A view of neutrino studies with the next generation facilities

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Neutrino physics is nowadays receiving more and more attention as a possible source of information for the long-standing investigation of new physics beyond the Standard Model. The rather recent measurement of the third mixing angle θ_{13} in the standard mixing oscillation scenario encourages the pursuit of what is still missing: the size of any leptonic CP violation, absolute neutrino masses and the characteristic nature of the neutrino. Several projects are currently running and they are providing impressive results. In this review, the phenomenology of neutrino oscillations that results from the last two decades of investigations is reviewed, with emphasis on our current knowledge and on what lesson can be taken from the past. We then present a critical discussion of current studies on the mass ordering and what might be expected from future results. Our conclusion is that decisions determining the next generation of experiments and investigations have to be strictly based on the findings of the current generation of experiment. In this sense it would be wise to wait a few years before taking decisions on the future projects. In the meantime, since no direct path forward is evident for the future projects, the community must be committed to their careful evaluation.

1. Introduction

The current scenario of the Standard Model (SM) of particle physics, being arguably stalled by the discovery of the Higgs boson, is *desperately* looking for new experimental inputs to provide a more comfortable theory. In parallel, experiments on neutrinos so far have been an outstanding source of novelty and unprecedented results. In the last two decades several results were obtained by studying atmospheric, solar or reactor neutrinos, or more recently with neutrino productions from accelerator-based beams. Almost all these results have contributed to strengthen the flavour-SM. Nevertheless, relevant parts like the values of the leptonic CP phase and the neutrino masses are still missing, a critical ingredient being the still undetermined neutrino mass ordering. On top of that the possibility of lepton flavour violation (if e.g. neutrinos are Majorana particles), is a very open issue, experimentally strongly pursued.

Even if the Standard Model can be easily extended with right-handed neutrinos to introduce Dirac mass terms, notwithstanding the lightness of the neutrino masses points to very small and unnatural Yukawa couplings. The latter issue is likely overcome by considering a Majorana neutrino mass and some choices of see-saw mechanisms. This peculiarity of neutrinos, compared to the other charged fermions, originates from the fact that they are neutral particles. The possible Majorana nature of neutrinos would correspond to lepton-flavour violation and a real portal for new physics beyond the SM. It is

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intriguing and wishful that studies on neutrinos could uncover some of the solutions to the open questions in fundamental physics. However, it might even happen that all our hopes are shattered in the end, and a coherent picture of SM will continue to hold, i.e. three flavours with no Majorana mass and a *natural* mass hierarchy, together with a tiny leptonic CP-phase.

Nevertheless, there are already some measurements that do not fit the standard 3-flavour neutrino-framework hinting instead at the possible existence of one (or more) *sterile* neutrino. That is a very wide issue, as well as one experimentally strongly pursued, too. If the existence of the dark matter and its possible interplay to neutrinos are additionally taken into consideration, thus the present picture turns out to be very stimulating.

From 2012 neutrino-oscillation physics entered a new era, as many applicable measurements were collected in the meantime. From one side, phenomenological fits were continuously improved by inputs given by those measurements. A coherent picture could be expected to emerge for the four most relevant missing pieces, namely, the CP-phase, the mass ordering, the octant of the largest mixing angle θ_{23}, and the presence or not of new sterile-like states at the eV mass scale. In any case the phenomenological scenario will be tested by the ensemble of inputs providing either a coherent or not-coherent picture. From the other side, many new experimental proposals were put forward, even if some of them not yet fully funded. In the context of the strategy the neutrino community is requested to take for the future, all of these proposed future projects must be carefully evaluated and perhaps even rejected in the event that the currently running experiments and approved projects will be able to confirm and complete the *standard* scenario by the year 2020–2025 (or less).

It would be unconceivable even to think to include in this short review descriptions of all the facts today known about neutrinos together with an exhaustive discussion of the whole set of experiments and proposals for the near future. Therefore a concise attitude is adopted, either referring to the bibliography or not including on purpose many results/studies/projects not so relevant to the mainstream of the discussion, which is instead focussed on the major issues according to the judgement of the author. The paper is organised as follows. In the next section an overview of the acquired phenomenological scenario for neutrinos is presented, while in the following one a critical discussion on the future determination of the mass ordering/hierarchy (MH) is depicted. A brief description of the major on-going experiments and fully funded proposals, useful to the mainstream, follows. In the last section some final considerations and conclusions are drawn. Several issues are just mentioned and not developed, as attempted measurements of individual neutrino masses, and the studies on the production and detection of the solar and supernova neutrinos.

2. Neutrino phenomenology in the last two decades and nowadays

The most famous *hunter* of neutrinos is probably Raymond Davis, Jr.. From the late sixties, with collaborators he looked at neutrinos coming from the Sun [1]. It took almost three decades to collect about 2000 solar electron-neutrino candidates in the Homestake experiment, much less (about 1/3) than what predicted by John N. Bahcall and collaborators (see, e.g., Ref. [2] for a discussion). Even if the neutrino deficit w.r.t. the solar models was unveiled quite soon [3], the dispute was finally settled by the confirmation of the neutrino oscillation. That was reached by the observation of the oscillations in both the atmospheric-neutrino sector by Super-Kamiokande (SK) in 1998 [4] through the ν_μ disappearance[1] and the solar sector by SNO in 2002 [5] through the measurement of the neutral current (NC) interactions, equally sensitive to all the neutrino flavours. The NC measurement confirmed the predictions of the solar model, and therefore the rightness of the deficit by Davis and Bahcall as due to a flavour changing of neutrinos from the Sun.

However, the just evident neutrino mass mixing was again puzzled by the simultaneous null result of CHOOZ in 1998 [6] that looked at neutrino oscillations at a very short distance (1 km) from an anti-ν_e reactor flux. The puzzle on flavours was clarified in 2002 after the KamLAND [7] measurement of the reactor-neutrino flux at an averaged distance of 180 km from several nuclear power plants. KamLAND showed evidence of the spectral distortion as function of L/E (distance over neutrino energy) providing insights of the 3-flavour structure. In Fig. 1 the (later) beautiful result by KamLAND is reported, with almost two complete oscillation cycles observed.

To better explain the general picture it is necessary to go back to the initial idea of Pontecorvo, who in 1957 introduced the concept of neutrino oscillation [9], further elaborated by Z. Maki, M. Nakagawa and S. Sakata in 1962 [10] and Pontecorvo himself in 1968 [11]. However, one had to wait until the measurement of KamLAND for a clear understanding of the mismatch between the diagonalisation of the charged lepton mass matrix and that of the neutrino mass matrix, similarly to what happens in the quark sector with the CKM matrix [12]. The mismatch is described by a unitary matrix, U_{PMNS} to honour the pioneering authors, that mixes the 3 flavour states ν_α, $\alpha = e, \mu, \tau$, of the weak interactions with the 3 mass eigenstates ν_i, $i = 1, 2, 3$:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \cdot \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}.$$

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[1] The correct inheritance of the physics measurements and results on atmospheric neutrinos is more articulated than here reported. More experiments were actually involved, see e.g. [24].
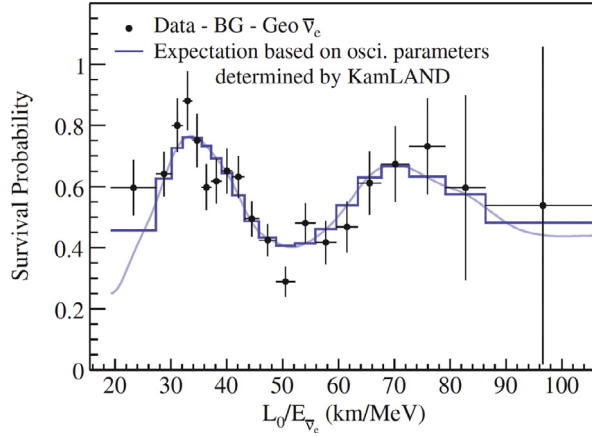
Fig. 1. The neutrino oscillation pattern measured by KamLAND from the reactor flux (Fig. 3 of [8]). The histogram shows the ratio of the observed background-subtracted $\overline{\nu}_e$ spectrum to the expectation for no-oscillation as a function of L_0/E. $L_0 = 180$ km is the effective baseline, as if all anti-neutrinos detected in KamLAND were due to a single reactor at this distance. The energy bins are equal probability bins of the best fit including all backgrounds (mainly accidentals, neutron related from α-decay of ^{210}Po and secondary interaction from ^{13}C(α, n)^{16}O, and geoneutrinos, attainable to a total of about 20% of the observed 1609 events, in the ratio 0.05:0.10:0.05, respectively). The curve shows the expectation based on the best fit oscillation parameters.

U_{PMNS} depends upon six real parameters: three mixing angles, θ_{12}, θ_{23}, θ_{13} that correspond to the three Euler rotations in a three-dimensional space, and three phases, δ, α_1, α_2. A suitable parameterisation is

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_1/2} & 0 \\ 0 & 0 & e^{i(\alpha_2/2)} \end{pmatrix}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. The phases δ ($\equiv \delta_{CP}$) and α_1, α_2 are Dirac-type and Majorana-type CP violating phases, respectively. The above description holds only for 3-flavours neutrinos, even if it can be extended to a basis with one or more neutrinos, either *steriles* i.e. neutrinos that do not couple to the weak interactions or neutrinos that do not contribute to the invisible width of the Z^0 boson. In that case one usually assumes that extra neutrinos mix with the standard neutrinos, i.e. that the mixing matrix is not degenerate. Applying the time evolution to the mass eigenstates in vacuum

$$|\nu_i(t)\rangle = \exp(-iE_i(t))|\nu_i\rangle,$$

and using the unitarity of the mixing matrix, the vacuum transition amplitudes and probabilities are obtained:

$$P_{\alpha\beta} = A^*_{\alpha\beta}A_{\alpha\beta} = \sum_{i,j=1}^{3} U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}e^{-i(E_i - E_j)t}$$

If ultra-relativistic neutrinos are taken then $E = \sqrt{p^2 + m_i^2} \simeq E + \frac{m_i^2}{2E}$, with $t \simeq L$, and the transition probabilities are expressed in terms of *frequencies*, defined as $\Delta m_{ij}^2 L/E$, where $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$, E is the neutrino energy[2] and L the travelled distance. In case of three flavours only two of such frequencies are independent, e.g. $\Delta m_{21}^2 = m_2^2 - m_1^2$ and $\Delta m_{32}^2 = m_3^2 - m_2^2$, usually named for historical reasons the *solar* (or δm^2) and the *atmospheric* (or Δm^2)[3] oscillation frequencies, respectively. Finally, the evolution in time brings to probabilities for survival (= 1 – disappearance probability) and appearance of a neutrino flavour with energy E over the distance L. In the simplest case where only two-flavours are involved the probabilities are described by an oscillation *amplitude* that depends on the mixing angle and an oscillation *frequency* that depends on the mass scales as well as the experimental constraints, L and E. The two-flavour approximation was widely used until 2002 when it was understood that this was not always appropriate.

Coming back to the CHOOZ/KamLAND results, the associated probabilities are obtained by convolution over the three flavours. If Δm^2 is expressed in eV^2, L in km and E in GeV, one gets (for a more exhaustive discussion see e.g. [14], chapter 3):

$$P(\nu_e \to \nu_\mu) = s_{23}^2 \sin^2 2\theta_{13} \sin^2(1.27 \times \Delta m^2 L/E) + c_{23}^2 \sin^2 2\theta_{12} \sin^2(1.27 \times \delta m^2 L/E),$$

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta_{13} \sin^2(1.27 \times \Delta m^2 L/E) - c_{13}^4 \sin^2 2\theta_{12} \sin^2(1.27 \times \delta m^2 L/E).$$

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[2] The issue on which energy to consider for the individual neutrino state is analysed and solved in [13].

[3] Sometimes, and usually in the phenomenological fits, Δm^2 is defined as $\Delta m^2 = m_3^2 - \frac{m_2^2 + m_1^2}{2}$.
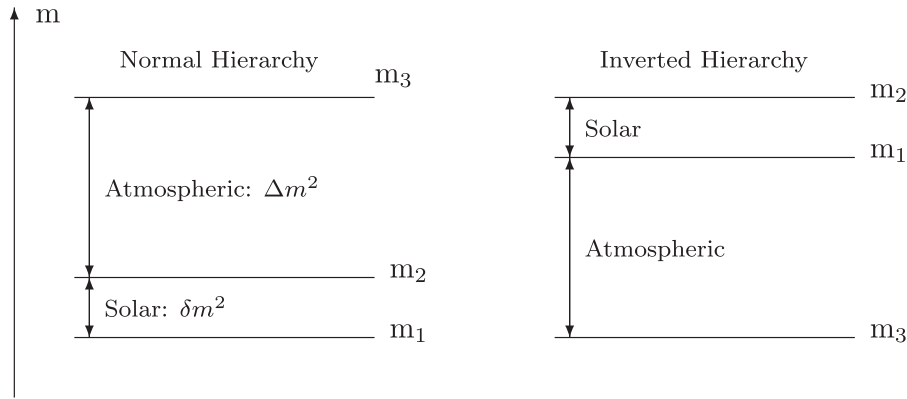
Fig. 2. Neutrino mass eigenstates for normal and inverted mass ordering (not to scale).

In the above expressions the CP-violating terms are omitted. Furthermore $|\Delta m^2| \gg \delta m^2$ is assumed, that is $\Delta m^2_{31} \simeq \Delta m^2_{32}$, so removing for a while the issue on the neutrino-mass ordering.[4] The assumption is justified by the fact that atmospheric mass-splitting $|\Delta m^2_{32}|$ is more than one order of magnitude greater than the solar mass-splitting δm^2_{21}.[5]

Analysing the probability expectations for $\nu_e \to \nu_\mu$ at the nuclear-reactor energies, the results of CHOOZ and KamLAND are explained by a) the low value of θ_{13} that allows the first term to be dropped, and b) the low value of δm^2 and hence the necessity for a rather large distance L to become sensitive to the second term (two orders of magnitude larger than the CHOOZ baseline of 1 km). Indeed Nature was very *vicious* with CHOOZ, since the value for θ_{13} was really "around the corner", and big enough to moderate the dreams/needs for new technologies. It is worth to outline that CHOOZ could probably catch θ_{13} by lowering its systematics effects. However, researchers did not find sufficient motivations to try to improve the detector just to gain few percents of phase-space. The next generation of reactor-experiments were developed under a general feeling of criticisms and disbelief.

The experimental and phenomenological scenario after 2002 was really exciting: θ_{23} and θ_{12} had been measured whereas the quest for the last missing mixing-angle θ_{13} prompted to a large variety of proposals, models and endless discussions. The possibility that $\theta_{13} = 0$, which implies no CP-violation in the leptonic sector, was widely parsed. That hypothesis had to be absolutely checked, even if very large-scale detectors and accelerators were plainly needed. In that period flourish of ideas on new accelerating beams like super-beams, beta-beams and neutrino-factories were developed (see e.g. [18]). Different strategies were set up, depending on the value of θ_{13}, below the limit set by CHOOZ, $\theta_{13} < 12^0$ (90% C.L.). In retrospect, besides the novelty of new techniques, always useful for future experiments, the lack of physics case (i.e. a very small θ_{13}) seems nowadays evident. That is a lesson that the neutrino community should learn: the request for always larger detectors and systems should be really motivated by a founded physics case. The prejudice that θ_{13} were very small was perhaps founded on the not proper appreciation of the (excellent) data analysis performed by CHOOZ, believed more conservative than it really was. Many papers investigated only the range to few degrees of θ_{13} (see e.g. [19]). This point is further discussed in later sections.

In the following 10 years, up to 2012, a large collection of measurements was gathered, all confirming the oscillation pattern and (almost) all consistent with the 3-neutrino framework. For example, the very recent (2015) observation of the ν_τ appearance from a ν_μ beam by the OPERA experiment [20], was largely expected and, in some respects, just delayed of about 10 years.[6]

Besides θ_{13} and the possible leptonic CP-violation there were other missing parts, for which the famous MSW effect must be recollected. In 1978 L. Wolfenstein [30] showed that the propagation of neutrinos is significantly modified in the presence of ordinary matter due to their interactions with electrons, protons, and neutrons. Moreover, the coherent forward elastic scattering amplitudes are not the same for all neutrino flavours, ν_e, ν_μ and ν_τ, since ν_e have additional contributions due to their charged current (CC) interactions with matter. After few years of studies (and corrections of mistakes) by several authors, finally in 1985 S.P. Mikheev and A.Yu. Smirnov [31] discovered resonant flavour transitions are possible when neutrinos propagate in a medium with varying density. That briefly accounts for the electron-neutrino oscillation

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[4] Mass ordering is associated to either $m_1 < m_2 < m_3$ or $m_3 < m_1 < m_2$. As far as oscillations are concerned the dependences on the mass ordering come from the interference between two effects. In vacuum the interference can be given by the joint atmospheric and solar oscillations (see later).

[5] Already towards the end of 2002, just after the SNO result, the two-flavour fits (see e.g. [15]) predicted the so-called large-mixing angle (LMA) as best fit with a *solar* mass-splitting around 10^{-5} eV^2, much smaller than the interval indicated for the *atmospheric* mass-scale by the first measurement from SK, $5 \times 10^{-4} < \Delta m^2 < 6 \times 10^{-3}$ eV^2 at 90% C.L., updated in 2004 as $1.5 \times 10^{-3} < \Delta m^2 < 3.4 \times 10^{-3}$ eV^2 at 90% C.L. [16]. The LMA was confirmed by including KamLAND results [17].

[6] Proposal for the OPERA experiment dated 1997 and the project was finally approved in 1999–2000, after the SK discovery on atmospheric neutrinos. Always in retrospect, it would have been wiser to stop the project for a couple of years, and hence to cancel it. OPERA may however be able to provide interesting insights on the presence of sterile neutrinos at Long-Baseline (LBL) [21,22].
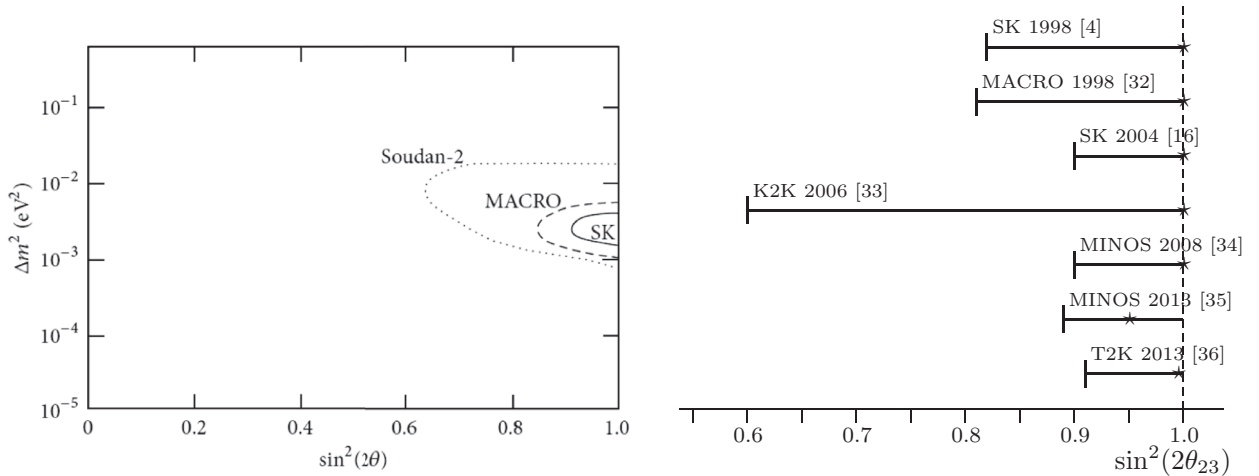
Fig. 3. Left: allowed regions for atmospheric neutrino oscillation, as measured by SK, MACRO [32] and SOUDAN-2 at 90% C.L., from 1998 up to year 2004. The figure is taken from [24]. Right: the most relevant measurements of $\sin^2(2\theta_{23})$, at 90% C.L., up to year 2013. The best fit, constrained to the physical region, is shown by the star. From around 2013 the angle θ_{23} began to be estimated with the $\sin^2(\theta_{23})$ variable.

pattern reported by the experiments on solar neutrinos. Similar effects are expected when neutrinos, in particular ν_e, travel inside the Earth at baselines of the order of at least 1000 km.

An aside effect of solar-matter effects is that $\delta m^2 > 0$. Instead, the sign of Δm^2 is not supplied, so far, by the atmospheric experiments. Thus, the structure of the neutrino mass–matrix is not fully determined, and several solutions are possible when estimating the value of δ_{CP}. In the framework of three neutrino-flavours, the two possible solutions for the mass ordering are usually named as Normal Hierarchy (NH) and Inverted Hierarchy (IH), for $\Delta m^2 > 0$ and $\Delta m^2 < 0$, respectively[7] (Fig. 2).

Together with δ_{CP} and the mass ordering, the third relevant ingredient for the evaluation of the full neutrino picture is the deviation, with sign, of the atmospheric mixing angle, θ_{23}, from $\pi/4$. The mixing angles θ_{ij} span two octants since $\theta_{ij} \in [0, \pi/2]$. While θ_{12} is around $33.5°$ within about 5%, θ_{23} has always been measured compatible with maximal mixing, i.e. $\theta_{23} = \pi/4$. Currently its error is around 10%. The maximal mixing corresponds to an equal contribution of ν_μ and ν_τ to the third neutrino mass state. Besides the solution of the mass ordering there are huge implications in cosmology and symmetry models: depending of their values θ_{12} and θ_{23} regulate the percentage of ν_e, ν_μ and ν_τ and the relative mass contributions. It is interesting to consider at the historical series of measurements of θ_{23}, as reported in Fig. 3.

From 2006 experiments on neutrino beams produced by accelerators, namely K2K [33], MINOS [34,35] and T2K [36], started to release results. From around 2013 the improvements in the precision of the measurements forced the analyses to be done within the full 3-flavours formalism, being also sensitive to its correlation with MH (therefore θ_{23} started to be estimated with the $\sin^2(\theta_{23})$ variable, see Fig. 3). Presently, the best estimation is provided by T2K [37], followed by MINOS [38].

Extended discussions on the neutrino oscillation in vacuum and in matter can be found nowadays in textbooks like e.g. [23], chapters 6 to 9. Some interesting papers on the historical perspectives, neutrino mass matrix and related issues are [24, 25] and [26] (we do purposely avoid to list notable papers before the measurement of θ_{13} in 2012 because the outlook differs before and after that date).

After the years 1998 and 2002 the third *annus mirabilis* for neutrino physics was 2012. Predictions of a large value for θ_{13}, i.e. close to the CHOOZ limit, were made in 2011 as preferred solution of the phenomenological fits [27], just before the discovery was claimed by the reactor experiments in 2012 [28]. The current estimated value of θ_{13} is $8.5°$ [29] with a combined 5% precision at $1\,\sigma$ level.

With the assessment of a non-zero and relatively large value of θ_{13} the possibility to measure CP-violation in the leptonic sector in a reasonable period of time is highly increased. In fact, the CP violation arises from the complex phase of the mixing matrix ($\delta_{CP} \neq 0, \pi$) and from the presence of at least three flavours that mix up ($\theta_{13} \neq 0$). The measurement of δ_{CP} may come from the different transition probabilities for neutrinos and antineutrinos.[8] However, if the *CPT* invariance holds, the transition probabilities for $\nu_i \rightarrow \nu_j$ and for $\overline{\nu}_j \rightarrow \overline{\nu}_i$ are equal. Hence no *CP*-violation can be observed in the

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[7] Most often the definition for the mass hierarchy is defined as the sign of Δm^2_{31} or $\frac{1}{2}(\Delta m^2_{31} + \delta m^2_{21})$, which better reflect the dependences in the transition probabilities. We keep Δm^2_{32} for an easier interpretation in the present context.

[8] While the *CP* conserving terms depend on $\sin^2(1.27 \times \Delta m^2 L/E)$, and it is the same for neutrinos and antineutrinos, the *CP* violating part depends on $\sin(2 \times 1.27\Delta m^2 L/E)$. The latter oscillates with a doubled frequency compared to the *CP* conserving part, being of opposite sign for neutrinos and antineutrinos. The need for at least three flavours is simply due to the fact that, even if one introduces an additional *CP* phase, the quartic invariant of the transition amplitude cannot become complex for two flavours. That is the reason, in passing, that forced Kobayashi and Maskawa to postulate three flavours in the quark sector due to the presence of *CP* violation in the hadronic flavour mixing.

disappearance mode ($i = j$). Instead, one needs to observe the transition among flavours, for both neutrinos and antineutrinos, to access CP violation.

The disentangling of the full picture in neutrino flavours/oscillations might come from detailed global fits, which are able to cover the whole set of available measurements. In such a case only joint analyses that include the different outcomes from all the experiments would produce reliable results. This sentence may appear too much blunt, in particular to experimentalists, who vigorously analyse their data to extract physical results. Our conclusion is motivated by the fact that either the neutrino interactions collected by single experiments usually provide quite limited datasets or correlation terms of the oscillations become too much large to be neglected. One example of analyses developed under questionable approximations, using only data of its own experiment, can be found in [39,40]. The consistent analysis in that case is the one performed in [21]. The correct approach seems finally accepted by the community, see e.g. [41–44]. However, for the author, there are still some points that need to be better clarified. Between them the most relevant one is the way to establish the mass hierarchy. It will be treated in the next section.

We end this short review on neutrino phenomenology by addressing the fourth point in the list of critical parameters to apprehend in the near future, the sterile neutrino. The experimental story of sterile neutrinos began in 1998 with the results of the LSND experiment [45].[9] At that time these results generated some confusion on global analyses, studies and proposals. Unfortunately the only experiment setup to confirm the result of LSND was inconclusive [48,49]. At the same time the quest and request of (at least one) sterile neutrino rose up, in particular for its possible contribution to dark matter. Furthermore, from around year 2010, additional experimental hints emerged from computations of the reactor neutrino fluxes [50] and the calibration of radioactive sources [51]. These neutrino *anomalies* could be coherently interpreted as due to the existence of a fourth sterile neutrino with a mass at the eV scale. Thus there were/are sufficient motivations to develop more projects and proposals [52].

So far, there is no demonstration of a sterile neutrino state with a mass around 1 eV. The current projects are attacking the issue mainly from three sides: oscillation behaviour at short distance (SBL beams) [53], deficit of $\overline{\nu}_e$ at nuclear reactors [54], and ν_e and $\overline{\nu}_e$ disappearance from radioactive sources [55]. Even if the real possibilities of these proposals remain challenging, they would either confirm or disprove the LSND result. Another proposal [56] being at the same time robust and able, in case of a positive outcome, to fully demonstrate the *sterile* origin of the anomalies (as originally addressed in [57]) was unfortunately not approved by FNAL.

The presence of sterile neutrinos, in particular at 1 eV, is a very open question that affects the results of the analyses. A good example is in [44], where the inclusion of a sterile state is shown to wash out the disentangling of the mass hierarchy. The final (negative) response on sterile neutrinos of 1 eV mass is expected in the next couple of years (by the year 2017–2018), when reactor and radioactive source experiments will start their (short) data taking. It is worth to outline that measurements sensitive to the possible presence of sterile neutrinos are also expected by IceCube [58] and LBL experiments, like e.g. MINOS+ [59].

It is clear that many relevant questions on neutrinos have not been considered here, since we focussed mainly on neutrino flavour oscillations. However it is important at least to mention the neutrinos from supernova bursts, the direct mass measurement, the solar and cosmological contributions of neutrinos. These four areas of investigations are undoubtedly relevant i.e. they are all worth *per se* the current and planned future activities. Other projects, not mentioned here, are probably not worth major effort, being interesting only as side results of more general items.

3. Discussion on the MH determination

The issue of the mass ordering has been highly debated in the last decade, but it gained in interest with the discovery of the relatively large value of θ_{13} in 2012. The convolutions between the three mixing angles and the mass parameters are such that measurements of the current experiments may become sensitive to the dependences of the oscillation probabilities to the sign of MH. Surely the MH determination will be a major point for the next experiments under construction. All the methods developed for establishing whether MH is normal or inverted are based on the computation of the difference of χ^2 with respect to the best-fit solutions of NH and IH ([60–62]). Even the Bayesian-statistics approach finally deals with that *indicator* [63]. The adopted expression is:

$$\Delta \chi^2 = \min \chi^2_{IH} - \min \chi^2_{NH},$$

where the χ^2, computed for a set of parameters, is defined from the likelihood $\mathcal{L}$ of the data according to $\chi^2 = -2 \log(\mathcal{L})$, thus making $\Delta \chi^2$ equivalent to the likelihood ratio of the best fit points in IH and NH cases.

In the frequentist approach several unappealing features are present, like in some cases the rejection of both NH and IH, and the fact that the chosen statistics cannot be approximated by a Gaussian one, or, in other words, Wilks' theorem [64] does not apply to a binary measurement. In the referred papers the second issue has been solved, the first has not. Moreover, the way the sensitivity is computed (how many σ's and which test-statistics is used) is matter of discussion. Further concerns relate to a more general consideration. So far in all the computations the interplay between the two

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[9] Actually, the first publication of LSND dated 1995 [46] and it immediately addressed the Δm^2 region around 1 eV^2. The possibility of a sterile origin was part of the discussion afterwards. However, the *sterile* hypothesis was seriously considered only after the discovery of the atmospheric oscillation in 1998, when it became clear that the LSND result did not really fit with both that observation and the solar neutrino deficit [47].

options, NH and IH, has been taken into account by computing the best-fit solutions for both options[10]. In this way they provide an answer to the question: what is the right hierarchy? The procedure mimics what has been developed for the Higgs search [66–68], based on the so-called CL_s [69] method, and that has been also positively applied in the search for sterile neutrinos [70,71]. However, if the neutrino phenomenology is described within the 3-flavour pattern, one of the two options has to be true and the other wrong, i.e. NH and IH are mutually exclusive. From a physical point of view we are not really interested in the wrong answer. In other words, we should prefer to identify the right hierarchy, forgetting about the wrong one. It is a discovery process, and not an exclusion one.

The confusion arose from the fuzzy evidence on the sign of MH that began to appear in 2012. It was believed more efficient/right to use statistical estimators sensitive to both options: [NH true – IH false] and [NH false – IH true], as exhaustively discussed in [60]. This approach may be justified for analyses based on set of data coming from a single experiments, but in this case the sensitivity is a quite delicate issue, as discussed in [62].[11] It is interesting to note that in the last 3 years confusion and disputes about the sensitivity on MH have been so large that some papers [72] even quoted the sensitivity for just the right solution itself. In the light of the present discussion that choice corresponds to the worst option. It would have been similar to establish the Higgs particle just by counting the initial handful of collected events, so reaching a 2–3 σ sensitivity instead of 5 σ. However the idea in [72] to introduce a continuous variable α might be interesting since it would allow to perform an analysis à la Higgs [67]. One should just keep in mind to evaluate the sensitivity to disprove IH, i.e. to compute the minimisation starting from $\alpha = -1$ following the notation of paper [72]. It is worth noting that [72,73] took the approach that is going to be proposed below. Unfortunately the same authors decided to go for the $\Delta\chi^2$ estimator in their recent paper [74].[12]

We suggest that today the issue can be approached in a more basic and straightforward way, more comfortable and more understandable for the community. A change in perspective is therefore desirable. In particular one must decide to investigate the sensitivity to either confirm or discard one of the two MH's. As experienced in many past measurements, it is usually much simpler to evaluate the discovery of a signal than to quantify its exclusion with an upper-limit. The key-point is to identify a good, and possibly optimal estimator for the test statistics. In this context the discovery of a new signal is founded on the exclusion of the no-signal hypothesis (H_0), and its sensitivity is given by the p-value of the test statistics on H_0. There is no need to include information on the signal hypothesis (H_1). The case of the Higgs search is somehow different. Looking for new particles the strength parameter μ that weights the cross-section of the new particle is introduced. The CL_s method is an optimal one [69], and μ is tested against the $\mu = 0$ hypothesis over the best solution $\hat{\mu}$ when looking for discovery. For NH/IH case an optimal test statistics has to be looked for. That cannot be μ, simply because the best fit is obtained over the sum *signal + background*, while NH and IH are mutually exclusive hypotheses.

It is also mandatory to check the consistency of the data among themselves. For example, if the estimation of θ_{13} by reactor experiments would not be consistent to that by LBL ones, let us say for less than 10% on the related phase space, it should make no sense to put them together to extract a solution for MH.

In the very recent release of results from NOvA [75,76] the right approach was chosen: the significance on MH has been computed separately for NH and IH, taking θ_{13} as estimated by reactor data. Despite the not so precise wording and the use of the biased technique of Feldman and Cousins [77],[13] we can extrapolate from Fig. 4 (from Fig. 4 of [76]) that NOvA disfavours IH at 3 σ (double-sided?) in the $0 < \delta_{CP} < 0.8\,\pi$ range, for the less restrictive data selection.

Since there are already hints and intuitive arguments that the option chosen by Nature be NH, the right approach should establish the exclusion of the IH possibility. Then, a simple goodness-of-fit test would be sufficient to disprove IH. Groups involved in global fits should start to quote the $\min\chi^2_{IH}$ and its probability, quantifying the p-value for IH, properly defined as $P(\chi^2 \geq \min\chi^2_{IH})$. It is worth to note that the conversion to the number of σ's has to be computed via the one-sided Gaussian-test convention, since the minimum χ^2 is computed.[14] In [78] a new estimator for MH is introduced as well as the subsequent perspectives for the determination of the mass hierarchy in the near future.

Following the NOvA results an interesting work was developed by A. Palazzo ([44]), who took into account also the effect of a sterile neutrino at the 1 eV mass scale. In the 3-flavour picture a consolidation of the preference for NH is extracted, disfavouring CP conservation with a statistical significance close to 90% C.L. However the preference is washed out in the 3+1 framework. The light sterile neutrinos constitute a potential source of fragility in the identification of the right mass

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[10] I saw only one study done by SK [65] that assumed as true the NH and computed the χ^2 for IH based on the best solution of NH. This goes along the approach suggested here, but it is incorrect as it does not exclude the possibility of degeneracy in case IH owns a different solution with a χ^2 at a similar level.

[11] Anyway, the approach on $\Delta\chi^2$ brought to uncorrected conclusions grounded in the discussion of type I and type II errors, which is not appropriate for the physics case under discussion. The neutrino community was led to think, perhaps by some of them trying to promote a (too much) robust experimental proposal, that the accurate sensitivity on MH should be extracted only via the contemporaneous evaluation of the two types of errors. That is right only when the two hypothesis are alike expected (!) and their PDF's overlap in the phase space.

[12] If people prefer to stay with the comparison between IH and NH, a surely better solution than the usual difference in χ^2_{min} is the evaluation of a modified *F*-test, where the test statistic is defined as $F = (\min\chi^2_{IH} - \min\chi^2_{NH})/\min\chi^2_{NH}$. F follows a Fisher distribution when the two compared models cannot be statistically distinguished. From the *F*-test a p-value can be extracted and the sensitivity computed in terms of σ's (in the one-sided procedure!). A further advice is to use always $-2\log\mathcal{L}_{max}$ instead of χ^2_{min} since there are non-Gaussian distributions in the set of random variables (that was not always done in the literature).

[13] It is well know that the Feldman and Cousins technique is biased by the amount of level of background [69].

[14] The distinction between one-sided and double-sided becomes not relevant when the 3 σ's limit and above is reached.
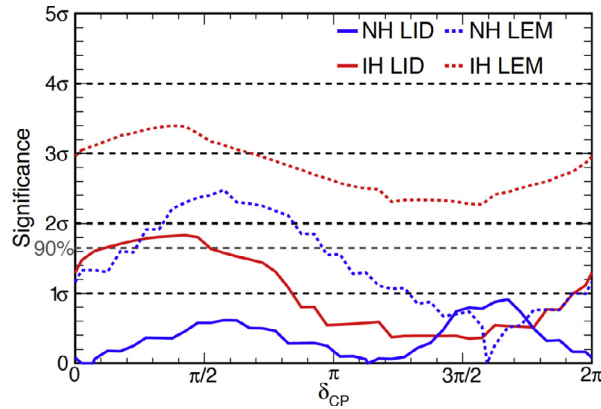
Fig. 4. The important result from NOvA on the statistical significance of MH as function of δ_{CP} (Fig. 4 of [76]). θ_{13} is constrained by the reactor measurements. Blue curves are obtained when the more restrictive data selection is applied, while the red curves corresponds to the less restrictive data selection. This result, if correctly interpreted, may become the turning point for the future. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

hierarchy, demonstrating the need of clarifying the sterile issue as soon as possible. We will address some further issues on the determination of MH in the conclusive section of the paper.

4. Relevant neutrino experiments for the near future

We will describe shortly the important features of two currently running experiments, T2K and NOvA, together with the already under construction JUNO experiment.

T2K experiment

T2K, a long-baseline experiment proposed in 2003 [79], had initially three clear goals, namely to measure θ_{13}, to refine measurement of θ_{23} and to search for sterile components in the ν_μ disappearance mode. Starting the data taking in 2009, T2K beautifully accomplished the first two tasks few years later [37,80], while it was not appropriate for the sterile search lacking the second detector at close distance. Indeed T2K includes SK as large detector at 295 km far and only one very close detector to the neutrino beam of the J-PARC facility. Its results (e.g. [37]) were obtained with a power steadily increased up to 220 kW that allowed the collection of 3.01×10^{20} protons on target (POT). The ν_μ beam, peaked at 0.6 GeV with a ~ 0.2 GeV wide-band is 4.4 mrad off-axis the SK detector. Systematic errors due to the detection (different targets at near and far sites) are currently the largest ones. Proposals to lower the systematics at 2–3% level are under scrutiny.

Even if conclusive results on the sterile neutrinos are missing, T2K is now in the optimal position for a substantial contribution to the measurement of the mass hierarchy and the CP phase [81]. That potentiality has been greatly recognised, and on July 15th the J-PARC Physics Advisory Committee approved the upgrade of the Main-Ring and a new plan aiming to reach 900 kW in 2020 (from the current 370 kW). The overall foreseen exposure (7.80×10^{21} POT), to be collected in a time-schedule of 5 years, is more than 20 times larger than what used until now for the published results.

Last summer the first study of $\overline{\nu}_\mu \to \overline{\nu}_e$ was reported by T2K [82]. Despite the handful of events not yet sufficient to discriminate background, it is highly expected that, depending on the value of δ_{CP} phase, a stronger evidence will be reached in one or two more years of data taking, starting to probe in a *direct* way[15] the CP symmetry in a LBL experiment.

NOvA experiment

NOvA [83] is a multi-purpose experiment located 14 mrad off-axis at 810 km from the neutrino beam source, NuMI (FNAL). The huge far detector has a mass of 14 kton, with 8.7 kton of active liquid scintillator mixed with oil. A similar detector, 0.3 kton, quite close to the beam, is also part of the system. Thanks to the off-axis position the two NOvA detectors can collect neutrino interactions at 2 GeV energy with a spread of about 0.25 GeV. Very efficient measurements of ν_μ and ν_e interactions are available. In particular ν_e CC candidates are selected in the restricted (released) 1.5(1.3) $< E <$ 2.7 GeV window, with expected background as low as 10%. The systematic effects related to the (very similar near and far) detectors are not relevant, while the major limitations come from the current knowledge of neutrino parameters, including its cross-sections.

NOvA was commissioned in 2014 and at the same time it started to take data. First results, based on 2.74×10^{20} POT were released in August this year [75], providing solid evidence of $\nu_\mu \to \nu_e$ oscillations. With ν_e and $\overline{\nu}_e$ data NOvA can obtain results on the whole set of interesting parameters, namely MH, δ_{CP} and the octant of θ_{23}. The overall foreseen exposure

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[15] The result on $\nu_\mu \to \nu_e$ at LBL depends tightly on both δ_{CP} and θ_{13}. By combining it with the θ_{13} measured at the reactors, which are practically independent of δ_{CP}, indirect information on δ_{CP} are extracted. By combining $\nu_\mu \to \nu_e$ and $\overline{\nu}_\mu \to \overline{\nu}_e$ at LBL a direct measurement can be obtained.

is 13 times larger, to be collected in about 6 years time-schedule, half ν and half $\overline{\nu}$. The NuMI beam reached already the power record of 520 kW. Major upgrades are foreseen that will bring the power operations at 700 kW.

JUNO experiment

The JUNO experiment consists of 20 kton target mass of Linear Alkyl-Benzene liquid scintillator, monitored by about 18,000 twenty-inch high-QE photomultipliers providing a 80% photo-coverage. It will allow an unprecedented $3\%/\sqrt{E}$ energy resolution at 1 MeV to detect $\overline{\nu}_e$ coming from reactor plants of about 20 GW power, at a first stage, 53 km far away. Approved in 2014, it is foreseen to start data taking in 2020. The current conceptual design report is in [84] and the physics achievements are described in [85].

With regards to the confirmation of MH at high significance, Juno will be relevant only if the time window 2020–2025 is kept on schedule. JUNO will allow to single out one of the missing fundamental parameters, MH, in an almost independent (and different) way of the others (no dependence on δ_{CP}, no strong dependence on three- vs. four-neutrino pattern, no dependence on θ_{13}, no dependence on matter effects). It is revealing that if the standard picture holds then the relevance of such confirmation on MH would not be so high (remember the case of the OPERA ν_τ).

Indeed, more than results on MH and increasing the accuracy on θ_{13} from 7% to 5% or better, not so interesting for the constraint on MH, JUNO will substantially reduce the uncertainties on the solar oscillation parameters, namely on θ_{12} and δm_{12}, by almost an order of magnitude in 6 years of data taking (see Section 3 of [85]). Therefore, the global significance on MH, and δ_{CP}, will significantly increase.[16] In any case JUNO, as the third big multi-purpose experiment, will join the team of T2K and NOvA, and they will beautifully crown the invaluable results to be gained within the next decade on neutrinos, i.e. for the year 2025.

5. Summary

Neutrino oscillation studies are entering the era of precise and refined measurements, mainly due to the success of the 3-flavour formalism. To confirm the scenario only three parameters have still to be determined with the sufficient precision, namely, the mass hierarchy of the neutrino mass states, the value of the CP phase and to which octant belongs θ_{23}. If the present hints for a Normal Hierarchy were confirmed by the next datasets to be collected by the current long-baseline experiments, T2K and NOvA, and if the already in construction JUNO experiment promptly released results in due time, a final disentangling of the full picture would probably be achieved. We also need Nature be not too devious. Its *deviousness* if any should not receive too much weight in view of the enormous costs that new projects have to face. The history of the recent physics discoveries, in particular in the field of neutrinos, is there to show that researchers should not be more weird than Nature.

There is however the possibility that this scheme be proved wrong/incomplete if a sterile neutrino state at 1 eV mass scale exists. That has to be clarified as soon as possible. A negative answer can be obtained quite soon by reactor and radioactive-source experiments. In case the answer were inconclusive or even positive, quite new scenarios have to be considered and fully new proposals be developed. Finally, an exciting period of data collection and analyses is foreseen in the next 3–5 years (by the year 2018–2020), provided the right approach be taken, as exemplified below for the mass hierarchy.

Mass hierarchy

The determination of the mass hierarchy is likely to be obtained in the near future. In Section 3 we briefly summarised the active studies performed mostly in the last 3 years. We concluded arguing that a new basic approach should be taken for the determination of MH. Provided that the current hints for NH are confirmed (a) by the next data on $\nu_\mu \to \nu_e$ by NOvA [75] and T2K [82], (b) by the potential contribution of new data analyses from the atmospheric neutrinos and (c) by the outcomes of the next global fits, and that all these new information are consistent among themselves at a C.L. as large as 90%, then the sensitivity on NH should be computed following the approach to disprove IH.

What would be the sufficient level of sensitivity is already matter of discussion. The usual rule of thumb in HEP is to consider a 5 σ level, and many proposals advised that value (e.g. the recent DUNE and HyperK studies). However this assumption is entirely a priori. Some recent papers already expressed some concerns about the reasonableness of that choice for the neutrino MH [86]. The rule of 5 σ for discovery of new particle states is an excellent way to establish the existence of new particles. Reason for that stands in the long history of particle physics (from about 1980), which proved that is the right value to choose, at least when the statistical analysis is properly done. Many signals at 3–4 σ were not eventually confirmed because of the Look-Elsewhere-Effect (LEE) and/or some hidden systematics. There are very few and very criticised examples where the systematics effects were so large, and so naively missed by researchers, that even the 5 σ rule was unacceptably disproven. In any case such examples occurred in individual experiments[17].

The situation is quite different for MH. There is obviously no LEE, the statistical issue is already extensively discussed, and, most relevant, results will come from the combinations of several different experiments and different physics channels.

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[16] A similar improvement in the MH significance will be also given by the increase precision on θ_{23} that will be provided by T2K and NOvA themselves.

[17] The famous case of pentaquarks around 2003–2006 was actually different. Many experiments looked at the possible presence of a new state around 1.5 GeV, with about half in favour and half against its existence. On top of an evident bandwagon effect, the statistical analysis was somewhat poor, at least in the first papers, and anyhow the controversy has not been clarified yet [87]. The recent discovery of pentaquarks by LHCb [88] may re-open the issue for the lower energy s-content pentaquarks.

The only assumption will be the 3-flavour neutrino pattern. Hence, we tend to conclude that already a 3 σ level, i.e. a probability of 99.73% in the Bayesian interpretation, would be sufficient. In fact what is meant with "sufficient level" is the persuasion that the result is right. For a new particle state a 5 σ level is needed. For a binary option, grounded on the assumption that the 3-flavour neutrino is the right model, a 3 σ level should be sufficient. Otherwise, to advocate higher levels one should address the possible origins of mistakes that would disprove the result.

To conclude this issue it can be ascertained that in three, maximum 4 years from now, pending the efficient running of NoVA and T2K and the confirmation of the current hints, the MH should be resolved [78]. The next generation of experiments, i.e. the JUNO experiment that is foreseen to start to take data in 2020, would confirm the result mainly by achieving better resolutions on the other parameters of the neutrino matrix (especially θ_{12} and δm^2) rather than by the direct measurement of MH. Nevertheless, the possible measurement of MH by JUNO would address different parts of the oscillation pattern, in particular there will be no dependence on the matter effect, therefore strengthening the global picture.

Unfortunately ICAL/INO is foreseen to reach a good sensitivity for MH in 10 years of running, i.e. in 2030. If the expected result on MH should be confirmed, ICAL would arrive too late for any quantitative contribution to this topics. On the same line the physics case for DUNE and HyperK, for what concerns MH, would be completely washed out in 3 years from now. The related issue on δ_{CP} would be also heavily affected, and DUNE/HyperK might totally miss the physics-case for these neutrino studies. Nevertheless, any incoherence that could originate between results from different experiments, validated by tensions arising from the global fits, would confute such conclusions. These arguments will be reported with more details and in a quantitative way in [78].

Conclusion

Neutrino Physics is a very attractive field of interest due to its role in the Standard Model and its still unknown parameters. In 2012 the discovery/measurement of a relatively large θ_{13}, the third neutrino mixing angle, had a two fold consequence: the first was to open a new era of precise determination of parameters, coherently described in the 3-flavour oscillation pattern; the second was just to increase the belief in the oscillation paradigm itself (worth to remind that the value of θ_{13} was foreseen with good precision by global fits). The two large experiments currently taking data, T2K and NOvA, will be able to collect a big harvest of results. The concurrence of JUNO that is foreseen to start taking data in 2020 would complement them.

The strategy for the next generation of experiments should necessarily take note of the results that will be reached in the next three to five years (by the year 2018–2020). From this point of view western countries should learn from the past and try to avoid non-optimal choices for the next future, as happened to Europe in the past two decades.

Acknowledgements

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