measurements in the near future. The asymmetry $A_{K^0\pi^+}$ is expected to be zero due to its pure penguin nature, and the current combined result, $A_{K^0\pi^+} = -0.017 \pm 0.016$ [40] supports the SM expectation. The crucial information for checking the sum rule of Eq. (57) is $A_{K^0\pi^0}$, where time-dependent CP asymmetry analysis is needed. The average asymmetry from BaBar and Belle is

$$A_{K^0\pi^0} = 0.01 \pm 0.10, \tag{58}$$

where the statistical uncertainty is around 0.13 for each experiment. If we assume Eq. (57) holds and $A_{K^0\pi^+} = 0$, the expected CP violating asymmetry of $B^0 \to K^0\pi^0$ is

$$A_{K^0\pi^0} = -0.129 \pm 0.027, \quad \text{(sum rule projection)} \tag{59}$$

which is sizeable and negative. The dominant uncertainty comes from $A_{K^+\pi^0}$, which will be reduced in the future. Since there is a π^0 in the final state for $B^0 \to K^0\pi^0$, only Belle II would be able to provide the update. To test the sum rule at the error of Eq. (59), assuming systematic errors will improve accordingly, one would need a data sample ∼20 times larger than at the B factories, which seems within reach at Belle II.

We have mentioned the rather large local asymmetries measured [148] by LHCb in three body charmless $B^+ \to K^+\pi^-\pi^+$, $K^+K^-K^+$, $\pi^+\pi^-\pi^+$ and $\pi^+K^-K^+$ decays. Given the complex rescatterings of the final states involved, a three body sum rule analogous to Eq. (57) may seem impractical. But the observed asymmetries seem paired in sign, i.e. $A_{CP}(B^+ \to K^+\pi^-\pi^+) \simeq +0.025$ vs $A_{CP}(B^+ \to K^+K^-K^+) \simeq -0.036$, and $A_{CP}(B^+ \to \pi^+\pi^-\pi^+) \simeq +0.058$ vs $A_{CP}(B^+ \to \pi^+K^-K^+) \simeq -0.12$. Assuming that these four modes reflect inclusive $b \to s\bar{q}q$, $s\bar{s}s$, and $b \to d\bar{q}q$, $d\bar{s}s$, where $q = u,\ d$, one can check a rather old perturbative two-loop sum rule [157], that inclusive asymmetries for charmless $b \to s$ and $b \to d$ each largely cancel between tree dominant and penguin dominant modes. A naive check using the LHCb result, without taking into account momentum-dependent K^+ vs π^+ detection efficiencies, we find the cancellation to work well for inclusive charmless $b \to s$ decays, but slightly less well with inclusive charmless $b \to d$ decays. But this check needs to be done in more detail across the Dalitz plot, while the V_{ub} and V_{td} values used in Ref. [157] should be updated.

In this vein, we recall that inclusive charmless $b \to s\bar{q}q$ (where q now stands for u, d and s) decay is predicted robustly at the 1% level, with $b \to d\bar{q}q$ slightly smaller. The rate for charmless $b \to s\bar{q}q$ is dominated by the timelike [158] penguin (Fig. 2(b) with the spectator quark line removed), while charmless $b \to d\bar{q}q$ penguin is tree-dominant. Direct measurement of these inclusive rates, which are at the same footing as $b \to s\ell^+\ell^-$ and $b \to u\ell\nu$, should be contemplated. Furthermore, as we will discuss $b \to s\gamma$ in a following subsection, we mention that the analogous lightlike penguin, $b \to sg$, where the emitted gluon is "on-shell" ($m_{g^*} < 1$ GeV), is predicted at the 0.2% level.[17] This is an important rare B decay that we do not have a good handle on, but hopefully can be addressed in the super B factory era to come. Just like the BaBar anomaly in the long neglected $B \to D^{(*)}\tau\nu$ mode, the difficulty to measure may be hiding some New Physics. Do we know it is not 1%, rather than 0.2%?

4.2. Semileptonic

As mentioned in Sections 2 and 3, there are unresolved tensions and even new physics hints in semileptonic B decays. The 2.9σ and 2.6σ discrepancies between exclusive and inclusive measurements of $|V_{cb}|$ and $|V_{ub}|$, respectively, need to be resolved by combined experimental and theoretical efforts, as currently experimental and theoretical uncertainties are on similar footing. Although it is unlikely to find New Physics by measuring semileptonic $b \to c\ell\nu$, $u\ell\nu$ decays, and that the discrepancies are likely due to our lack of knowledge about nonperturbative hadronic effects, V_{cb} and V_{ub} are fundamental parameters, which need to be precisely determined to test the CKM paradigm. Measuring $|V_{cb}|$ and $|V_{ub}|$ is one of the topmost missions in flavor physics, and certainly a subject of high priority at Belle II, with many workshops and private meetings dedicated to their determination.

The combined 3.9σ discrepancy from SM in R_D and R_{D^*} in $B \to D\tau\nu$, $D^*\tau\nu$ decays is very enticing, and needs to be further addressed and confirmed with better significance. The analysis method is already proven to work and measuring τ polarization [136] in $B \to D^*\tau\nu$ is another triumph. Besides requiring more data, the systematic uncertainties that arise from the unmeasured backgrounds, such as $B \to D^{**}\ell\nu$ and hadronic B decays, need to be reduced by measuring these branching fractions. With both Belle II and LHCb data, the R_D, R_{D^*} anomaly should be confirmed or disproved by early 2020s.

Two possible hints of New Physics from $B \to K^{(*)}\ell^+\ell^-$ decays are discussed in Section 3.3. The P_5' and $R_K^{(*)}$ issues should reach a conclusion with LHCb, CMS, ATLAS and Belle II data by early 2020s. A similar decay that sheds light on New Physics is $B \to K^{(*)}\nu\bar{\nu}$, where the branching fractions in SM are estimated with smaller theoretical uncertainty due to absence of long-distance effects from electromagnetic penguin contributions. Besides revealing the same possible New Physics that contributes to the decay $B \to K^{(*)}\ell^+\ell^-$, the requirement of missing four-momentum with missing mass, owing to the unobserved $\nu\bar{\nu}$ pair, can imitate the signature of other weakly interacting particles, such as low mass dark matter particles, right-handed neutrinos, or SUSY particles. Therefore, The decay rates or other observables should be measured as a function of the dineutrino momentum squared (q^2). Technically, background contaminations differ for each q^2 bin, while binning also depends on available statistics. Therefore, it is better to optimize the event selections bin by bin, rather than throwing a common requirement for all events.

Experimentally, much like $B \to \tau\nu$ decay, the $B \to K^{(*)}\nu\bar{\nu}$ decays are identified by fully reconstructing the accompanying B meson, and requiring a reconstructed $K^{(*)}$ meson without any other particle in the event. Although tagging the other B reduces the efficiency, the kinematic variable q^2 can be resolved as $q^2 = (P_{\rm beam} - P_{\rm Btag} - P_{K^{(*)}})^2$. BaBar has performed a study with 10 bins in q^2 [160], and the result shows a small excess (6 events with 2.9 background) in the low q^2 bins, $0 < q^2/m_B^2 < 0.3$ for the $B^+ \to K^+\nu\bar{\nu}$, although the lowest bin for both $K^+\nu\bar{\nu}$ and $K^{*0}\nu\bar{\nu}$ indicate excess. Since the significance for four $B \to K^{(*)}\nu\bar{\nu}$ decay channels are low, the branching fraction upper limits at 90% C.L. are given from 3.2×10^{-5} to 11.5×10^{-5} [160], even a two-sided bound, and potentially an order of magnitude higher than SM expectations. Although the significance of the excess is small, one should make diligent updates in the future. Belle should perform a binned analysis with existing data and compare with BaBar, while the order of a hundred events is expected by the end of Belle II, and the situation can be better clarified. A model discussion of the BaBar hint is given in Section 5.2.

[17] Given the agreement of $b \to s\gamma$ with SM, there is little room for large enhancement. But enhancement up to 10% level [159] had been entertained in the CLEO era, though the original reasons have by now evaporated.

4.3. Leptonic and radiative

Leptonic Decays

Unlike charmless hadronic B decays, purely leptonic B decays are largely free from hadronic uncertainties, and are therefore ideal for New Physics search. For neutral B mesons, the decay $B_s^0 \to \mu^+\mu^-$ is finally observed by CMS and LHCb, as discussed in the highlight section, Section 3.2. As already commented there, the LHC experiments can eventually measure the even rarer $B^0 \to \mu^+\mu^-$ with the Phase II upgrade, if the value is at SM expectation. But discovery could come earlier if the rate is enhanced, and background issues understood. Belle II is expected to accumulate around 5 to 10×10^{10} $B\overline{B}$ pairs, which is at the threshold of evidence according to the SM expectation at 10^{-10} level, Eq. (41). But again, if the rate is enhanced, Belle II can also be part of the game, in a much cleaner environment. So, the LHC experiments should strive to overcome background issues for $B^0 \to \mu^+\mu^-$.

Searches for $B_{(s)}^0 \to \tau^+\tau^-$ have been conducted by BaBar and LHCb using 2.32×10^8 $B\overline{B}$ pairs and 3 fb^{-1}, respectively. The obtained upper limits are all at 10^{-3} level [161,162]. These are around four to five orders higher than SM expectations, $\mathcal{B}(B^0 \to \tau^+\tau^-) = (2.22 \pm 0.19) \times 10^{-8}$ and $\mathcal{B}(B_s^0 \to \tau^+\tau^-) = (7.73 \pm 0.49) \times 10^{-7}$ [94]. Unless there is a breakthrough in analysis technique, the experimental measurements for $B_{(s)}^0 \to \tau^+\tau^-$ cannot reach the SM value in the next 10 years. Searching for $B_{(s)}^0 \to e^+e^-$ is more challenging since it is helicity suppressed and expected to be extremely small. Lepton flavor violating neutral B decays such as $B_{(s)}^0 \to \mu\tau$ can also be entertained.

The situation for the charged B case would have a neutrino in the final state, with $B^- \to \tau^-\bar{\nu}$ already observed, which was a highlight at the B factories. The decay of $B \to \tau\nu$ proceeds through $b\bar{u}$ weak annihilation, and the decay branching fraction is estimated rather well,

$$\mathcal{B}^{\rm SM}(B^- \to \tau^-\overline{\nu}_\tau) = (0.75^{+0.10}_{-0.05}) \times 10^{-4}, \tag{60}$$

based on inputs from lattice calculations and unitarity-constrained $|V_{ub}|$ value from other measurements [85]. Both BaBar and Belle studied $B \to \tau\nu$ decays as data accumulated, and earlier measurements [163–165] hinted at higher central values compared with Eq. (60), which drew a lot of attention from both theory and experiment communities. With improvements in reconstruction and tagging techniques, analyses based on the final Belle data set [166,167] show a weaker tension with SM. The combination of the best available measurements is compatible with Eq. (60) within 1.4σ,

$$\mathcal{B}(B^- \to \tau^-\bar{\nu}_\tau) = (1.06 \pm 0.19) \times 10^{-4}. \tag{61}$$

Instead of showing an anomaly in the decay of $B \to \tau\nu$ itself, the result provides a strong constraint on the extended Higgs sector, using the simple multiplicative factor [168] in the (SUSY type) two Higgs doublet model, 2HDM-II,

$$\mathcal{B}^{\rm 2HDM\text{-}II} = r_H\, \mathcal{B}^{\rm SM}(B^- \to \tau^-\bar{\nu}_\tau), \qquad r_H = \left(1 - \tan^2\beta\, \frac{m_{B^+}^2}{m_{H^+}^2}\right)^2, \tag{62}$$

where $\tan\beta$ is the ratio of vacuum expectation values of the two Higgs doublets, which, like m_B and m_{H^+}, is also a physical parameter in the model. A large swath in the m_{H^+} vs $\tan\beta$ plane is excluded by Eq. (61). The measurement is expected to improve with the upcoming Belle II data. If the current central value stays, it might hint at a deviation from SM. Otherwise, it would put an even stronger constraint on the charged Higgs sector, or on any similar New Physics process. This is an unusual tree level effect. But the situation is a little volatile, because the BaBar anomaly seems to rule out 2HDM-II, and perhaps the extended Higgs sector is more intricate, which we would return to comment in Section 8.3. Regardless, measurement of $B \to \tau\nu$ would provide a useful probe in the future.

We remark that, assuming SM, the $|V_{ub}|$ value determined from the $B \to \tau\nu$ branching fraction is similar to the combined inclusive and exclusive $|V_{ub}|$ determination, both in value and uncertainty. The next in line is $B^- \to \mu^-\bar{\nu}$, which is simpler to measure, except the branching fraction is more suppressed. The expected ratio of branching fractions for the $\mu\nu_\mu$ and $\tau\nu_\tau$ modes depends simply on the B meson and lepton masses for SM or the popular 2HDM-II,

$$\mathcal{B}(B \to \mu\nu_\mu)/\mathcal{B}(B \to \tau\nu_\tau) = m_\mu^2(m_B^2 - m_\mu^2)/m_\tau^2(m_B^2 - m_\tau^2). \quad \text{(SM \& 2HDM II)}. \tag{63}$$

Using the current averaged branching fraction of Eq. (61), one gets the expected branching fraction $\mathcal{B}(B^- \to \mu^-\nu_\mu) = (4.87 \pm 0.87) \times 10^{-7}$.

Since the muon momentum in the B rest frame is monoenergetic with vanishing recoil mass, one can utilize this information with other variables to identify the signal. Both BaBar [169] and Belle [170] searched for $B \to \mu\nu_\mu$ decay and the obtained branching fraction upper limits are at the 10^{-6} level. On closer scrutiny, however, Belle's limit of

$$\mathcal{B}(B \to \mu\bar{\nu}) < 1.7 \times 10^{-6}, \quad (\text{Belle},\ 277\text{M}\ B\overline{B}) \tag{64}$$

at 90% C.L. is based on only a fraction of the full data, while the BaBar limit of

$$\mathcal{B}(B \to \mu\bar{\nu}) < 1.0 \times 10^{-6}, \quad (\text{BaBar},\ 468\text{M}\ B\overline{B}) \tag{65}$$

at 90% C.L. uses their entire data set. Belle has not made a general full update with the 772M $B\bar{B}$ at hand, but instead analyzed full data using [171] hadronic B tag (full reconstruction) on the other B meson. This is, however, counterproductive, since the efficiency of B tag is at 10^{-3} level, and the reason it was developed was for events like $B \to \tau\nu$ and $B \to D^{(*)}\tau\nu$ where there is missing mass in addition to missing energy. Taking the tagging efficiency hit on such a rare decay, no wonder the limit obtained with 2.9 times the data used in Ref. [170], is poorer, i.e. worse than Eq. (64). The method used by BaBar, and the 2007 paper by Belle, dates to CLEO, which reconstructs all the energy and momentum of the other B more loosely, demanding no extra missing energy, hence total missing mass is zero. In this way, the low efficiency of full reconstruction is avoided.

Judging from the BaBar limit, Eq. (65), is only a factor of two above SM, assuming Belle can do similarly with a larger data set, and given the tendency of enhancement in $B \to \tau\nu$ (Eq. (61)), a hint for $B \to \mu\nu$ may well emerge with B factory data, especially if one could combine the Belle and BaBar data sets. Even with the few $\times 10^{10}$ $B\bar{B}$ events in the future of Belle II, one should avoid using full reconstruction of the accompanying B meson for this channel. Such a requirement will simply kill most of the signal events. The point has been made in the FPCP2012 theory summary [172], but unfortunately it still has not been followed through by Belle. We could further emphasize that measurement of $B \to \mu\bar{\nu}$ is a pursuit in itself. After all, the BaBar anomaly points beyond the usual 2HDM-II, and a deviation of $\mathcal{B}(B \to \mu\nu_\mu)/\mathcal{B}(B \to \tau\nu_\tau)$ from Eq. (63) would be rather interesting.

Note Added: As announced at EPS 2017, Belle has finally performed [173] a regular (untagged) search for $B \to \mu\nu$ with full data set, finding the value $\mathcal{B}(B \to \mu\bar{\nu}) = (6.46 \pm 2.22 \pm 1.55) \times 10^{-7}$, which has a significance of 2.4σ. Belle and BaBar should now follow through with a B factory combination as suggested in Ref. [172]. It seems that, before long, $B \to \mu\nu$ search would be an end in itself at Belle II.

Radiative Decays

Radiative B decays provide important information on flavor physics in SM, but are also sensitive to New Physics in the penguin loop. $B \to K^*\gamma$ was the first electromagnetic penguin $b \to s\gamma$ transition to be observed, in 1993 by CLEO [174]. The inclusive $B \to X_s\gamma$ branching fraction were subsequently measured by CLEO, then BaBar and Belle, providing a star constraint on H^+ bosons, *from flavor physics*. Technically, the $B \to X_s\gamma$ signal is identified by requiring a minimum energy cut on photon candidates in the e^+e^- center of mass frame, and then either reconstruct X_s with a kaon plus one to four pions, or estimate the number of background events from off-resonance data for $e^+e^- \to q\bar{q}$ ($q = u, d, s, c$ quarks), and background photons from π^0 as well as η decays in B events. The measured branching fractions are then extrapolated to $E_\gamma > 1.6$ GeV using the method given in Ref. [175]. The averaging is then performed by HFAG, which currently gives

$$\mathcal{B}(B \to X_s\gamma)|_{E_\gamma > 1.6} = (3.49 \pm 0.19) \times 10^{-4}, \tag{66}$$

where the systematic uncertainty dominates. This inclusive rate provides a rather stringent bound on charged Higgs mass that is independent of $\tan\beta$ in 2HDM-II. Using the current NNLO calculations [176], Eq. (66) gives the limit of

$$M_{H^+} > 480\ \mathrm{GeV/c^2}, \quad (b \to s\gamma,\ \mathrm{2HDM\ II}) \tag{67}$$

at 95% C.L. for 2HDM-II. While sophisticated NNLO calculations, requiring the participation of a large number of theorists (18 for Ref. [176]), is needed for proper numerical extraction, the reason that the above limit is powerful is similar to why $b \to s\gamma$ receives large enhancement [177,178] from QCD corrections: suppression of the leading contribution by power GIM cancellations, where QCD alleviates at α_s order by bringing in logarithms, or H^+ effects which can therefore have large impact. The independence of $\tan\beta$ is even more subtle. It is because the dipole transition is of $\sigma_{\mu\nu}m_bR$ form, where R is the right-handed projection. To extract an m_b, it could be by Yukawa coupling of the H^+ to top at one side of the loop, while to bottom at the other side to give m_b factor. Such terms would bring, in 2HDM-II, a $\cot\beta$ factor to counterbalance a $\tan\beta$ factor, hence become $\tan\beta$ independent.[18] It so happens that the effect carries the same sign as short distance SM effect, so always enhances the amplitude [179,180], hence sensitively probes m_{H^+}, independent of $\tan\beta$.

In fact, a recent Belle update [181] of $B \to X_s\gamma$ is slightly lower than SM expectation, which results in a more stringent bound [182] of $m_{H^+} > 570$ GeV in 2HDM-II, raising issues also of theory–experiment correspondence (such as photon energy E_γ cut). Such high masses are not yet sufficiently probed by direct search. Together with challenges to 2HDM-II such as from the BaBar anomaly, perhaps one should broaden scope in considering $b \to s\gamma$ bound on charged Higgs mass.

Let us refrain from discussing inclusive and exclusive $B \to X_d\gamma$ modes, as it is quite challenging for the inclusive case. Many exclusive channels of $b \to s\gamma$ decays have been measured, including mesons or baryons from s-quark fragmentation, and measuring the M_{X_s} spectrum for inclusive process. Inclusive and exclusive $B \to X_{s,d}\gamma$ decays are a mainstay for the B factories and Belle II. What we wish to stress is that, during the B factory era, the experimental effort that measured $B \to \tau\nu$, and the joint experiment–theory co-development for studying inclusive $b \to s\gamma$, were two highlights that together provide much more stringent constraints on the charged Higgs boson H^+ than is currently achieved at the LHC by direct search. This will continue for a while into the Belle II era. It is a domain that LHCb has less to say. But we caution again that, with the BaBar anomaly in $B \to D^{(*)}\tau\nu$, perhaps the enlargement of the Higgs sector is different from 2HDM-II that is championed by SUSY, and we look forward to further developments in the coming few years.

[18] This effect therefore does not [179] happen for 2HDM-I.

4.4. Dark connection

The overwhelming evidence for dark matter and the long standing muon $g-2$ anomaly may suggest particle physics beyond SM. The extension of SUSY is a natural possibility that offers to explain both effects. However, lack of evidence from LHC direct search may imply that it might be some other extension. Recently, models of the dark sector introduce a new hidden U(1) interaction, under which the dark matter particles are charged [183–185]. An Abelian gauge field, the dark photon A', couples the dark sector to SM particles through kinetic mixing [186] with hypercharge of SM. Since the dark sector models do not by themselves constrain the dark photon mass, dark sector particles with masses ranging from a few MeV/c^2 to few GeV/c^2 can be probed at B factories.

LHC, B factories, beam dump and kaon experiments have all conducted searches for dark sector particles. The high luminosity B factory with a clean e^+e^- environment is an ideal place for dark particle search. The searches for dark particles by BaBar and Belle can be divided into four different categories:

1. Search for dark photon in $e^+ + e^- \to \gamma + A'$ with A' decaying into two charged leptons or invisible final states [187,188].
2. Search for dark Higgs (h') in Higgsstrahlung process, $e^+ + e^- \to A' + h'$, $h' \to A'A'$ [189,190].
3. Search for muonic dark force, $e^+ + e^- \to \mu^+\mu^- Z'$, $Z' \to \mu^+\mu^-$, where Z' is a dark boson coupling only to 2nd and 3rd generation leptons [191].
4. Search for dark vector gauge boson in $U' \to \pi^+\pi^-$ in $D^0 \to K_S^0\eta$, $\eta \to U'\gamma$ [192].

Although no signals were observed in any search, much parameter space has been either excluded or given upper limits. We will give some explicit discussion of dark boson search with kaons in Section 5.2, which also relates back to B physics as probe. There, we would also give some further discussion on the Dark connection of flavor physics.

The goal of B factory experiments is to study Flavor Physics and CP Violation, aiming at finding New Physics. The large $e^+ + e^- \to q + \bar{q}$ sample provides opportunities to do other physics. Some may be within Standard Model, such as QCD fragmentation, but some may provide a clean probe for New Physics. Dark sector search certainly belongs to the latter case. The extension to the Standard Model in the Dark direction may have impact on flavor physics. If some dark sector model is confirmed, it is important to understand flavor physics and the dark sector model together in a coherent way.

5. Strange

As mentioned in the Introduction, kaon physics is the granddaddy and progenitor of much of FPCP. Yet it has not run out of its bag of tricks, namely $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$, the pursuit of which is quite at the forefront of NP probes [193], and constitutes a precision test of SM. These two processes are induced by the Z-penguin (and associated box diagram) in SM and theoretically clean, i.e. precisely what is left from nondecoupled top after GIM cancellations, and does not suffer much from hadronic uncertainties [194],

$$\mathcal{B}^{\rm SM}(K^+ \to \pi^+\nu\bar{\nu}) = (8.4 \pm 1.0) \times 10^{-11}, \tag{68}$$

$$\mathcal{B}^{\rm SM}(K_L \to \pi^0\nu\bar{\nu}) = (3.4 \pm 0.6) \times 10^{-11}. \tag{69}$$

These predicted SM rates do depend on CKM angles and CPV phase ϕ_3, the dependence of which we do not exhibit. We focus on the experimental pursuit, plus a twist that was uncovered recently that augments the prospects for the K_L process. Besides these and other topics, it is interesting that ε'/ε has resurfaced again to be on the forefront. This is due to decades of improved lattice calculations, which we mention only briefly.

5.1. $K^+ \to \pi^+\nu\bar{\nu}$ and NA62

In the long experimental pursuit at Brookhaven, E787 and E949 observed altogether seven $K^+ \to \pi^+\nu\bar{\nu}$ decay events, giving the value [195,196]

$$\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu}) = \left(17.3^{+11.5}_{-10.5}\right) \times 10^{-11}, \quad \text{(E787/E949)} \tag{70}$$

which is on the high side of, but not inconsistent with, Eq. (68). As shown in Fig. 10, the seven signal events are in two signal boxes straddling a rather "bright" region that reflects $K^+ \to \pi^+\pi^0$ background events. The mild excess of Eq. (70) motivated the NA62 experiment at CERN SPS, with the main goal of making 10% (assuming Eq. (68)) measurement of $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$, or a total of $\sim 10^{13}$ K^+ decays.

NA62 opts for K^+ decay in flight, compared with decay at rest for E787/E949. The signal boxes are quite similar to E787/E949, however, and require precision kinematic reconstruction of $m^2_{\rm miss} = (P_{K^+} - P_{\pi^+})^2$. Upstream there is kaon ID for the K^+ beam, with precision measurement of the 75 GeV beam momentum using Si pixel beam tracker. Timing is of essence, as the K^+ then enters a 65 m long decay region, and downstream one has charged particle tracking with excellent timing, $e/\mu/\pi$ PID, and hermetic photon rejection. One selects a solo downstream track that matches the beam track and calorimeter energy, with $p_{\pi^+} < 35$ GeV to optimize $K^+ \to \mu^+\nu$ rejection, as well as photon rejection since $p_{\pi^0} > 40$ GeV from $K^+ \to \pi^+\pi^0$ decay. After detector commissioning in 2014, low intensity runs were conducted in 2015 for detector

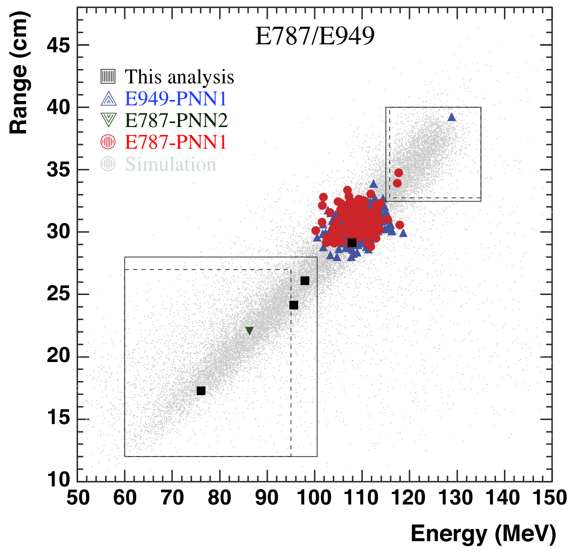

Fig. 10. Seven $K^+ \to \pi^+\nu\bar{\nu}$ events observed by E787/E949 on either side of $K^+ \to \pi^+\pi^0$ backgrounds.
Source: Courtesy E949 Collaboration, from Ref. [196], copyright APS.

quality verification. The upshot is that timing and kinematic resolutions, as well as μ/π separation (by RICH detector) are all close to design, while calorimeter studies are still ongoing. With the low statistics of 2015 run, photon rejection already gives π^0 rejection at $O(10^6)$, where target is $O(10^8)$. With commissioning over, the 2016 data should be enough to address the latter target.

NA62 has an approved run program until end of 2018, i.e. until LS2 of LHC.[19] The 2016 data (which is up to 30% nominal beam intensity) is expected to reach SM sensitivity, the 2017 data should be able to surpass the precision reached at Brookhaven considerably, while by end of 2018, the data should allow NA62 to reach 10% precision of SM (i.e. $O(100)$ events). Thus, one can expect that, by time of LS2, our knowledge of $K^+ \to \pi^+\nu\bar{\nu}$ should improve dramatically. NA62 could confirm the enhancement suggested by Eq. (70) — New Physics, indicate a significant deviation from Eq. (68), or confirm this SM prediction. There is much to look forward to.

With active charged particle and photon detection plus PID capabilities, NA62 can do a lot more than $K^+ \to \pi^+\nu\bar{\nu}$ search, and with a total of 10^{13} K^+ decays, the topics studied depends much on trigger bandwidth hence strategy. Let us discuss a few examples. What first comes to mind is 3-track trigger with dileptons, namely $K^+ \to \pi^+\ell^\pm\ell'^\mp$ and $K^+ \to \pi^-\ell^+\ell'^+$. While LFV search with $\ell = \mu$ and $\ell' = e$ is obvious, let us discuss the dimuon cases.

NA48/2, the predecessor to NA62, has recently published [197] their search for the LNV process $K^+ \to \pi^-\mu^+\mu^+$,

$$\mathcal{B}(K^+ \to \pi^-\mu^+\mu^+) < 8.6 \times 10^{-11}, \quad \text{(NA48/2)} \tag{71}$$

at 90% C.L., probing for Majorana neutrino via $K^+ \to \mu^+ N_4$ with $N_4 \to \pi^\mp\mu^\pm$ [198]. Dimuon resonances have also been searched for in LNC $K^+ \to \pi^+\mu^+\mu^-$ decay [197]. Nothing was found, but the point is that, with 50 times the expected number of K^+ than NA48/2 and with better detector resolution, NA62 can vastly improve the above bound, as well as exotic particle search. We further note that $K^+ \to \pi^+\mu^+\mu^-$ and $\pi^+e^+e^-$ may show correlations [199] with the NP hint in P_5' and R_K observables in $B \to K^*\mu^+\mu^-$ and $K\ell^+\ell^-$, and are worthy of pursuit. These kaon modes, however, have long distance effects to disentangle, which demand a lot of studies.

Another example we give concerns rare/exotic π^0 decays. With 10^{13} K^+ decays and with $K^+ \to \pi^+\pi^0$ at ∼21%, NA62 is a "π^0 factory". Again, its predecessor NA48/2 has searched for the dark photon A' via $\pi^0 \to \gamma A'$, followed by prompt $A' \to e^+e^-$ decay. Such a dark photon with γ–A' mixing parameter $\varepsilon \sim 10^{-3}$ and mass below 100 MeV could possibly explain the muon $g-2$ anomaly. Altogether, 17 million $\pi^0 \to \gamma e^+e^-$ decays were fully reconstructed by NA48/2, with no signal for A' observed. Combined with other experiments, the NA48/2 result [200] practically rules out the dark photon as an explanation of muon $g-2$ (assuming $A' \to e^+e^-$ decay is predominant), as illustrated in Fig. 11. The NA48/2 bound, however, is limited by irreducible $\pi^0 \to \gamma A'$ Dalitz decay background, so only modest improvement is expected with the larger NA62 data. Furthermore, an e^+e^- trigger may need to be scaled down. There is also $\pi^0 \to \nu\nu$, or invisible π^0 decay that in principle can be probed with π^+ tagging. We defer the discussion to the next subsection.

Beyond LS2 of the SPS/LHC complex, NA62 can further improve the studies mentioned above. There is further thought for a year-long run in "beam dump" mode to probe for hidden sector candidates in MeV–GeV range, which we do not discuss any further here.

[19] Long Shutdown 2, scheduled for 2019 and 2020.

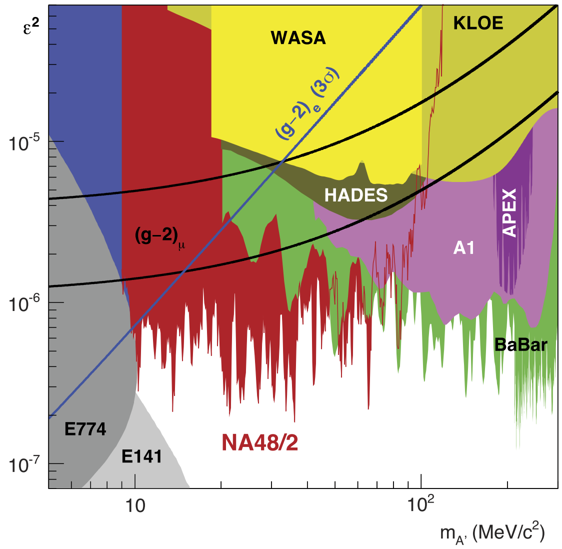

Fig. 11. Combined experimental bounds rule out the dark photon A' as explanation for muon $g-2$ (curved band), with NA48/2 [200] closing the last window in $m_{A'}$–ε^2 plane. These results assume A' couple to quarks and decay dominantly to SM fermions.
Source: Courtesy NA48/2 Collaboration, from Ref. [200].

5.2. $K_L \to \pi^0 \nu\bar{\nu}$ and KOTO

If NA62 evolved from NA48, then the KOTO experiment currently running at KEK's J-PARC facility descended from KTeV, with E391A as intermediate step. The current best direct search limit [201],

$$\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu}) < 2.6 \times 10^{-8}, \quad \text{(E391A)} \tag{72}$$

at 90% C.L. is still the one from E391A, the first dedicated $K_L \to \pi^0 \nu\bar{\nu}$ search experiment. Eq. (72) is, however, 3 orders of magnitude above the SM expectation, Eq. (69), and a factor of 20 weaker than an indirect bound extracted from the E949 result on $K^+ \to \pi^+ \nu\bar{\nu}$, Eq. (70), assuming isospin and lifetime ratio factors [202],

$$\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu}) < 1.4 \times 10^{-9}, \quad \text{(E949/GN)} \tag{73}$$

which illustrates how difficult things are for KOTO.

Compared with $K^+ \to \pi^+ \nu\bar{\nu}$ decay, $K_L \to \pi^0 \nu\bar{\nu}$ is indeed a really challenging mode to search for. There is no way to detect the incoming K_L (without risking conversion to K_S), which decays in flight. One knows only the beam direction and energy profile, and the appearance of just a single π^0 in the detector, plus nothing else. More specifically, the experimental signature is 2γ, plus "nothing", plus missing p_T. One actually *assumes* the detected 2γ are from π^0 decay, and from π^0 direction one calculates the decay vertex z along beam direction, and hence the p_T of the π^0. A signal box is defined in z and p_T, and the remaining job, in addition to getting as many primary beam protons on target as possible, is to understand and reject background. This is quite the poor-man's $K \to \pi \nu\bar{\nu}$ process, with no luxury of "kinematic control". Besides moving from KEK PS to J-PARC, the main changes from E391A to KOTO are the reuse of CsI crystals from KTeV for signal photon detection, more hermetic photon veto for $K_L \to \pi^0\pi^0$ suppression, and waveform digitization.

If the path for long-lived neutral kaon rare decay search is long and tortuous, it suffered further some bad luck. After just 100 h into data taking in 2013, the beam was stopped by an unrelated accident at J-PARC, and data taking did not resume until two years later. The hiatus of two years, however, was perhaps a mixed blessing. Analyzing the meager 2013 data, one event was found in the signal box when 0.34 ± 0.16 events were expected. The main backgrounds were beam halo neutrons that hit the CsI calorimeter, and physics backgrounds, e.g. from $K_L \to \pi^+\pi^-\pi^0$ events where the charged pions go down the beampipe. Steps were taken, both in added hardware and in software improvement, to mitigate these problems (the only approach since E391A period), and these were tested with the first equivalent amount of data taken in 2015, leading to improved understanding hence further control of background. Only by the KAON2016 conference did KOTO announce their final result [203] of 5.1×10^{-8} at 90% C.L. based on 2013 data, which is weaker than Eq. (72) by E391A. The paper, however, announced also a first direct search limit,

$$\mathcal{B}(K_L \to \pi^0 X^0) < 3.7 \times 10^{-8}, \quad (\text{KOTO}, m_{X^0} \simeq m_{\pi^0}) \tag{74}$$

which is more than a factor of 6 improvement [203] of an indirect result from $K^+ \to \pi^+ X^0$ search by E949 [196], where X^0 is an invisible object of π^0 mass. The three bounds are shown in Fig. 12.

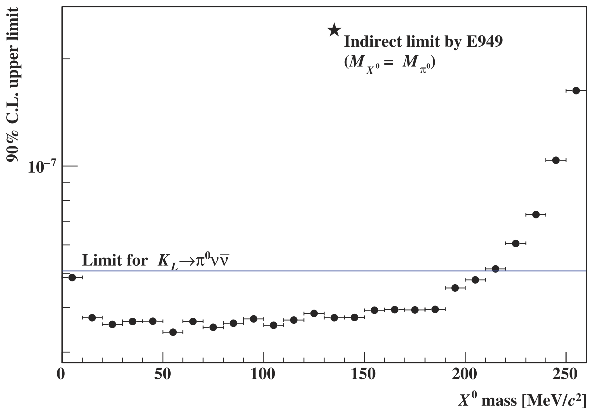

Fig. 12. Upper limit on $\mathcal{B}(K_L \to \pi^0 X^0)$ vs m_{X^0} at the 90% C.L., compared with limit (horizontal line) on $\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu})$ and the indirect limit (star) by E949. *Source:* Courtesy KOTO Collaboration, from Ref. [203].

The new interest in the process of Eq. (74) needs some explanation. It reflects some longstanding oversight in the arduous planning of a K_L experiment, self-hypnotized by the "Grossman–Nir bound". The main target for NA62 and KOTO are to measure the SM processes $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L \to \pi^0 \nu\bar{\nu}$, respectively. The two branching ratios are related by the Grossman–Nir bound [202] of

$$\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu}) < 4.3 \times \mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu}), \quad \text{(Grossman-Nir)} \tag{75}$$

which is rather solid as it is based on isospin symmetry and the fact that the K_L process is CP violating while K^+ process is a bulk measure, with the numerical factor dominated by the lifetime ratio. It is by inserting the E787/E949 upper bound implied by Eq. (70) into this relation that one arrives at Eq. (73), which we shall refer to as the "GN bound". While the Grossman–Nir bound of Eq. (75) is robust, it is the numerical "GN bound" of Eq. (73) that everyone seems to be conditioned by, though it is a thorn-in-flesh for KOTO people in their heroic efforts, as if their "business" would not really start until the magical number of Eq. (73) is breached.

Inspecting the detection methods of $K_L \to \pi^0 \nu\bar{\nu}$ and $K^+ \to \pi^+ \nu\bar{\nu}$, with the fact that the $\nu\bar{\nu}$ pair is not actively detected but known rather just as missing energy–momentum, a loophole was recently pointed out [204] that allows KOTO to have NP discovery potential above the "GN bound" of Eq. (73). Recall that the K^+ experiments use kinematic control to define signal boxes around $K^+ \to \pi^+ \pi^0$ decay, the reason being that the 21% branching fraction makes the latter too "bright" to behold. Even then, photon veto is critical for π^0 rejection. Thus, E787/E949 and NA62 all avoid the m_{π^0} window, which KOTO, without the luxury of kinematic control, can turn disadvantage into opportunity. In fact, and as already mentioned in Section 5.1, E949 had tried putting a bound on $K^+ \to \pi^+ X^0$ with X^0 in the π^0 mass window, by effectively searching for $\pi^0 \to$ *nothing* with π^+ tagging. Unfortunately, the bound [205]

$$\mathcal{B}(\pi^0 \to \nu\bar{\nu}) < 2.7 \times 10^{-7}, \quad \text{(E949)} \tag{76}$$

at 90% C.L. results in $\mathcal{B}(K^+ \to \pi^+ X^0) < 5.6 \times 10^{-8}$ [196] that is ~200 times worse than the one on $K^+ \to \pi^+ \nu\bar{\nu}$. This is the indirect bound on $K_L \to \pi^0 X^0$ (deduced by reversing Grossman–Nir argument) that the recent direct search [203] by KOTO has surpassed with small 2013 data set, Eq. (74).

What type of NP could this be? In general, any "dark" X^0 with mass $m_{X^0} \sim m_{\pi^0}$ emitted from $K \to \pi$ transition that goes through the detector undetected, would slip through NA62 but can be caught by KOTO. As existence proof, Ref. [204] gave an explicit model based on gauged $L_\mu - L_\tau$ symmetry that was originally [206] motivated by the P_5' anomaly (Section 3), but redirected [207] to account for muon $g-2$. The latter demanded the Z' to be lighter than 400 MeV.[20] To couple to quarks, one introduces a vector-like quark U that carries U(1)$'$ charge, which mixes with normal u-type quarks. A weak boson loop then converts [208] the effective Z' couplings of u-type quarks into sdZ' and bsZ' couplings (see Fig. 13[left]).

Stringent constraints from narrow dimuon bump search by LHCb [209] in $B \to K^* \mu^+ \mu^-$ disfavors the Z' above dimuon threshold (Z' decay should be rather prompt [208]). Below $\mu^+\mu^-$ threshold, only $Z' \to \nu\bar{\nu}$ is allowed, and interestingly, there is a mild hint [160] from BaBar in $B \to K^{(*)} \nu\bar{\nu}$ search for a K^+ or K^{*0} recoiling against an unobserved low mass system.[21] A similar analysis has not been carried out yet by Belle. Within the model, the BaBar result does favor the range between Eqs. (72) and (73), as can be seen from Fig. 13[right], which has specific parameters $m_{Z'} = 135$ MeV, $g' = 10^{-3}$ for the Z'

20 This light Z' cannot be behind P_5', but it is just right for kaon physics.

21 Comparing with kaon decays, note that there are no worries of $B \to K\pi^0$ background, as it is also a penguin loop process, and the detection environment is very different.

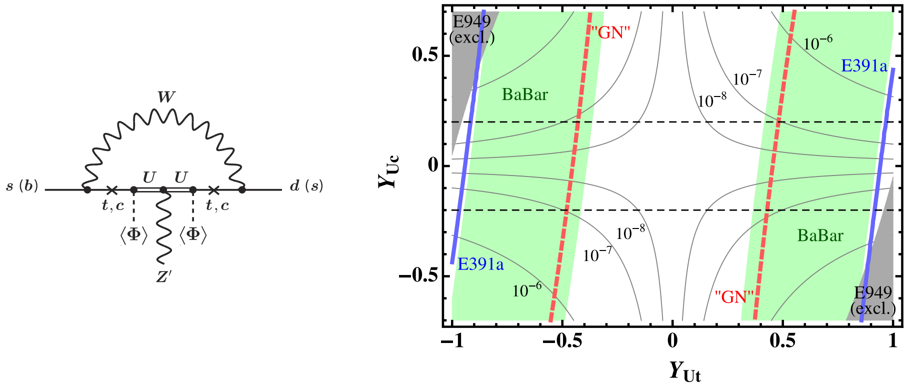

Fig. 13. [Left] Induced $s \to dZ'$ and $b \to sZ'$ transitions with $L_\mu - L_\tau$ charged vector-like U quark; [right] for $m_{Z'} = 135$ MeV ($Z' \to \nu\bar{\nu}$ 100%), the BaBar allowed 2σ range of $\mathcal{B}(B \to K^{(*)}Z')$ span the E391A bound (solid) of Eq. (72) and "GN bound" (thick-dashed) of Eq. (73), while a weaker indirect bound by E949 on $K^+ \to \pi^+ Z'$ is at the corner of the plot in the exotic Y_{Uc}–Y_{Ut} Yukawa plane, where horizontal lines indicate Cabibbo angle strength, and $\mathcal{B}(t \to cZ')$ contours are in the backdrop.
Source: Ref. [204], copyright APS.

gauge coupling, and $m_U = 2$ TeV. This example illustrates the genuine discovery potential for the $K_L \to \pi^0 X^0$ process with 2015 data, which is 20 times larger than 2013 data, and should allow KOTO to explore $K_L \to \pi^0\nu\bar{\nu}$ down to the GN bound of Eq. (73) with improvements stated in Ref. [203].

We note that, as KOTO continues to take data, and with new measurements of $K^+ \to \pi^+\nu\bar{\nu}$ by NA62 coming soon, the suggestion of Ref. [204] still holds, that the $K_L \to \pi^0 X^0$ process could still be discovered above the $K_L \to \pi^0\nu\bar{\nu}$ rate implied by a new $K^+ \to \pi^+\nu\bar{\nu}$ upper bound through Eq. (75).

The above loophole originates from the "brightness" of $K^+ \to \pi^+\pi^0$ decay, such that K^+ experiments use kinematic control to exclude events with missing mass in the m_{π^0} region. While this means extra business for KOTO to pursue, is there anything that can be done by NA62? The answer is yes, by perfecting their measurement of $\pi^0 \to \nu\bar{\nu}$ (or, *nothing*), which is indeed on NA62's non-$\pi\nu\bar{\nu}$ agenda. Recall the bound of Eq. (76). The limit is not so good because of photon detection inefficiency [205], such that a small fraction of $\pi^0 \to \gamma\gamma$ events could mimic $\pi^0 \to$ *nothing*. With non-$K_{\pi 2}$ background expected at ~3 events, E949 saw 99 events [205], and attributed the excess to photon detection inefficiencies. These could come from calorimeter sampling fluctuations for lower energy photons, or photonuclear interactions (e.g. with neutrons) for higher energy photons. For NA62 to compete in $K^+ \to \pi^+ X^0$ detection, investment should be made in understanding the photon detection inefficiency, to see how much they can effectively improve the bound of Eq. (76) on $\pi^0 \to$ *nothing*. Roughly speaking: how well can NA62 stare into the brightness?

Back on the main pursuit of $K_L \to \pi^0\nu\bar{\nu}$ search, the 2013 proton beam power for KOTO was 24 kW, but 42 kW has already been attained since. The plan is to increase to 100 kW by 2019. Together with further photon/neutron discrimination by adding photon sensors in front of CsI crystals to measure shower depth, KOTO should be able to push below 10^{-10} [210]. To attain a genuine measurement of SM prediction of Eq. (69), i.e. $O(100)$ events, KOTO had an ambitious machine and detector upgrade plan called Step 2 in its proposal. But judging from the timeline so far, this probably would not occur until HL-LHC era, i.e. 2025 and beyond. In part because of this, and in part planning for their own HL-LHC era run plan, preliminary design studies have started in NA62 [211] for a 5 year run at the SPS targeting 2026 start, to observe $O(60)$ $K_L \to \pi^0\nu\bar{\nu}$ events (assuming Eq. (69)).

It seems that rare kaon search would continue beyond another decade, but major progress is expected within the next few years, either by discovery of NP in $K^+ \to \pi^+\nu\bar{\nu}$, $K_L \to \pi^0\nu\bar{\nu}$ or $K_L \to \pi^0 X^0$ processes, or confirming SM in the K^+ mode.

Dark Addendum: Not a Dark Photon?

We have seen that KOTO has a unique probe into a dark boson X^0 with mass around the π^0, and an example given was gauged $L_\mu - L_\tau$ symmetry with heavy vector like quarks. A dark object at specific mass, and a built-up model to hide it, all sounds "fantastic". We wish to make a little detour to illustrate some points, even though some of it is not about FPCP, but perhaps "the flavor of Dark Matter".

Dark Matter (DM), that the bulk of matter of the Universe (let alone the "Dark Energy" that predominates in the current era) are not seen, but "felt" through their gravitation effect, is one of the main motivations for New Physics beyond SM. The premium example is the so-called WIMP (weakly interacting massive particle), with the lightest SUSY particle offering an ideal candidate. Alas, both direct search for SUSY at LHC, and DM search with a plethora of means, turn out empty handed so far. We cannot go deep into this Dark subject, but wish to extend our discussion of B and K physics, in particular the $K \to \pi +$ *nothing* searches that we have just discussed.

The NA48/2 experiment claims to have closed the window for a dark photon A' explanation of the muon $g-2$ anomaly, see Fig. 11. But there is a catch: A' has to decay via e^+e^- dominantly. This may seem plausible, as $m_{A'} < m_{\pi^0}$ in the remnant window that NA48/2 covered, which corresponds to $\varepsilon \sim 10^{-3}$. The ε parameter induces "kinetic mixing" [186] between γ and A', and 10^{-3} is typically viewed as some loop-induced value. It was then pointed out [212] that A' could decay invisibly, e.g. $A' \to \chi\chi$ where χ is some light dark matter particle, which would then evade the bounds from NA48/2 and other experiments, therefore still provide an explanation of muon $g-2$. This motivated a fixed target experiment, NA64 at SPS, that rose to the challenge of directly searching for such an A'. Utilizing 100 GeV electron beam in $e^-Z \to e^-ZA'$, where Z denotes a high mass nuclear and A' is produced via kinetic mixing with bremsstrahlung photon, but decays invisibly [212] hence would lead to large missing energy. An early summer 2016 run with 2.75×10^9 electrons-on-target (EOT),[22] saw no such events [213], excluding the invisible A' with mass below 100 MeV as an explanation of the muon $g-2$ anomaly. A small region above 100 MeV but below the dimuon threshold [212] remains. NA64 completed a run in Fall 2016 that accumulated an order of magnitude more EOT, and should be able to cover this region. Subsequent to Ref. [213], however, the BaBar experiment has ruled out [188] the invisibly decaying dark photon explanation of muon $g-2$.

But there is another catch: what if A' is not a dark "photon", that it does not couple to electrons, but only to muons to induce the $g-2$ anomaly? The $L_\mu - L_\tau$ model provides an excellent and motivated example. Because of the built-in cancellation, the μ and τ loop induced γ–Z' kinetic mixing is quite suppressed, $\varepsilon \sim 10^{-5}$, which is way out of reach for NA64. Stimulated by models such as $L_\mu - L_\tau$, there is a proposed concept to use μ-beam to conduct the analogous $\mu Z \to \mu ZZ'$ [214] with missing energy from $Z' \to$ *nothing*. However, it does not seem feasible that it can be realized before LS2 of SPS/LHC, i.e. to run before 2021. But the KOTO experiment can probe, and is already probing this relatively exquisite Z' scenario, albeit depending on vector-like quark assumptions [208]. On the other hand, while the muon beam can test the generic $L_\mu - L_\tau$ model, KOTO has its own advantage, too: any Dark X^0 from $K_L \to \pi^0 + X^0$ can be probed, so long X^0 is not far from π^0 mass, and decays invisibly. To be a bit whimsical, since the devil hides in the details, this seems like a long standing little detail that ...

Our point is to illustrate that dark photon and dark boson search is ongoing, and flavor physics offers its own angle. Let's hope $K \to \pi +$ *nothing*, and its sister studies in $B \to K^{(*)} +$ *nothing*, could shed light on *Dark* physics beyond SM.

5.3. ε'/ε

The measurement of direct CPV in kaon decay, namely ε'/ε, was a true *tour de force* of experimental physics, culminating in the measured values of $(28.0 \pm 3.0\,(\text{stat}) \pm 2.8\,(\text{syst})) \times 10^{-4}$ by KTeV [215] and $(18.5 \pm 4.5\,(\text{stat}) \pm 5.8\,(\text{syst})) \times 10^{-4}$ by NA48 [216] in 1999, using subsets of data taken in the late 1990s. This is 35 years after the original observation of indirect CPV [5]. The final result, combining [17] the two experiments with full data, gives

$$\text{Re}(\varepsilon'/\varepsilon)|_{\text{exp}} = (16.6 \pm 2.3) \times 10^{-4}, \tag{77}$$

which is 3 orders of magnitude smaller than $|\varepsilon_K| \simeq 2 \times 10^{-3}$ itself. DCPV in kaon decays is really small. This is in contrast with DCPV in $B \to K\pi$ decays, measured [217,218] at strength of 10% and only 5 years after ε'/ε, and just 3 years after the measurement [20,21] of indirect CPV in $B^0 \to J/\psi K_S$, which is order one. CPV, including DCPV, effects in B system are large.

Interest in the measurement of ε'/ε rose after the discovery of the b quark [8], allowing theoretical estimates with three generations of quarks, where early predictions were at the 10^{-2} level. As DCPV involves the interference between K^0 and $\overline{K^0}$ amplitudes to different final state isospin [219], the trick is to measure the double ratio,

$$R = \frac{\Gamma(K_L \to \pi^0\pi^0)/\Gamma(K_S \to \pi^0\pi^0)}{\Gamma(K_L \to \pi^+\pi^-)/\Gamma(K_S \to \pi^+\pi^-)} \simeq 1 - 6\,\text{Re}(\varepsilon'/\varepsilon), \tag{78}$$

plus other ingenious ways to reduce error. However, by the time early measurements pushed below the percent level, the heaviness of the top quark, first hinted by the sizable B^0–$\bar{B}^0$ mixing [11], implied an enhanced electroweak penguin [25] that could cancel against [62] the earlier estimates based on the strong penguin, resulting in a much smaller ε'/ε value. It was this trick of Nature that caused Winstein to lament [220] that perhaps we will not be able to distinguish between the three generation SM and the "superweak" model of Wolfenstein [221], which predicted $\varepsilon'/\varepsilon = 0$. The experimental effort, of course, went on regardless of this. The result of Eq. (77) therefore deals the deathblow to the superweak model, and is a triumph of kaon physics.

The case would be settled as that, because although the top mass became precisely known, the strong penguin amplitudes involve various hadronic matrix elements that are hard to estimate precisely. Thus, ε'/ε does not seem like a good observable to probe for NP, despite its suppressed value due to accidental cancellations, which affirms our point of avoiding probes that are susceptible to large hadronic effects. However, motivated in part by the latter, and the ability to put the QCD Lagrangian on the spacetime "lattice" and use the computer to "measure" the involved path integrals, a recent result from lattice QCD motivates us to put ε'/ε back as a measurable to watch, and the comparison is between lattice predictions versus experimental measurement.

Lattice QCD started in the 1970s to approach the fundamental problems of QCD itself. But practitioners started to tackle the measurements of, e.g. $K \to \pi\pi$ hadronic matrix elements since the 1980s. In an effort that parallels the experimental

[22] A relative low number compared to low energy muons discussed in the next section.

measurement of ε'/ε, after 30 years of development, the RBC+UKQCD collaboration announced recently [222] the lattice QCD measurement of

$$\mathrm{Re}(\varepsilon'/\varepsilon)|_{\mathrm{latt.}} = \mathrm{Re}\left\{\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]\right\} = (1.38 \pm 5.15\,(\mathrm{stat})\ \pm 4.59\,(\mathrm{syst})) \times 10^{-4}, \tag{79}$$

where A_0 and A_2 are the $I = 0$ and 2 decay amplitudes to $\pi\pi$ final states, respectively, δ_I are the strong phase shifts, and $1/\omega = \mathrm{Re}A_0/\mathrm{Re}A_2 \simeq 22.5$ is from experiment. A slightly earlier result [223] had calculated A_2 to good ($\sim$10%) precision, supporting the $1/\omega$ value.[23] The main result in Eq. (79) calculated $\mathrm{Re}A_0$ on one hand that is in agreement with experiment, and on the other hand, the computation of $\mathrm{Im}A_0$ predicts ε'/ε. The numerical result, consistent with zero, is in striking contrast with experiment, Eq. (77), but the disagreement is only at 2.1σ level, in good part because of lattice errors. Although some theorists have taken this "anomaly" very seriously [193], the RBC+UKQCD collaboration itself projects that improvements in statistics, larger lattice volume and more lattice spacing values, would take a $\sim$5 year or so effort to pan out. If the discrepancy stays, it would turn into a true anomaly in the kaon sector. Would this motivate a renewed effort for experimental measurement? Maybe, but we caution further that, on the lattice side, a second, independent crosscheck would also be desirable.

6. Muon and EDM

The muon was the harbinger of fermion generation repetition, and at $\sim$200 times the electron mass but with a rather long lifetime (compared with τ), it became a handy, versatile tool that probes across subfield boundaries. For instance, the proton size problem [224] was recently revealed by precision study of muonic hydrogen [225]. We cannot do justice to "muon physics", but would focus on rare decays or (charged) lepton flavor violation, anomalous magnetic moment (muon $g-2$), then crossover to comment on electric dipole moment searches. That is, we focus on potential probes for NP. For a more comprehensive discussion on precision muon physics, see Ref. [226].

6.1. LFV $\mu \to e$ processes

We now know that lepton flavor is violated (LFV) in neutral lepton sector in the form of neutrino mixing, or that neutrinos have mass (true sign of NP). But so far we have not observed any LFV in charged lepton sector.

The $\mu \to e\gamma$ transition is analogous to $b \to s\gamma$ (and certainly preceded it conceptually), but the "penguin" loop is extremely suppressed by neutrino mass (at the eV level, compared with $m_t \cong 173$ GeV) and is practically negligible. It is thus a very good probe of NP, and has been pursued ever since the muon became known to mankind. The current bound [227] of

$$\mathcal{B}(\mu \to e\gamma) < 4.2 \times 10^{-13}, \quad (\mathrm{MEG}) \tag{80}$$

at 90% C.L. is the final result of the MEG experiment, and improves the result of the previous MEGA experiment [228] by a factor of 30. Earlier MEG results are documented in PDG [17].

The signal is rather distinct: with μ^+ decay at rest, one has back to back e^+ and photon in coincidence and with equal energy. As an nth generation experiment in the long quest for $\mu \to e\gamma$, the MEG experiment has a large Liquid Xenon photon detector, opposite an ultra-light drift chamber accompanied by fast timing counters all placed in an innovative gradient field COBRA magnet. Energy and angular resolution plus timing work together to suppress the main background of accidental coincidence between a regular $\mu \to e\nu\bar{\nu}$ decay and another that emits a photon, or from $e^+e^- \to \gamma\gamma$. For this reason, one uses a DC rather than pulsed muon beam. The result of Eq. (80) corresponds to 7.5×10^{14} stopped μ^+ accumulated during 2009–2013.

With null observation at the LHC, traditional MSSM and SUSY-GUT model expectations for $\mu \to e\gamma$ are now somewhat mute. But given that pockets (cracks) of parameter space remain, it would be interesting to watch the correlation [229] between $\mu \to e\gamma$ and muon $g-2$ in SUSY context. Motivated by large neutrino mixing [17] and still in context of SUSY, linking with flavor symmetries suggest [230] $\mathcal{B}(\mu \to e\gamma)$ at the 10^{-13} level, while linking [231] with seesaw and baryogenesis-through-leptogenesis still leaves some parameter space to be probed. From the latter perspective, however, $\tau \to \mu\gamma$ may not be observable at Belle II. In any case, the MEG-II upgrade is currently under construction, and is expected to enter physics run for 3 years starting 2017. The aim is to improve the MEG limit by another order of magnitude, to the 4×10^{-14} level. Besides increased beam current, there is a factor of two improvement in resolution in all aspects, e.g. changing from 2″ PMT to SiPM for the LXe photon detector.

If the $\mu \to e\gamma$ dipole transition process probes loop effects, a different class of experiments such as $\mu N \to eN$ and $\mu \to e\bar{e}e$ probe contact 4-fermion interactions. The current limit on $\mu \to e$ conversion is held by SINDRUM II [232],

$$R_{\mu e} = \frac{\Gamma(\mu + (A,\ Z) \to e + (A,\ Z))}{\Gamma(\mu + (A,\ Z) \to \nu_\mu + (A,\ Z-1))} < 7 \times 10^{-13}, \quad (\mathrm{SINDRUM\ II}) \tag{81}$$

[23] Suggesting the cancellation of two major contributions to A_2, but no cancellation for A_0, as the source of the $\Delta I = 1/2$ rule, that $1/\omega \simeq 22.5$ is a large number.

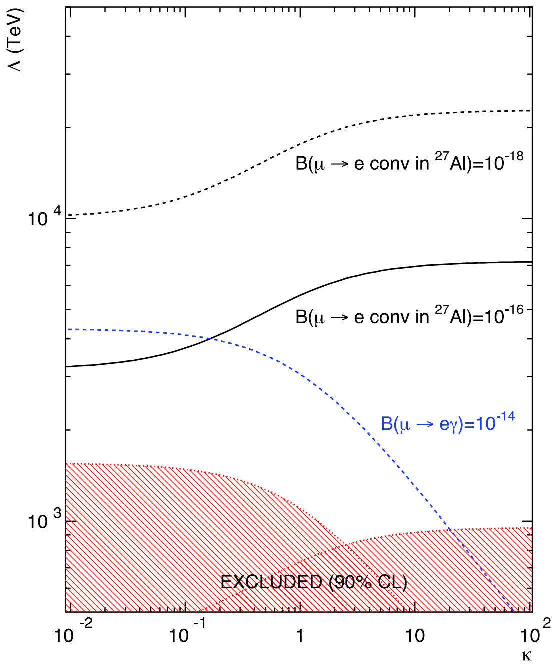

Fig. 14. Mass scales probed by $\mu \to e\gamma$ and $\mu \to e$ conversion, where the κ parameter "dials" between dipole-like (small κ) to contact-like (large κ) interactions. MEG-II aims to push the $\mu \to e\gamma$ excluded region close to the lower dotted curve, while COMET and Mu2e aim to push the $\mu \to e$ conversion excluded region beyond the solid curve.
Source: Courtesy A. de Gouvêa, updated from Ref. [237].

at 90% C.L., with muonic atoms formed on gold nucleus. The conversion electron has energy basically the same as the muon mass. The limit of Eq. (81) is a bit dated, and improvement with same method appears stiff, since the SINDRUM II experiment already used ∼1 MW proton beam power. The next generation experiments are based on the idea [233] of using a strong solenoidal B field to confine soft pions then collect the decay muons, which drastically reduces the requirement on proton beam power. Thus, two new experiments, aiming for a staggering *4 orders of magnitude* improvement, invest on superconducting solenoids. Background from beam pion capture, followed by nuclear γ decay with γ conversion resulting in e^+, is mitigated by using pulsed proton beam and waiting out the prompt decay. Muon decay in orbit, where the tail of energy distribution above $m_\mu/2$ can enter the signal region, constitutes the main background, and detector resolution is key.

The Mu2e experiment is under construction at Fermilab. It aims for commissioning the solenoids in 2020, and physics run starting 2021 for 3 years to accumulate a total of 10^{18} stopped muons, with the goal of reaching $R_{\mu e} < 10^{-16}$ [234]. The COMET experiment [235] at J-PARC takes a staged approach aiming for fast start. For COMET Phase I, detector and facility preparation is underway, aiming for 2018 run start that could reach below 10^{-14}. Phase II would use a more sophisticated S-shaped muon transport solenoid, higher beam power and improved detector, and aims for run start in 2022, also with a goal to reach below 10^{-16}. Both experiments would use muonic atoms formed on Al, as well as 8 GeV protons. There is another experiment, DeeMe [236] at J-PARC, that is based on a different approach. It would use 3 GeV proton beam at ∼1 MW beam power on a thick target for pion production, decay *and* muon stopping, then collect the electrons from $\mu \to e$ conversion and use a second beamline as part of the "spectrometer". DeeMe aims for physics run in 2017, and could reach below 10^{-13}, with improvement possible by optimizing target and running longer.

Thus, there is a robust ongoing program for $\mu \to e\gamma$ and $\mu \to e$ conversion search. If the former is related to SUSY and neutrino mixing, the latter has even broader coverage of possible NP, as it probes new contact interactions including Z', extra Higgs, heavy neutrinos, leptoquark, compositeness, etc., i.e. similar to the LHC, with higher reach in NP scale, probing up to 10^4 TeV. We cannot do justice to the potential physics contact, but refer the reader to Ref. [237] for a review. We just use a figure (Fig. 14) from this reference to illustrate the NP scale probed by these experiments.

Finally, analogous to μeqq contact terms probed by $\mu \to e$ conversion, $\mu^+ \to e^+e^+e^-$ could probe μeee contact terms. The old but current limit [17] at 10^{-12} is from SINDRUM almost 30 years ago. The Mu3e experiment [238] at PSI aims to push the limit down to 10^{-16} by ultimately having 2×10^9 muon decays *per second*. Phase I running with 10^8 muon decays per second could start in 2017 or 2018. Note that, analogous to Fig. 14 for $\mu \to e\gamma$ vs $\mu \to e$ conversion, in the dipole-like region, $\mu^+ \to e^+e^-e^+$ should be scaled by α when compared with $\mu \to e\gamma$ in rate. However, for contact-like interactions, $\mu^+ \to e^+e^+e^-$ wins over $\mu \to e\gamma$ in energy scales probed. As an explicit model that illustrates the probing power of $\mu \to e$ transitions, let us take a recent study [239] of warped extra dimensions, where KK excitations of multi-TeV now seems beyond the reach of LHC. After applying $\mu \to e\gamma$ and $\mu \to e$ conversion as the most stringent constraints, the combination of MEG-II, Mu3e, and certainly Mu3e and COMET as well, could explore the effects of extra dimensions on LFV beyond $m_{\rm KK} > 20$ TeV.

6.2. Muon g - 2

It can be said that particle physics started in 1947 in part with Schwinger's seminal calculation of $a_e \equiv g_e/2 - 1 = \alpha/2\pi$ at one loop, the "anomalous" gyromagnetic ratio (i.e. deviation from $g = 2$). In flavor language, one could have called this the first "penguin" diagram. It is pretty amazing that, 70 years later, we still have a "muon $g-2$ anomaly", now meaning deviation between experiment and SM prediction,

$$\Delta a_\mu = a_\mu(\text{Expt}) - a_\mu(\text{SM}) = (274 \pm 76) \times 10^{-11}, \tag{82}$$

where the experimental value is measured by BNL-E821 [240], $(11659208.0 \pm 5.4 \pm 3.3) \times 10^{-10}$, and the SM expectation is from the recent update [241] of hadronic vacuum polarization (HVP) contribution. The more than 3σ deviation has persisted for more than a dozen years, and could be handily explained [242] by MSSM. Although SUSY has not been sighted yet at the LHC, the persistent discrepancy has motivated serious efforts to remeasure a_μ, as well as to refine the theoretical calculation.

After relocating and refurbishing the BNL muon storage ring, Fermilab-E989 (also called Muon g-2) experiment [243] has meticulously shimmed the magnetic field to 3 times better uniformity than at BNL, and fully around the 44 m storage ring circumference. With improved detectors and other technologies, and with over 20 times more muons, the aim is to improve the experimental error by a factor of 4, from 540 ppb down to 140 ppb. Initial physics run should commence in 2017 and continue until 2019. But one also has to improve the theory with a concerted [244] effort. The QED (calculated to $O(\alpha^5)$!) and electroweak uncertainties are under control, but improvement of hadronic uncertainties are critical. For HVP contribution, which enters at $O(\alpha^2)$ level, we have already mentioned the recent update [241] that utilize $e^+e^- \to$ hadrons data. For hadronic light-by-light (HLbL) scattering contribution that enters at $O(\alpha^2)$ level, recent lattice progress [245] seems to halve both the numerical value and (statistical) error compared with models [246]. If so, it would heighten the "anomaly". However, systematic errors from finite volume and lattice spacing are still under investigation, which could increase [245] the value. But in any case the two approaches are independent, with unrelated systematic errors. Note that lattice is also catching up [247] with the numerical HVP contribution.[24]

We cannot do full justice to this important subject that is somewhat outside of our scope, but look forward to major progress on the lingering muon $g-2$ anomaly in the coming few years.

6.3. Electric Dipole Moments (EDM)

We have given only a cursory account of the long-standing muon $g-2$ anomaly, in part because of its flavor conserving nature. Unlike magnetic moments, however, electric dipole moments of nondegenerate particles violate CP,[25] and is thus close to our core subject. But again we give only a cursory account, as the experimental studies are both diverse and rather specialized. For instance, measurements of neutron EDM, d_n, typically utilize trapped ultracold neutrons (UCN), while the extraction of electron EDM, d_e, involves molecular and atomic (and even nuclear, e.g. mercury EDM) physics. On the particle physics side, SM effects from CPV phase in CKM matrix contribute only at rather high loop order, and current experiments are many orders of magnitude away from SM expectations, i.e. $d_n^{\rm SM} \sim 10^{-32}\ e$ cm, $d_e^{\rm SM} \sim 10^{-40}\ e$ cm. Thus, discovery of EDM would definitely imply NP, albeit in a rather indirect way. We will briefly discuss the two cases of d_n and d_e as examples, referring to more specialized reviews, such as Ref. [249], for more detail.

If new CPV phases in SUSY were of $\mathcal{O}(1)$, and SUSY particles have mass around the weak scale, then neutron EDM should have appeared above the $10^{-24}\ e$ cm level. The current experimental bound [250], from ILL in France, gives (final update from 2006 result [17])

$$d_n^{\rm Expt} < 3.0 \times 10^{-26}\, e\,\text{cm}, \quad \text{(ILL)} \tag{83}$$

at 90% C.L., which means either CPV phases are small in MSSM, or SUSY breaking scale is considerably higher than 1 TeV. Judging from the lack of evidence for SUSY at the LHC so far (at neither 8 nor 13 TeV), the latter is becoming more and more likely. But it also means that *d_n could appear at any time*. Of course, the smallness of d_n already gave the puzzle of an extremely small $\theta_{\rm QCD}$, and the possibility of axion as explanation. However, neutron EDM also probes quark EDMs d_u, d_d as well as the corresponding chromo-dipole moments, hence fascinating for theorists.

On the experimental side, there is a world-wide race now to reach below $10^{-27}\ e$ cm using UCN sources, to improve Eq. (83) by two orders of magnitude. For example, the ILL setup was moved to PSI, where a dedicated UCN source was built. The latter moderates spallation neutrons through heavy water, then solid D_2 crystals. The PSI nEDM experiment [251] began data taking in 2015 and will last until 2017. The sensitivity should reach below $10^{-26}\ e$ cm. The n2EDM experiment to follow targets reaching below $10^{-27}\ e$ cm, where data taking could start in 2020. Across the Atlantic, the SNS nEDM experiment [252] at the Oak Ridge SNS (Spallation Neutron Source) adopts a different and novel [253] approach, using

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24 For those interested in following lattice developments, including K, D and B meson physics, we refer to the Flavor Lattice Averaging Group (FLAG) [248].

25 It is T-violating, hence CP-violating by CPT invariance.

superfluid ^{4}He as both the UCN moderator, as well as the high voltage insulator to sustain high electric field. It further uses ^{3}He as co-magnetometer and superconducting shield, to control and measure magnetic field systematics. With demonstration phase close to completion, large scale integration and commissioning should converge at ORNL by 2019. The sensitivity, assuming 3 years of running, aims at 2×10^{-28} e cm by the early 2020s.

We note that there is ongoing R&D to pursue proton EDM, d_p, measurement using a storage ring. This bears some analogy with muon storage ring study of $g-2$ (which would measure d_μ parasitically), but would be an "all electric" ring with no B field. The target is to reach sensitivity of 10^{-29} e cm, but schedule is not yet clear.

The current leading edge of charged particle EDM search is that of the electron, d_e, where the limit from the ACME experiment [254] gives,

$$d_e^{\rm Expt} < 8.7 \times 10^{-29}\, e\,{\rm cm}, \quad ({\rm ACME}) \tag{84}$$

at 90% C.L. ACME utilizes polar thorium monoxide (ThO) molecule, which has internal effective electric field $E_{\rm eff}$ of order 84 GV/cm. We cannot describe the methodology here, which uses molecular beams and lasers, but since this is a first generation experiment of the type, and there are other approaches (e.g.^{199}Hg, ^{224}Ra, etc.) as well, the result of Eq. (84) stands further improvement. As an example of the theories probed, we quote the study [255] of 2HDM-II with CPV in Higgs potential, which induces mixing between *CP*-even and *CP*-odd neutral Higgs bosons. It is found that, at present, the ThO result on d_e poses the most stringent constraint, while neutron and mercury constraints are less stringent, and furthermore suffer from hadronic and nuclear matrix element uncertainties. However, given the expected progress, this indirect probe of NP scales would be complementary to the direct search at the LHC. Note that 2HDM-II naturally follows from, but does not necessarily imply, SUSY. Together, these two types of NP provide sufficient motivation for the continued quest of EDMs, and the NP-CPV phases carried by scalar particles are being probed by current EDM searches. If a discovery is made, one would need a lot of improvement in hadronic and nuclear matrix element estimates to disentangle the underlying NP [249] from the multiple probes.

7. Tau/charm

In this section we put the discussion of tau and charm physics together, in part because each discussion is short, and also because "tau/charm factory" are often discussed together due to similar production energy for an e^+e^- collider.

The third generation τ lepton provides a unique and clean probe of lepton universality and charged lepton flavor violation, which extends from the previous section. The τ lepton itself is also a powerful tool for searches in high energy collisions. For example, the lepton-flavor violating $h \to \tau\mu$ decays (next section) can arise from several possible extensions to SM, including additional Higgs doublets at tree level, or new vector bosons at loop level. Studying τ lepton decays can also probe fundamental SM parameters, such as measuring the CKM element $|V_{us}|$ by comparing τ decay widths to hadrons with and without strange quark. From detailed comparisons with perturbative QCD corrections, parameters such as the strong coupling constant α_s at τ mass can be extracted.

Many recent contributions come from lepton collider experiments, in particular the B factory experiments Belle and BaBar, which each have several hundred million τ-pair events produced in a relatively clean environment. New contributions from LHC experiments are expected as well.

The second generation up-type quark, charm, is the third heaviest known quark, with mass slightly below the τ lepton. A highlight of charm physics is that four quark or "molecular" states, as well as pentaquark states, emerged from discoveries of exotic charmonia. But we shall refrain from discussing this rich field of exotic charmed meson and charmonium spectroscopy, as these do not pertain to the physics beyond SM that we are interested in, and refer the reader to the companion review [32].

The charm system is in principle an excellent place to look for New Physics contributions, in the sense that the decay branching fractions of FCNC processes as well as CPV asymmetries are very small in SM. Any nonzero value from experimental measurement can be a strong hint of physics beyond SM. However, compared with B physics where leading decays are suppressed by $|V_{ub}| \ll |V_{cb}| \sim 0.04$, the potential NP effects are hampered by fast and unhindered decay rates as the leading weak decay is Cabibbo allowed, $|V_{cs}| \cong 1$. Thus, charm as a probe for NP is not particularly promising. On the other hand, knowledge of charm quark decays are useful for many studies in the B physics sector, for example the CKM angle γ (ϕ_3) measurements via the various "*DK*" methods rely on *D* decays as part of the program.

Reflecting the Cabibbo allowed leading decay but Cabibbo suppression of loop processes, unlike the bottom or strange systems, the unitary triangle for charm is very squashed, resulting in rather tiny *CP* violation effects. CPV in the charm sector is restricted to Cabibbo-suppressed (CS) decay modes (or through D^0–$\overline{D}^0$ mixing, discussed in Section 7.3). The expected DCPV asymmetries in SM are of order 10^{-3}, but the estimations are not very robust due to large uncertainties from penguin amplitudes and other hadronic effects. New Physics can enhance the DCPV asymmetries, and it had been commonly assumed that finding a DCPV asymmetry at 10^{-2} level could be a hint for new CPV sources. However, recent estimates [256,257] suggest that DCPV asymmetries of $\mathcal{O}(10^{-2})$ can be accommodated within SM. Thus, finding DCPV asymmetries larger than $\mathcal{O}(10^{-2})$ is needed for establishing NP in the charm sector. Nevertheless, one should probe every possible corner experimentally.

Since the new millennium, measurements of CPV in charm sector have been dominated by Belle and BaBar, and more recently joined by the LHCb experiment. The experimental precision has been pushed to the level of $O(10^{-2})$ with uncertainties of a few per mille. Finding a truly sizable CPV asymmetry in charm sector would be difficult to accommodate by SM, but the current results are still within the expectations from hadronic uncertainties.

In this section, we first discuss briefly recent tests for lepton universality and lepton flavor violation in the τ sector (deferring $h \to \mu\tau$ to Section 8.3). We then discuss the most recent results associated with the charm quark, focusing mainly on *CP* violation.

7.1. New Physics in τ decay

In SM, the charged current weak interaction couples with the same strength to all lepton flavors. Tests of lepton universality can probe this assumption and provide a strong constraint to the model extensions, for example to some lepton-specific two Higgs doublet models proposed to solve the muon $g-2$ anomaly. Although the direct tests of lepton universality with τ decays do not show any hint of deviation from SM expectation, there are some unresolved tensions in semileptonic B meson decays, such as $B \to D^{(*)}\tau\nu$ and $B \to K\ell\ell$, as introduced in Section 3.

The charge current induced leptonic decay width is

$$\Gamma(\tau \to \ell\nu\overline{\nu}_\ell) = \frac{B(\tau \to \ell\nu\overline{\nu}_\ell)}{\tau_\tau} = \frac{G_\tau G_\ell m_\tau^5}{192\pi^3} f\left(\frac{m_\ell^2}{m_\tau^2}\right)\left(1+\frac{3}{5}\frac{m_\tau^2}{M_W^2}\right)\left[1+\frac{\alpha(m_\tau)}{2\pi}\left(\frac{25}{4}-\pi^2\right)\right], \tag{85}$$

where $G_\ell = g_\ell^2/4\sqrt{2}M_W^2$ for $\ell = e,\ \mu$ are the Fermi coupling constants, f is a phase space factor, and we have made explicit the radiative corrections [258]. By inserting the ratios of partial widths and the measured τ lifetime, the latter significantly improved by a Belle measurement [259], the following ratios of coupling constants are obtained by HFAG 2016 [82] using purely leptonic processes,

$$\left(\frac{g_\tau}{g_\mu}\right) = 1.0010 \pm 0.0015, \quad \left(\frac{g_\tau}{g_e}\right) = 1.0029 \pm 0.0015, \quad \left(\frac{g_\mu}{g_e}\right) = 1.0019 \pm 0.0014. \quad \text{(HFAG)}. \tag{86}$$

If semi-hadronic processes such as $\tau \to (K,\pi)\nu_\tau$ or $(K,\pi) \to \mu\overline{\nu}_\mu$ are also considered, the ratio g_τ/g_μ can be further combined to 1.0000 ± 0.0014, which is in remarkable agreement with lepton universality, providing a very strong constraint to New Physics models in the lepton sector.

Just like the pursuit of $\mu \to e$ transitions, observing lepton flavor violation (LFV) would be an unambiguous signal of New Physics. LFV processes are absent at tree level in SM, and can occur only through tiny neutrino masses at loop level, highly suppressed by the equivalent of the GIM mechanism. The decay rate for $\tau \to \mu\gamma$ is negligible in SM, while the stringent $\mu \to e\gamma$ bound of Eq. (80) dampens one's hope. But several NP scenarios can in principle increase the rate for this mode, or some other decay channels, to more accessible values.

The sensitivity to a particular τ decay channel is very model dependent. Let us mention a few examples. Consider Higgs-mediated decays in SUSY seesaw models [260], where LFV can arise through the renormalization of soft SUSY breaking terms. But as we have no evidence for SUSY so far, LHC Run1 results seem to push expectations for $\tau \to \mu\mu\mu,\ e\mu\mu$ towards 10^{-9} [261], which are not quite within experimental reach in the near future. By introducing additional heavy right-handed Majorana neutrinos, or additional left-handed and right-handed neutral singlets [262], the branching fractions of $\tau \to \mu\gamma,\ e\gamma$ and $\tau \to \mu\mu\mu,\ eee$ can be raised to approximately $O(10^{-10})$–$O(10^{-8})$, which is still pertinent. Adding a non-universal gauge boson Z' in topcolor-assisted technicolor models, the branching fraction of $\tau \to eee$ or $\mu\mu\mu$ can be as large as 10^{-8} within a range of parameter space [263]. But it is not clear whether these models still stand with LHC data, and correlation with the stringent MEG limit on $\mu \to e\gamma$ is a concern. Finally, in the 2HDM-III that we would discuss in Section 8.3, because of flavor changing neutral Higgs couplings and other new Yukawa couplings, $\tau \to \mu\gamma$ might be close [264] to the current experimental limit. Suffice it to say that many proposals can raise the LFV branching fractions to the level of $O(10^{-10})$–$O(10^{-8})$, and also generate other possible LFV B meson or Higgs decays. While NP models are now more and more constrained by LHC search and $\mu \to e\gamma$, the experimental study of rare τ decays provide complementary information, and should continue in any case.

The summary of experimental upper limits from HFAG [82] is given in Fig. 15, which shows a plethora of search modes, especially as compared with rare muon decays of the previous section. Most of the existing measurements are from Belle and BaBar. The most stringent limits are for three light charged lepton final states, i.e. combinations of e or μ, with an entry of $\tau \to \mu\mu\mu$ from LHCb. The best single limits are from Belle with a data set consisting of 7.2 million τ pairs [265]. The analysis was carried out in a very clean environment with expected background level of < 0.2 events. No candidate events were observed in any combination of lepton flavors. Given that all measurement errors are statistics dominant, combinations of limits have been carried out by HFAG based on the standard CL_s method. Due to possible statistical fluctuations of single results, the combined limit could be weaker than from individual experiments. Most of the combined limits are in the range of 10^{-8}–10^{-7}, which are still 1–2 orders of magnitude higher than estimates by various NP models, as illustrated in the previous paragraph. The exception is $\tau \to \mu\gamma$, in conjunction with $h \to \mu\tau$ search at the LHC, as discussed further in Section 8.3.

The upcoming Belle II program, together with advances from LHCb, should provide substantial update to these tests of LFV, and more theoretical attention is encouraged.

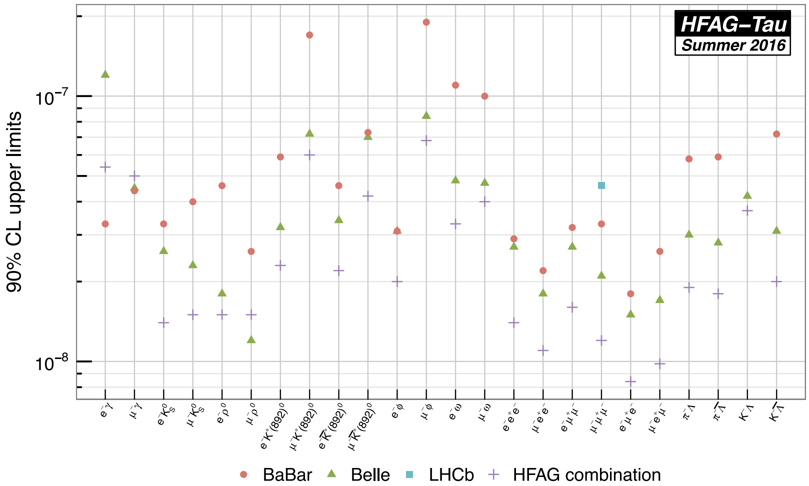

Fig. 15. Upper limits on LFV τ decays, including published individual limits and combined values (marked by +). Due to statistical fluctuations of single results, the combined limits are not necessarily tighter.
Source: HFAG 2016.

7.2. Direct CPV in charm

Non-zero CPV asymmetry requires at least two interfering amplitudes, and with both weak and strong phase differences. In SM, DCPV in charm sector can appear in single Cabibbo suppressed decays, such as interference between $c \to d$ tree and the highly suppressed $c \to u$ penguin processes. Consider a typical D meson decay with amplitude $\mathcal{A}(D \to f)$, and $\mathcal{A}(\overline{D} \to \bar{f})$ for anti-D meson decay. The DCPV asymmetry can be represented by a time-integrated asymmetry in the form of

$$A_{CP} = \frac{\Gamma(D \to f) - \Gamma(\overline{D} \to \bar{f})}{\Gamma(D \to f) + \Gamma(\overline{D} \to \bar{f})}. \tag{87}$$

For the CP eigenstate two-body final states $f = K^+K^-$ and $\pi^+\pi^-$, A_{CP} can be expressed in two terms: the component associated with DCPV in the decay amplitudes and the component associated with indirect CPV in the mixing, or from interference between mixing and decay. The indirect CPV term is in general independent of the decay channels, hence the difference in CPV asymmetries between $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ decays, $\Delta A_{CP} = A_{CP}(D^0 \to K^+K^-) - A_{CP}(D^0 \to \pi^+\pi^-)$, is a good observable of DCPV. The LHCb experiment caused a splash when it announced evidence [30] for nonzero ΔA_{CP}, which subsequently faded with more data [31]. The most recent measurement from LHCb [266] tags the flavor of charm by the charge of the pion from $D^{*+} \to D^0\pi^+$ decays, and gives

$$\Delta A_{CP} = -0.10 \pm 0.08 \pm 0.03\%, \tag{88}$$

which unfortunately is still consistent with zero, and in any case at the per mille level.

As mentioned, the interest is in finding CPV asymmetries above the percent level. Following the formulation in Ref. [267], the time-dependent observable ΔA_{CP} can be expressed by

$$\Delta A_{CP} = \Delta a_{CP}^{\rm dir}\left(1 + y_{CP}\,\overline{\langle t\rangle}/\tau\right) + a_{CP}^{\rm ind}\,\Delta\langle t\rangle/\tau, \tag{89}$$

where $\Delta\langle t\rangle$ is the difference of mean decay time between KK and $\pi\pi$ final state and $\overline{\langle t\rangle}$ is their average, τ is the true lifetime of the D^0 meson, and $y_{CP} = (0.866 \pm 0.155)\%$ [40] is a CPV observable related to the mixing parameter y (see the next subsection for a more detailed discussion). $\Delta a_{CP}^{\rm dir}$ and $a_{CP}^{\rm ind}$ measure direct and indirect CPV, and the current world best average from HFAG [82] is

$$\Delta a_{CP}^{\rm dir} = (-0.137 \pm 0.070)\%, \quad a_{CP}^{\rm ind} = (0.056 \pm 0.040)\%. \quad \text{(HFAG16)}. \tag{90}$$

The precision of these measurements has reached $O(10^{-3})$ already, but the central values are within 2σ from the null CPV hypothesis, as can be seen in Fig. 16.

There are many other potential measurements in hadronic two-body D decays. For example, the $D^0 \to K_S^0K_S^0$ decay can have an enhanced CP violation even within the CKM picture. An estimate [268] of penguin annihilation and exchange amplitudes gives an upper bound on DCPV at 1.1%, while the most recent measurement from Belle has attained an uncertainty of 1.5% [269], but with no indication for DCPV. Many other measurements have uncertainties of a few percent, so there is still plenty of room to look for CPV and New Physics.

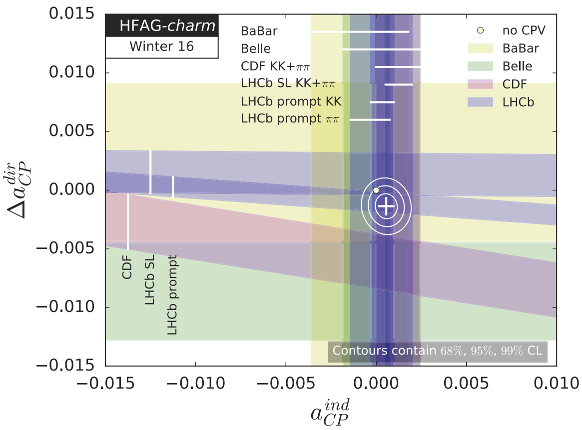

Fig. 16. Combination plot for $\Delta a_{CP}^{\text{dir}}$ and a_{CP}^{ind} from HFAG, with 68%, 95%, and 99.7% C.L. contours around the best fit value marked by +.
Source: http://www.slac.stanford.edu/xorg/hfag.

Looking for DCPV with three-body or multi-body charm decays may have better potential, since CPV can occur in the full decay phase space, and local asymmetries can be larger than the integrated ones (e.g. in effective two-body decays as discussed above). In addition to a direct comparison between positively and negatively tagged distributions, a more generic amplitude analysis can also be applied and has direct access to the phase information of the decay amplitudes. The triple-product correlations evaluated from the independent vectors from a multi-body decay can also be studied and provide a different probe of T-odd asymmetries. Note that multi-body charm decays are also essential for the measurement of CKM angle ϕ_3.

From an experimental point of view, the multitude of multi-body decays provide a measurement bonanza. The analyses require advanced techniques such as Dalitz analysis, and modeling of decay amplitudes with kinematic variables of the decay daughters. In a recent study of three-body $D^0 \to K_S K^{\pm}\pi^{\mp}$ decay by LHCb [270], the magnitude and phase information is studied in the Dalitz plot, and a search for time-integrated CPV shows no clear evidence, within an uncertainty of few %. Given the complexity of multibody decays, several new experimental methods are being developed. For example, a model-independent technique, called "energy test", is introduced by LHCb to study the Cabibbo-suppressed decay $D^0 \to \pi^+\pi^-\pi^+\pi^-$ [271]. The method provides an unbinned description of the decay, and can be used to look for local excess of CPV asymmetries. The technique is used to examine both P-even and P-odd CPV effects. The P-even test separates the events according to flavor, D^0 vs $\overline{D}^0$, for comparison. The P-odd test distinguishes the events according to both flavor and sign of the triple products. The core method is built on a test statistic T, which is used to compare the average distances of events in phase space. If the events in both samples are identical, the value of T should be randomly distributed around zero. Based on the resulting T values and the simulated test statistic distributions, a p-value of (0.6 ± 0.2)% (or 2.7 standard deviations) is found in the P-odd CPV test, while the P-even test shows no hint.

All of these results on DCPV tests can be substantially improved in the near future with a much larger data set from (HL-)LHC and Belle-II. In particular, the experimental sensitivities are approaching SM-allowed upper bounds, and discovery of CPV effects in the charm system could still be just around the corner. But as stressed in the Introduction, we do not consider DCPV, be it in B or D decays, as promising probes for NP.

7.3. *Indirect CPV in charm*

The mixing of neutral charm mesons is well-established. The two mass eigenstates $|D_1\rangle$ and $|D_2\rangle$ of a neutral charm and anticharm meson system can be expressed as linear combinations of flavor eigenstates $|D^0\rangle$ and $|\overline{D}^0\rangle$,

$$|D_1\rangle = p\,|D^0\rangle + q|\overline{D}^0\rangle, \quad |D_2\rangle = p\,|D^0\rangle - q|\overline{D}^0\rangle, \tag{91}$$

with complex coefficients satisfying $|p|^2 + |q|^2 = 1$. The mixing parameters are defined as

$$x = 2(m_2 - m_1)/(\Gamma_1 + \Gamma_2), \quad y = (\Gamma_2 - \Gamma_1)/(\Gamma_1 + \Gamma_2), \tag{92}$$

where $m_{1,2}$ and $\Gamma_{1,2}$ are the masses and decay widths of the two mass eigenstates, respectively. CP violation is established if a non-unity value is found for the parameter λ_f,

$$\lambda_f = -\left|\frac{q}{p}\right|\left|\frac{\overline{A}_{\bar{f}}}{A_f}\right| e^{-i\phi}, \tag{93}$$

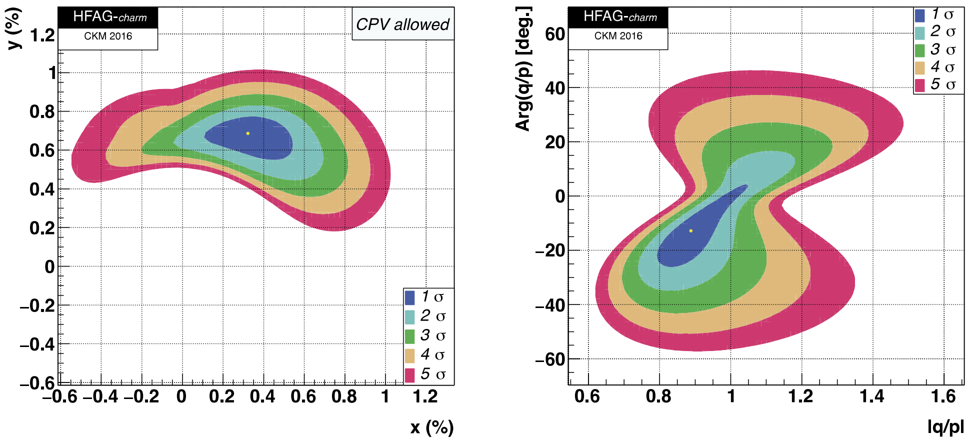

Fig. 17. Two dimensional contours for [left] charm mixing parameters x and y, and [right] the CPV parameters $|q/p|$ and ϕ. The best fit point is indicated as a white dot, and the 1σ–5σ contours are shown.
Source: HFAG 2016.

where A_f and $\bar{A}_{\bar{f}}$ are the amplitudes for D^0 and $\overline{D}^0$ decaying into final state f and $\bar{f}$, and $\phi = \arg(q/p)$ is the CPV weak phase. If $|q/p|$ deviates from unity, or if phase ϕ is nontrivial, one has indirect CPV.

The decay $D^0 \to K^-\pi^+$ proceeds through a Cabibbo-favored (CF) tree diagram. Decay to the charge-conjugate final state, $D^0 \to K^+\pi^-$, proceeds through a Doubly Cabibbo-suppressed (DCS) process, or a mixing process together with a CF decay, $D^0 \to \overline{D}^0 \to K^+\pi^-$. The relative rates of DCS to CF decays is expressed as

$$R(t)^{\pm} \approx R_D^{\pm} + \sqrt{R_D^{\pm}}y' \frac{t}{\tau} + \frac{(x'^{\pm 2}) + (y'^{\pm})^2}{4}\left(\frac{t}{\tau}\right)^2, \tag{94}$$

where the $+$ and $-$ signs in the exponent denote the decays of D^0 or $\overline{D}^0$, respectively. The primed parameters, x' and y', are rotated from the x and y parameters by a strong phase difference between CF and DCS amplitudes. The most recent measurement from LHCb [272] using double-tagged $\overline{B} \to D^{*+}\mu^- X$, $D^{*+} \to D^0\pi^+$ events, gives

$$\begin{aligned}
&R_D^+ = (3.474 \pm 0.081) \times 10^{-3}, \;\; R_D^- = (3.591 \pm 0.081) \times 10^{-3},\\
&(x'^+)^2 = (0.11 \pm 0.65) \times 10^{-4}, \;\; (x'^-)^2 = (0.61 \pm 0.61) \times 10^{-4},\\
&y'^+ = (5.97 \pm 1.25) \times 10^{-3}, \quad y'^- = (4.50 \pm 1.21) \times 10^{-3}. \quad \text{(LHCb, Run1)}.
\end{aligned} \tag{95}$$

These results, together with many other recent measurements, provide strong constraints on $|q/p|$ and ϕ, in particular for the region near $\phi = 0$. A recent measurement using four-body $D^0 \to K^+\pi^-\pi^+\pi^-$ decay has also been carried out [273]. The analysis measures the ratio between DCS and CF in bins of decay time,

$$R(t) \approx (r_D^{K3\pi})^2 - r_D^{K3\pi} R_D^{K3\pi} y'_{K3\pi} \frac{t}{\tau} + \frac{x^2 + y^2}{4}\left(\frac{t}{\tau}\right)^2, \tag{96}$$

where the measured $r_D^{K3\pi}$ is the phase space averaged ratio of DCS to CF amplitudes, the coherence factor $R_D^{K3\pi}$ covers the phase difference between DCS and CF amplitudes, $y'_{K3\pi}$ is rotated from the mixing parameters x and y with the average strong phase difference $\delta_D^{K3\pi}$. These parameters have been most precisely determined to date as $r_D^{K3\pi} = (5.67 \pm 0.12) \times 10^{-2}$ and $R_D^{K3\pi} y'_{K3\pi} = (0.3 \pm 1.8) \times 10^{-3}$, and are essential for the CKM ϕ_3 angle measurement.

Fig. 17 shows the two dimensional contours from HFAG [82] for the mixing parameters x and y, and the CP parameters $|q/p|$ and ϕ. Charm mixing is firmly established ($>11.5\sigma$ away from no mixing hypothesis) from the combined fit. However, the fit also shows no indication of CP violation, i.e. within 1σ from the CP-conserving reference point $|q/p| = 1$ and $\phi = 0$.

A complementary study for CPV in the charm system can be achieved by measuring the two observables A_Γ and y_{CP}. The observable A_Γ measures the decay time asymmetry of D^0 and $\overline{D}^0$ decaying into the same CP eigenstate f, including K^+K^- and $\pi^+\pi^-$. The explicit definition is given by

$$A_\Gamma = \frac{\Gamma(D^0 \to f) - \Gamma(\overline{D}^0 \to f)}{\Gamma(D^0 \to f) + \Gamma(\overline{D}^0 \to f)} = \frac{1}{2}\left[\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right) y\cos\phi - \left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right) x\sin\phi\right], \tag{97}$$

which contains the CPV effects in mixing and in interference. Given there is no hint for DCPV asymmetry in charm measurements, A_Γ is mainly a pure probe of indirect CP violation. It relates to the time-dependent asymmetry $A_{CP}(t) \approx$

$a_{CP}^{\rm dir} + a_{CP}^{\rm ind}\, t/\tau$, where $a_{CP}^{\rm ind}$ is just $-A_\Gamma$ in the limit of no DCPV. The observable y_{CP} measures the difference of effective lifetime for decay into CP eigenstate and decay into a flavor-specific state, such as $D^0 \to K^-\pi^+$,

$$y_{CP} = \frac{\tau(D^0 \to K^-\pi^+)}{\tau(D^0 \to K^+K^-)} - 1 = \frac{1}{2}\left[\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right) y\cos\phi - \left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right) x\sin\phi\right], \quad (98)$$

in absence of DCPV. If CP is conserved, the y_{CP} observable is equal to the mixing parameter y.

With a simultaneous fit to $D^0 \to K^+K^-$, $\pi^+\pi^-$, and $K^-\pi^+$ candidates from the final Belle data set [274], the result is

$$y_{CP} = (+1.11 \pm 0.22 \pm 0.09)\%, \quad A_\Gamma = (-0.03 \pm 0.20 \pm 0.07)\%. \quad \text{(Belle)}. \quad (99)$$

The measured values are consistent with world averages[26] and show no indication of CP violation. LHCb has recently updated the A_Γ measurements with two different methods. Unbinned maximum likelihood fits have been carried out to measure effective lifetimes with $D^0 \to K^+K^-$ and $\pi^+\pi^-$ events from 2012 data [275]. The observable A_Γ is measured separately in K^+K^- and $\pi^+\pi^-$ channels, and then combined with 2011 results:

$$A_\Gamma(K^+K^-) = (-0.14 \pm 0.37 \pm 0.10) \times 10^{-3}, \quad A_\Gamma(\pi^+\pi^-) = (0.14 \pm 0.63 \pm 0.15) \times 10^{-3}. \quad \text{(LHCb)}. \quad (100)$$

Another analysis is carried out with $D^0 \to K^+K^-$ and $\pi^+\pi^-$ events with full LHCb Run 1 sample [276], measuring the asymmetries in bins of proper decay time, $A_{CP}(t) \approx a_{CP}^{\rm dir} + a_{CP}^{\rm ind}\, t/\tau$. The detection biases are corrected using control samples of $D^0 \to K^-\pi^+$ events. The analysis finds consistent results,

$$A_\Gamma(K^+K^-) = (-0.30 \pm 0.32 \pm 0.14) \times 10^{-3}, \quad A_\Gamma(\pi^+\pi^-) = (0.46 \pm 0.58 \pm 0.16) \times 10^{-3}, \quad \text{(LHCb)} \quad (101)$$

which is the most precise measurement of CPV in charm to date. By combining with other existing measurements, the average from HFAG

$$A_\Gamma = (-0.032 \pm 0.026)\%, \quad \text{(HFAG)} \quad (102)$$

is not so different from Eq. (99), and consistent with no CP violation at few per mille level. We do not have any hint for NP so far in charm sector.

It is expected that some of these studies can be further improved both in statistics, by including a much larger data set from (HL-)LHC, and in systematics, by introducing more robust modeling and methodology. In any case, these studies are also essential for the ϕ_3 CKM phase angle measurement, and further investigations are necessary.

8. Top/Higgs

We now discuss a subfield that was not traditionally regarded as part of flavor physics (and associated CPV): top and Higgs decay. These involve the two heaviest particles we know, and the champion top quark receives mass from the runner-up Higgs field, with a Yukawa coupling $\lambda_t \cong 1$ that generates most FPCP effects within SM. But the top quark itself decays before it can hadronize into a "top meson", and the extremely short lifetime makes rare top decays a somewhat depressed field. Nevertheless, FCNC $t \to cZ$ decays have been searched for since [17] the Tevatron discovery of the top. Furthermore, the P_5' anomaly suggests there could be analogous top FCNC tcZ' couplings of a new Z' boson.

With the discovery of the Higgs boson $h(125)$, however, it is the top in conjunction with the Higgs that is drawing attention: $t \to ch$ decay, which is possible because $m_h < m_t$. As these are the newest particles we have uncovered, from a purely experimental point of view, the flavor-changing neutral Higgs (FCNH) tch coupling is of fundamental interest, *because it can be probed directly*. There is no such tree level coupling within SM after mass diagonalization, and the loop-induced effect can be ignored for all practical purposes. In 2HDM extensions, prejudice towards FCNH couplings had been to "remove before birth", by the so-called *Natural Flavor Conservation* (NFC) condition of Glashow and Weinberg [277], where one invokes a discrete symmetry to forbid FCNH (thereby 2HDM-I and 2HDM-II). But it was pointed out long ago [278,279] that, given the "trickling off" flavor pattern observed in quark masses and mixings, Nature does have her schemes regarding flavor, while NFC itself turns out "anthropological" (human, not "Natural"). FCNH involving 3rd generation fermions in a 2HDM extension is *natural*, which was thereby called 2HDM-III [280]. It should in fact be called the Standard 2HDM, or SM2 for short, as it just follows SM without *ad hoc* assumptions. Now that ATLAS and CMS experiments search directly for $t \to ch$ (Section 8.2) and $h \to \mu\tau$ (Section 8.3) decays, these are in fact true, current frontiers, where discovery could emerge at anytime.

8.1. Top FCNC: $t \to cZ^{(\prime)}$

By the GIM mechanism, there is no tcZ coupling at tree level in SM, with loop effects further suppressed by GIM cancellation. The case is worse than charm: whatever makes the corresponding B decays favorable, the situation is turned upsidedown for top. Unlike the prolonged B lifetime, the top is the shortest-lived ever of all known particles, while if the

26 The current HFAG fit is $y_{CP} = (0.835 \pm 0.155)\%$ [82], consistent with the input used for arriving at Eq. (90), and consistent with the Belle measurement of Eq. (99) as well.

nondecoupling of the top quark makes rare B decays interesting, the near degeneracy of the d, s and b quark masses at the M_W and m_t scales means GIM cancellation is very effective. This is what we mentioned earlier that $t \to cZ$ decay is negligible for all practical purposes within SM. New Physics might have saved the day if there were new particles at the weak scale. Alas, we have not found any at the LHC so far, neither at 8 TeV nor at 13 TeV collision energies, with scales pushed to a few TeV, i.e. considerably higher than the weak scale.

The current world best limit is from CMS Run 1 data [281],

$$\mathcal{B}(t \to cZ) < 0.05\%, \quad \text{(CMS)} \tag{103}$$

at 95% C.L., with slightly weaker but comparable limit from ATLAS [282]. This result translates into "a constraint on the KK gluon to be heavier than 1.1 TeV" [281], which is actually quite good for a bound coming from an indirect rare top decay probe. However, it should be clear that if there were KK gluons at 1 TeV scale, they would have been discovered by direct search at the LHC already. Indeed, compared with pre-LHC estimates, warped extra dimension benchmarks for $t \to cZ$ have moved down [283] to 10^{-5} level with LHC Run 1 data, and suppressed by top compositeness scale M_*^{-4}, which means the number would shrink further with 13 TeV bounds on M_*. Other models, even in most favorable considerations in say R-parity violating SUSY, the situation is no better [284]. The projections from theory seem to be beyond reach even at the HL-LHC with 3000 fb^{-1}, where estimates by CMS [285] and ATLAS [286] are at 0.01% order.

We note that limits on $t \to cg$ and $c\gamma$ are better, and in particular, single top production processes can effectively probe smaller branching fractions than $t \to cZ$, e.g. $\mathcal{B}(t \to ug) < 4 \times 10^{-5}$ [287] from ATLAS, with $t \to cg$ limit five times weaker. In our view, however, modeling for FCNC involving gluons or photons would be in general much more contrived, while $t \to u$ transitions do not seem favorable compared to $t \to c$ transitions from the known flavor pattern. Search, of course, should continue. What may be more interesting is tZ associated production through the process,

$$g + c \to t + Z, \tag{104}$$

from tcZ coupling. A recent study by CMS [288] using 8 TeV data, sets comparable limit on $\mathcal{B}(t \to cZ)$ as Eq. (103). In this connection, we remark that there may well exist a weaker coupled Z' boson associated with $t \to c$ transitions. While the left-handed tcZ' coupling suggested by the P_5' anomaly is too weak to be directly probed at the LHC, the anomaly has inspired the suggestion of a right-handed tcZ' coupling that is not much constrained by B decays [208]. A recent study which could be, but not necessarily, related to the gauged $L_\mu - L_\tau$ model, suggests [289] that tZ' associated production analogous to Eq. (104) might lead to discovery with even 100–300 fb^{-1} at 13–14 TeV collision energies.

Our point is to emphasize that, despite some pessimism for the $t \to cZ$ probe, the top quark might still be a key to potential New Physics discoveries in the future.

8.2. Top FCNH: $t \to ch$

Tree level tch coupling is certainly absent in SM, and GIM cancellation is effective at the loop level, such that $t \to ch$ decay is again negligible in SM for all practical purposes. The usual 2HDM-I and 2HDM-II follow the NFC condition [277] of Glashow and Weinberg, such that FCNH is again absent at tree level: each type of fermion charge receive mass from one and only one Higgs doublet, so the Yukawa and mass matrices are always simultaneously diagonalized. With 2HDM-II automatic in MSSM, it became the most familiar two Higgs doublet model. Cheng and Sher, however, offered some dissent [279] a decade after the NFC condition. Generalizing the Fritzsch ansatz [278], they noted that if the generic Yukawa matrices for multi-Higgs models follow an $\sqrt{m_i m_j}$ -like pattern, then low energy FCNC can be naturally suppressed. It was then pointed out [280] that the tch coupling would be the most interesting, with $t \to ch$ the one to watch if $m_h < m_t$.

This is now indeed the case, and we advocated [290] $t \to ch$ search at the LHC. Sure enough, efforts within ATLAS [291] on $t \to ch(\to \gamma\gamma)$ search, or in association with CMS [292] and in multi-lepton final states, were already ongoing, and within two years of Higgs boson discovery, the limits already reached the sub-percent level [293,294], which is quite remarkable. Both ATLAS and CMS embarked on $t \to ch$ search in $t\bar{t}$ events with $h \to \gamma\gamma,\ WW^*,\ \tau^+\tau^-$, as well as $b\bar{b}$, and the Run 1 combined limits for 8 TeV collisions are,

$$\mathcal{B}(t \to ch) < 0.46\%, \ \ 0.40\% \quad \text{(LHC Run 1)} \tag{105}$$

at 95% C.L. for ATLAS [295] and CMS [296], respectively.[27] On the one hand, these sub-percent limits are rather impressive, as they could have already resulted in discoveries, and could still in principle emerge tomorrow, and all this on the backdrop of the prevailing NFC prejudice beforehand. On the other hand, judging from the time it took the experiments to conduct their study, the dominant $h \to b\bar{b}$ final state [298] seems to suffer from serious background, and it remains to be seen how improvements would unfold with Run 2 data at 13 TeV. Based on the cleaner $h \to \gamma\gamma$ mode, however, ATLAS projects [299] a final reach at HL-LHC of $\mathcal{B}(t \to ch) < 1.5 \times 10^{-4}$ at 95% C.L.

27 Based on 36.1^{-1} data at 13 TeV, ATLAS has recently reported [297] the new limit of $\mathcal{B}(t \to ch) < 0.22\%$ at 95% C.L.

It should be noted that, with h^0 denoting the 125 GeV boson, all data point towards the decoupling or alignment limit, that h^0 is very close to the SM Higgs boson. In the context of 2HDM, it means that its non-standard couplings, such as tch, are modulated by a small mixing angle with the exotic CP-even Higgs boson H^0,

$$\rho_{ct}\ \cos(\beta-\alpha)\,\bar{c}th^0, \tag{106}$$

where we have kept the 2HDM-II notation of $\cos(\beta-\alpha) \simeq 0$ as the h^0–H^0 mixing angle, and ρ_{ct} is the FCNH Yukawa coupling of heavy Higgs H^0.[28] Thus, FCNH for h is "protected" to be small, which is a generalization from the Cheng–Sher argument. Ref. [290] stressed that $\rho_{ct} \sim 1$ is possible, while the analogous ρ_{cc} and ρ_{tt} could also be $\mathcal{O}(1)$, and are not well constrained. Given our poor handle on $h \to c\bar{c}$ measurement, 2HDM-III highlights the importance of precision measurement of $h \to \tau\tau,\ c\bar{c}$, and gg with more data. The latter could be enhanced by ρ_{tt}, but $h \to WW^*,\ ZZ^*,\ \gamma\gamma$ must be close to SM expectation because $\cos(\beta-\alpha) \sim 0$. As for the Yukawa coupling ρ_{bb} of heavy Higgs H^0, it enters $b \to s\gamma$ loop via the charged Higgs, modulated by CKM matrix elements, and also receives a chiral enhancement factor m_t/m_b, and has been shown [290] to be ∼0.01 if ρ_{ct} is sizable. Hence $h \to b\bar{b}$ is also SM-like. The Yukawa pattern discussed here can be checked at LHC Run 2. Note that for large $\rho_{ct} \sim 1$, the heavy neutral Higgs bosons H^0 and A^0 could be searched for in $t\bar{c}$ final states [300], opening up a new search program.

8.3. Standard 2HDM: $h \to \mu\tau$ (and CPV)

In the previous subsection, we have given an almost experimental account on $t \to ch$ search at the LHC, to make clear that ATLAS and CMS would simply do it, regardless of "doctrines" such as NFC, or prejudices from MSSM (i.e. 2HDM-II). In this subsection, we broaden the view to advocate that 2HDM-III [280] with FCNH should in fact be called the "Standard 2HDM", or SM2 for short, in the same way we treat SM: let Nature have her say.

Motivated in part by the tch discussion, as well as the difficulty of Higgs search at the Tevatron, $h \to \mu\tau$ search was suggested [301,302] at the turn of the millennium.[29] Fast forward to LHC Run 1, it is remarkable that, reconstructing τ leptons in the electronic and hadronic decay channels, CMS uncovered [303] a 2.4σ effect with 8 TeV data,

$$\mathcal{B}(h \to \mu\tau) = \left(0.84^{+0.39}_{-0.37}\right)\%, \quad \text{(CMS 8 TeV)} \tag{107}$$

or $\mathcal{B}(h \to \mu\tau) < 1.51\%$ at 95% C.L., which aroused quite some interest. The corresponding bound [304] from ATLAS with 8 TeV data is 1.84% at 95% C.L., which is not inconsistent. Alas, an early peek by CMS at 13 TeV with 2.3 fb^{-1} data [305] did not quite support Eq. (107), finding $\mathcal{B}(h \to \mu\tau) < 1.20\%$ when 1.62% was expected.[30] It could be that the 8 TeV excess was an upward fluctuation, but it could also be that the early 13 TeV result had a downward fluctuation. Suffice it to say that $\mathcal{B}(h \to \mu\tau) \sim 0.5\%$ is perfectly possible. Unfortunately, neither CMS nor ATLAS updated this Higgs decay mode at ICHEP 2016 held in Chicago, nor at 2017 Winter conferences. But clearly the issue can be clarified with the large amount of 2016 data at 13 TeV, and with full Run 2 data.

The importance of $h \to \mu\tau$ direct search at LHC had been emphasized [306,307] before the CMS study [303] that gave the result of Eq. (107), and in fact provided strong motivation for it. Let us discuss the development of thoughts and studies. After the $t \to ch$ suggestion [280] within 2HDM-III, impact of FCNH in the lepton sector was explored [308] for $\mu \to e\gamma$ which involves $\mu\tau h$ and τeh couplings, and the importance of the two-loop diagram was emphasized. This brings in a top loop with ρ_{tt} coupling [290] to heavy Higgs boson H^0, which was mentioned below Eq. (106). That is, an effective $H\gamma^*\gamma$ correction is attached to the $\mu \to \tau \to e$ line, which could compete with the one loop diagram for $\mu \to e\gamma$, as the latter is suppressed by three chirality flips, while the former has only one. The fact that two-loop diagrams may in fact dominate in these transitions were originally pointed out by Bjorken and Weinberg [309], but it is often called the Barr–Zee mechanism [310], from the independent but similar diagrammatic discussion for electron EDM. With discovery of Higgs boson h^0 in 2012, and following the earlier suggestion [306] that $h^0 \to \mu\tau$ is still allowed even at $\mathcal{O}(10\%)$, Ref. [307] converted the detailed formulas for $\mu \to e\gamma$ [308] to the case of $\tau \to \mu\gamma$ for a more thorough study. While confirming the suggestion of Ref. [306], the authors of Ref. [307] pointed out further that direct search for $h^0 \to \mu\tau$ at the LHC would quickly give rise to a better bound, if not discovery, of finite $\mu\tau h$ coupling, or

$$\rho_{\mu\tau}\ \cos(\beta-\alpha)\,\bar{\mu}\tau h^0, \tag{108}$$

that is more stringent than coming from $\tau \to \mu\gamma$. In fact, the CMS study [303] took over the banner plot of Ref. [307], which we display in Fig. 18[left]. The plot compares $\tau \to \mu\mu\mu,\ \mu\gamma$ indirect bounds with direct bound from $h \to \mu\tau$. We note that $|Y_{\mu\tau}| = |\rho_{\mu\tau}\ \cos(\beta-\alpha)|$, reflecting the $Y_{\mu\tau}$ notation used in Ref. [307], which treated the FCNH of h^0 directly. This may be an oversimplification, while we have not distinguished between $\rho_{\mu\tau}$ and $\rho_{\tau\mu}$ for simplicity of discussion. We stress that exotic $h^0 \to \mu\tau$ decay is naturally suppressed by a small h^0–H^0 mixing angle (called "alignment"). If a sizable rate at the level of Eq. (107) is confirmed, it implies a sizable $\rho_{\mu\tau}$, but if $h^0 \to \mu\tau$ disappears, it could just be due to a small $|\cos(\beta-\alpha)|$, and should not diminish the possibility of FCNH $\mu\tau H^0$ coupling.

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[28] The physical ctH^0 coupling would be modulated by $\sin(\beta-\alpha) \simeq \pm 1$.

[29] Another motivation is the observation of large ν_μ–ν_τ mixing in the late 1990s. But this does not really translate to large $h \to \mu\tau$, because of the extreme lightness of neutrinos and the large mixing angles of the PMNS matrix (CKM counterpart in lepton sector). We are not sure that neutrino masses are generated by the same Higgs mechanism. But again, the experiments can simply do it once $h(125)$ is discovered, while Nature can have her say.

[30] Corresponding to $\mathcal{B}(h \to \mu\tau) = \left(-0.76^{+0.81}_{-0.84}\right)\%$, a negative central value, which tends to affect theorists.

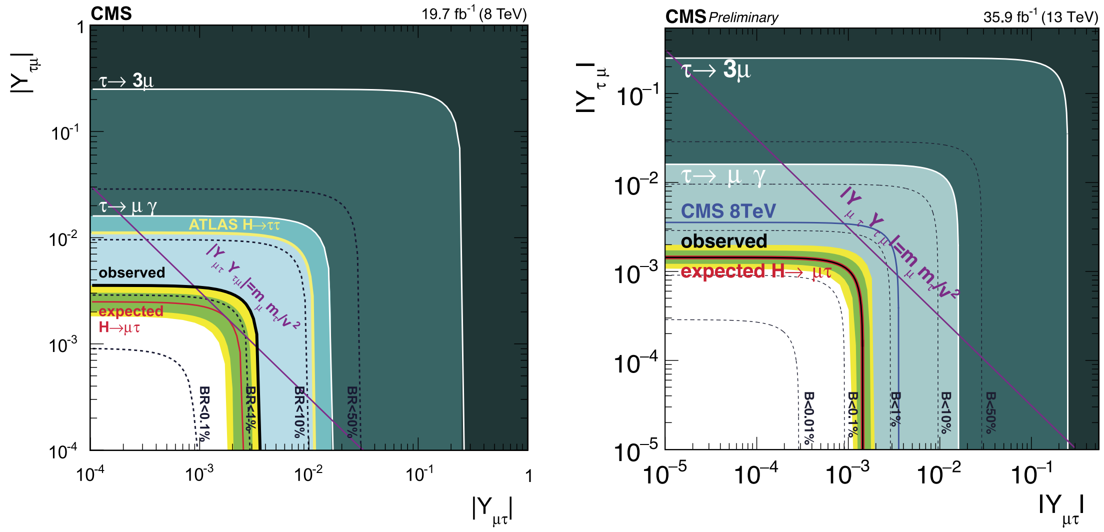

Fig. 18. Comparison of CMS results for $h^0 \to \mu\tau$ (h^0 marked as H in plot) with $\tau \to \mu\gamma$, 3μ bounds in the $|Y_{\mu\tau}|$–$|Y_{\tau\mu}|$ coupling plane. The heavy (thin) solid contour is the observed (expected), dashed contours are marked with $\mathcal{B}(h^0 \to \mu\tau)$ values, while thin solid diagonal line corresponds to the "Cheng–Sher" value of $|Y_{\mu\tau}Y_{\tau\mu}| = m_\mu m_\tau/v^2$. [Left]: 8 TeV result, hinting at excess (observed above expected); [Right] 13 TeV result (2016 data), which rules out 8 TeV hint (lower observed coinciding with expected). See text for further discussions.
Source: Courtesy CMS Collaboration, from Refs. [303] and [311].

Note Added (Post LHCP 2017): At the LHCP 2017 conference held in Shanghai, the CMS experiment unveiled the result of 2016 data set at 13 TeV, with a total of 35.9 fb^{-1}, which does not support the 8 TeV result of Eq. (107). The new bound (see Fig. 18[right]) is [311]

$$\mathcal{B}(h \to \mu\tau) < 0.25\%, \quad (\text{CMS 13 TeV},\ 35.9\ \text{fb}^{-1}) \tag{109}$$

at 95% C.L. We have kept our discussion to give a sense of excitement/disappointment and development.

2HDM-III, or SM2?

We have by now introduced the flavor changing couplings ρ_{ct} (Eq. (106)), $\rho_{\mu\tau}$ (Eq. (108)), as well as the flavor conserving couplings ρ_{cc}, ρ_{tt}, of the heavy Higgs boson H^0. These couplings mix into the coupling of h^0 by the small mixing parameter $\cos(\beta-\alpha)$, which means that the ρ_{ij} couplings of the physical H^0 boson should be modulated by a $\sin(\beta-\alpha)$ factor. Likewise, the SM couplings of h^0 would pick up a $\sin(\beta-\alpha)$ factor, while these mix into H^0 with a $-\cos(\beta-\alpha)$ factor. The h^0–H^0 mixing arises from the Higgs potential, where data indicates that $|\cos(\beta-\alpha)|$ is quite small. Our stated context is the 2HDM, but we have debunked the myth of NFC. The observed light h^0 boson (with small H^0 admixture) gives rise to VEV, and hence the three other components of the mass-giving doublet become Goldstone bosons that are absorbed into the longitudinal parts of the massive $W^\pm$, W^0 gauge bosons. The apparent decoupling/alignment indicates that the second Higgs doublet is considerably heavier, with a scalar H^0, a pseudoscalar A^0, and charged scalar $H^\pm$ components that are not too far apart in mass. We shall assume *CP* is conserved within the Higgs sector.[31] The key point is that, with the simultaneous diagonalization of the fermion mass matrices and Yukawa matrices of the mass-giving doublet, the $\sin(\beta-\alpha)$-dependent couplings of h^0 are flavor diagonal. For the second Higgs doublet, however, the Yukawa matrices *cannot* be simultaneously diagonalized with the mass matrices, hence are naturally flavor changing. This general phenomenon of FCNH is now being probed at the LHC, as described above, although Nature seems to have been hiding it by a small $|\cos(\beta-\alpha)|$.

Interest in 2HDM-III, which we now prefer calling SM2, in fact grew already after BaBar announced the $B \to D^{(*)}\tau\nu$ anomaly [129], claiming that it cannot be explained by 2HDM-II. It was then pointed out [138,139] that the BaBar anomaly could possibly be explained by the charged Higgs boson $H^\pm$ of 2HDM-III,[32] i.e. 2HDM that allows for FCNH at tree-level. It thus became quite an active field. But a charged scalar, or scalar operator, explanation for the BaBar anomaly has been challenged [144,145] recently by argument of the B_c lifetime, i.e. the $B_c \to \tau\nu$ fraction. Thus, in this backdrop, it is the direct searches such as $t \to ch$ or $h \to \mu\tau$ that would be the unequivocal tests for the presence of an extra Higgs doublet, attesting to the power of direct search. Discovering these modes, of course, does not directly prove the existence of a second doublet with FCNH couplings, as other models are possible (see e.g. Refs. [312] and [313]).

Why would we like to call 2HDM-III the "standard" 2HDM, or SM2? Yukawa couplings are provided, but not predicted in SM, and we just learn their values from Nature by experimentation. Now that a mass-giving Higgs doublet seems affirmed,

31 Otherwise it could have easily been detected by neutron EDM.

32 As well as other NP such as leptoquarks, see Section 3.4.

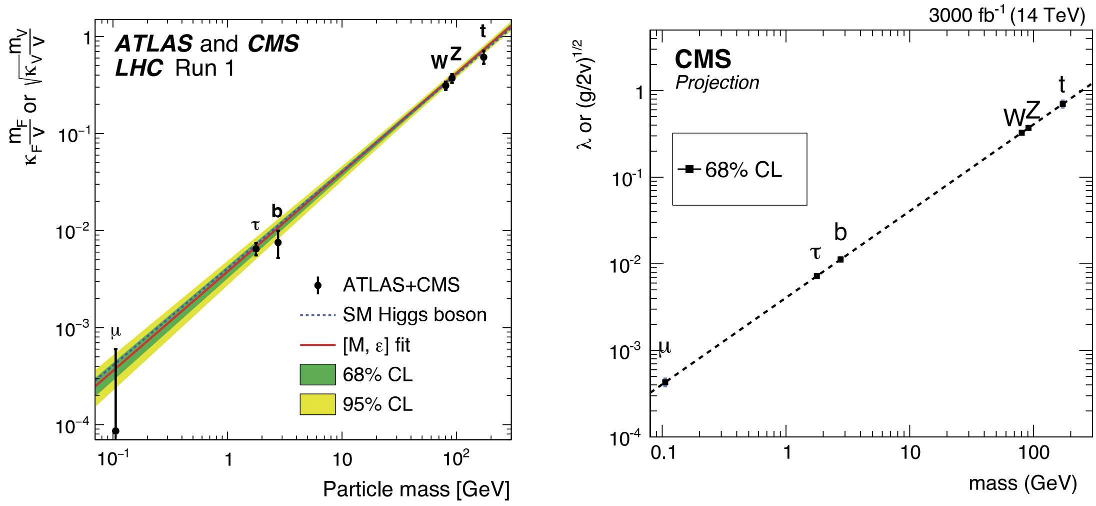

Fig. 19. Mass vs Yukawa coupling "linear plot": [left] combined Run 1 measurement, and [right] CMS projection for HL-LHC. Note the absence of charm. *Source:* Left figure, courtesy ATLAS and CMS Collaborations, from Ref. [318]; right figure, courtesy CMS Collaboration, from Ref. [319].

given the generation repetition of fermions, it seems rather plausible that Nature would provide a second Higgs doublet. In contrast, an extra scalar singlet could be conveniently added for model building, but it would be relatively arbitrary as it is not related to electroweak symmetry breaking, while triplets have issue with electroweak precision measurables. Although 2HDM traditionally comes with NFC that eliminates FCNH couplings, but in comparing with SM, we should "learn their values from Nature by experimentation". It is in this sense that it should be called SM2, since FCNH is general, while Nature has apparently opted for decoupling/alignment, such that the exotic component of $h(125)$ is quite suppressed. There is a second reason why we would like to call it SM2. With a myriad of new Yukawa couplings, including FCNH couplings such as ρ_{ct}, $\rho_{\mu\tau}$ that we have discussed, it was pointed out recently [314] that the extra ρ_{tt} coupling, which can carry CPV phase (since phase freedom has already been used in establishing the CKM matrix), could be sufficient for electroweak baryogenesis, i.e. generating baryon asymmetry of the Universe (BAU) by weak scale physics. Besides ρ_{tt}, the second Higgs doublet also supplies the needed first order phase transition by Higgs sector couplings. If so, given the known deficiency of SM towards this very important effect for our own existence, it may well be Nature's design to achieve matter–antimatter asymmetry of the Universe through a second Higgs doublet. Calling it SM2 would only be fitting.

With the myriad of new Yukawa couplings and associated CPV phases, SM2 would greatly enrich the future FPCP program. For example, on the direct search front, given $\rho_{tt} \neq 0$, even if one is in the decoupling/alignment limit, there is still $gg \to H^0$ production via a top triangle loop. Assuming $m_{H^0} > 2m_t$, although establishing a resonance in $t\bar{t}$ may be challenging because of interference with $gg \to t\bar{t}$ background [315–317], one could search for, and perhaps discover the H^0 (and A^0) boson through FCNH $H^0 \to t\bar{c}$ [300] search. The Higgs–Top flavor physics may be just starting.

9. Conclusion

Taking on the topic of Flavor Physics and *CP* violation, we have covered a large swath of ground. The CKM edifice that governs charged current weak interactions, including through interesting loop effects such as box and penguin diagrams, shows no sign of fraying. As it stands so well through the B factory era, and by now the first round of LHC, it largely convinces the FPCP practitioner of the "reality" (and, well, complexity) behind, or underneath the edifice: Yukawa couplings and their structure are very real! The CKM matrix elements, including the CPV phase(s), are due to Yukawa couplings, which are dynamic. These are the same couplings that generate fermion masses through the Higgs mechanism, which is nicely illustrated in the mass–coupling linear plot, Fig. 19 from the LHC. The lefthand side of Fig. 19 summarizes our current knowledge of direct measurements [318] of Yukawa couplings from Higgs boson h decays or interactions. While the muon case is not yet really measured, and the charm quark is also absent (difficulty to charm-tag and measure $h \to c\bar{c}$), the apparent linear relation between mass and coupling is truly remarkable. But it is just the more recent echo from hadron collider experiments that discovered the h boson, where FPCP physicists have known since observation of B^0 mixing, that the dynamical, nondecoupled top mass effect is there.

The righthand side of Fig. 19 is the projection [319] of the mass–Yukawa linear plot to the end of the HL-LHC running, ca. 2035, where the absence of charm is still conspicuous. Methods would supposedly be developed to cover this. Impressive as Fig. 19 is, the current CKM measurement as reflected in Fig. 1 is no less impressive, and probably more beautiful. And, with advent of B physics agenda at Belle II coming soon, and with the excellent performance of LHCb (and also CMS and ATLAS), even before HL-LHC starts, the improvement of Fig. 1 would already be comparable to Fig. 19[right]. What is more, just like the pursuit of New Physics by direct search at the LHC, FPCP has a broad program to indirectly probe New Physics. The aim of this review is not just for the current status, but to give the prospect for the early 2020s, i.e. by time of LHC Run 3.

Perhaps the CKM fit would show some cracks? The CPV phase of ϕ_s may still be more promising, and LHCb is the experiment to watch, which is luckily crosschecked by CMS and ATLAS. $B_s \to \mu^+\mu^-$ would become better measured, for sure. But would we discover $B^0 \to \mu^+\mu^-$ with Run 2 data? If so, it would necessarily mean New Physics beyond SM. The world, however, would need to be convinced that LHCb and CMS have backgrounds under control.

The P_5' and BaBar anomalies look quite enticing, and would certainly improve soon, although the BaBar anomaly would really need Belle II for a more convincing probe with high statistics. Concurrently, the $B \to \tau\nu$ and $\mu\nu$ modes should be scrutinized to give us a better picture of what type of New Physics, if any, could it be. The recent update by CMS show consistency of P_5' with SM. Though with larger errors than LHCb, it is a cautionary note to be taken, but it also makes Run 2 exciting on this front. On the other hand, the very recent LHCb result based on Run 1 data finds R_{K^*} to be consistent with R_K (despite the issue of lower bin fr R_{K^*}), which together offer important and theoretically clean probes of New Physics. We not only await Run 2 results to unfold, but also the imminent turn-on of Belle II.

B physics is not just composed of these highlights. There is a large number of more traditional B factory subjects, some of which can be studied by LHCb, but some would need Belle II data to scrutinize, such as semileptonic, leptonic (with neutrino) and radiative decays. We would soon start to enjoy the influx of such information. For instance, is there anomaly in the $B \to \tau\nu, \mu\nu$ rate or not? Is the mild hint of $B \to K^{(*)}\nu\nu$ above SM expectation anywhere real? How about ΔS, i.e. deviation of indirect CPV for $b \to s\bar{q}q$ modes from $b \to c\bar{c}s$ modes? What would be the outcome of the $B \to K\pi$ direct CPV sum rule test?

We are in a lucky period where major progress is expected in the next few years on $K^+ \to \pi^+\nu\nu$ and $K_L \to \pi^0\nu\nu$. NA62 should measure the K^+ mode with some precision by 2020, while KOTO should start to see the evidence for the K_L mode. Perhaps New Physics would unfold? KOTO has some chance to uncover a Dark particle with mass of π^0. In the same time frame, diligent work by lattice theorists may expose a new problem, or anomaly, in ε'/ε, and at the same time demonstrate the coming of age of lattice itself in FPCP.

Turning to low energy, we will see major development in both $\mu \to e\gamma$ (MEG-II) and $\mu \to e$ conversion (COMET and Mu2e), with prospect for new measurements, if not discovery. This program has some parallel in $\tau \to \mu\gamma$, $\mu\mu\mu$, eee with Belle II, while LHCb can also pursue the all charged track processes. Back to muons, finally we expect to see the muon $g-2$ anomaly scrutinized experimentally again, and there is great anticipation for the outcome from the new Muon g-2 experiment at Fermilab. At even lower energy, there is good promise for new measurements of electron and neutron EDM, and new developments maybe even for the proton. This is the far, but truly important reach, of the CPV quest under FPCP banner.

Returning to rather high energies, ATLAS and CMS are vigorously pursuing $t \to cZ$ (and hopefully start on $t \to cZ'$) and $t \to ch$ search, where there is better prospect for improvement in the latter with Run 2 data. Despite the Run 1 hint for $h \to \mu\tau$ was not confirmed at 13 TeV, $h \to \mu\tau$ could still reemerge with full Run 2 data. If so, or if $t \to ch$ is discovered, it would exclude 2HDM-II, the favorite two Higgs doublet model from SUSY, as it would call for tree level flavor changing neutral Higgs couplings. But if these are not seen, it would add another support to alignment, that h is an SM-like Higgs particle. However, from the BaBar anomaly, interest in $\tau \to \mu\gamma$, top and Higgs FCNH couplings, to electroweak baryogenesis, we find this General 2HDM, which we would prefer calling SM2 if any genuine evidence emerges, appealing. It would provide many new observables and parameters to test and check, and usher in a new era for FPCP.

We have put forth many questions on many fronts, many of which could soon make progress. We can only hope that some genuine New Physics would emerge from at least one of these probes.

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