$K_{\mu 3}$ Decay: Tests for Time Reversal and the Two-Component Theory*

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The $K_{\mu 3}$ decay is investigated with special attention to possible muon polarization normal to the decay plane $[\boldsymbol{\sigma}_\mu\cdot(\mathbf{p}_\pi\times\mathbf{p}_\mu)]$ which can be detected by observing an "up-down" asymmetry in the electron distribution in the subsequent parity-nonconserving muon decay. If the K particle is spinless or is unpolarized, the presence of such polarization would indicate that invariance under time reversal is violated in the $K_{\mu 3}$ decay. This polarization is calculated field-theoretically for the spinless K particle. In addition a sensitive test for the two-component theory of the neutrino is proposed for the decay configuration in which the three decay products are collinear.

I.

RECENT important developments concerning weak interactions were originally motivated by the perplexing situation created by the decay interactions of K particles. Now that the violation of invariance under spatial reflection, P, and under charge conjugation,[1] C, has been established for processes that bear no relevance to K particles, it is worth examining what we can learn further about noninvariances under symmetry operations from the study of K particles. Experiments that throw light on the question of invariance under time reversal, T, are of particular interest.

In the decay of $K_{\pi 2}(\equiv\theta)$ and $K_{\pi 3}(\equiv\tau)$, all the information we can possibly obtain about symmetry properties is precisely what has led to the $\tau-\theta$ puzzle, provided that the K particle is spinless *or* that the K particle, if it has spin, is unpolarized at the time of its decay. The decay of $K_{\mu 2}$ resembles that of the charged pion in the sense that the muon is longitudinally polarized if P and C are not conserved. No test for time reversal is possible since the only invariant we can construct is of the form $\mathbf{p}_\mu\cdot\boldsymbol{\sigma}_\mu$ which remains unchanged under T.[2]

In the decay of $K_{\mu 3}$ (or K_{e3}) the situation is the same as that in the $K_{\mu 2}$ case, so long as the pion momentum is not observed (or, equivalently, insofar as the decay plane is not defined), although the magnitude of the longitudinal polarization is likely to be smaller, especially for nonrelativistic muons. On the other hand, if the pion momentum is measured, the muon polarization (a pseudovector defined as usual as the average muon spin in units of $\hbar/2$) is, in general, not purely longitudinal. However, unless invariance under time reversal is violated, the muon polarization necessarily lies in the decay plane (i.e., the plane determined by $\mathbf{p}_\pi$ and $\mathbf{p}_\mu$) provided that the K is spinless or unpolarized. A nonvanishing component of the polarization vector in the direction normal to the decay plane would establish the nonconservation of T and C, as can be seen from the transformation property of $\boldsymbol{\sigma}_\mu\cdot(\mathbf{p}_\mu\times\mathbf{p}_\pi)$.[2,3]

Experimentally the decay plane of the $K_{\mu 3}^{\pm}$ can be defined, if we observe, in addition to the muon, the electron pairs created by the two γ rays which, in turn, arise from the π^0 decay. The subsequent parity-nonconserving decay of the muon can be used as a "natural analyzer" of the muon polarization. Components of the muon polarization parallel to the decay plane result in forward-backward and right-left asymmetries in the distribution of the decay electron, whereas any up-down asymmetry is due to a polarization of the muon in the direction normal to the decay plane. That such a test for time reversal is feasible in a heavy-liquid (e.g., xenon) bubble chamber where the radiation length is short and the stopping power is large was pointed out by Glaser.[4]

II.

To see the whole situations more quantitatively, let us recall that an arbitrary beam of spin-$\frac{1}{2}$ particles is completely specified if we know four real quantities, namely, the intensity, I, and the three components of the polarization, $\langle\boldsymbol{\sigma}\rangle$; or, equivalently, if we know the density matrix, ρ. In our case, the state of the muon "beam" arising from the $K_{\mu 3}$ decay is characterized by

$$\begin{aligned}\rho_\mu&=I_\mu(1+\boldsymbol{\sigma}_\mu\cdot\langle\boldsymbol{\sigma}_\mu\rangle)/2\\&=I_\mu[1+\boldsymbol{\sigma}_\mu\cdot\{A\hat{p}_\mu+B(\hat{n}\times\hat{p}_\mu)+C\hat{n}\}]/2,\end{aligned}\tag{1}$$

where

$$\hat{p}_\mu=\mathbf{p}_\mu/|\mathbf{p}_\mu|,\quad \hat{n}=(\mathbf{p}_\pi\times\mathbf{p}_\mu)/|\mathbf{p}_\pi\times\mathbf{p}_\mu|.$$

I, A, B, and C are functions of the quantities specifying the kinematics of the decay system (e.g., of $|\mathbf{p}_\mu|$ and $\theta_{\pi\mu}$ or of $|\mathbf{p}_\pi|$ and $\theta_{\pi\nu}$).

Assuming that the muon stops and then decays, we obtain the following electron distribution:

$$I_e\propto f(\eta)[1+\alpha(\eta)\{r_l A\hat{p}_\mu+r_t B(\hat{n}\times\hat{p}_\mu)+r_t C\hat{n}\}\cdot\hat{p}_e],\tag{2}$$

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[1] We assume the validity of the TCP theorem throughout.

[2] In order that a naive approach to time-reversal invariance of this kind be valid, it is essential that there be no final-state interaction. In the $K_{\mu 2}^{\pm}$ and $K_{\mu 3}^{\pm}$ decays, because of the absence of final-state interactions, there is no counterpart of the Ze^2m/p term that appears in β decay. For detailed discussion on the rules for detecting noninvariance under T see T. D. Lee, ["Conservation Laws in Weak Interactions" (Lectures at Harvard University, March, 1957), available as Nevis Report-50, Columbia University (unpublished)].

[3] Strictly speaking, this statement is true for charged $K_{\mu 3}$ only. For $K_{\mu 3}^0\rightarrow\mu^{\pm}+\pi^{\mp}+\nu$, the statement is valid only insofar as we neglect a term of the order 1/137.

[4] D. A. Glaser (private communication).

where η is the momentum of the decay electron in units of its maximum value (≈ 52 Mev/c) and $\hat{p}_e$ is a unit vector along the electron momentum; r_l and r_t are the depolarization factors for longitudinally and transversely polarized beams, respectively. There is no reason *a priori* to believe that r_l and r_t are equal, but since the main depolarization effect takes place after the muon is slowed down, it is likely that the muon has "forgotten" its original direction by that time, in which case $r_l = r_t$. This can be checked experimentally by studying the π–μ–e sequence for muons produced in the backward direction from pions *in flight*.[5] r_l and r_t are, of course, independent of the initial muon energy. $f(\eta)$ and $\alpha(\eta)$ are defined in such a way that a beam of 100% polarized muons would give the electron distribution

$$I_e \propto f(\eta)\{1+\alpha(\eta)\cos\varphi\},$$

where φ is the angle between the muon polarization and the electron momentum. If the muon direction and the spin direction coincide in pion decay (as in the case of the *positive* muon within the framework of the two-component theory with lepton conservation), $\alpha(\eta)$ is on the average negative, and its magnitude is particularly large for large values of η, whereas it may even have the opposite sign with rather small magnitude for $\eta < \frac{1}{2}$ according to the two-component theory.[6] Thus the muon decay is a good analyzer only when η is fairly large, and we may select those decay events in which the electron momentum exceeds some cutoff value η_0. The asymmetry coefficient C that characterizes the violation of time-reversal invariance in the $K_{\mu 3}$ decay can now be expressed in terms of experimentally observable quantities;

$$C = \frac{2}{r_t \bar{\alpha}(\eta_0)} \left(\frac{\mathfrak{U} - \mathfrak{D}}{\mathfrak{U} + \mathfrak{D}} \right), \qquad (3)$$

where $\mathfrak{U}$ and $\mathfrak{D}$ refer to the number of electrons with momenta greater than η_0 decaying above and below the decay plane, respectively, and

$$\bar{\alpha}(\eta_0) \equiv \int_{\eta_0}^{1} f(\eta)\alpha(\eta)d\eta \bigg/ \int_{\eta_0}^{1} f(\eta)d\eta.$$

III.

So far our results have been completely general and are applicable regardless of the spin of the K except that, if the K has spin and is polarized, I, A, B, and C further depend on the state of this polarization, and an observation that $C \neq 0$ would not necessarily imply that invariance under T is violated. We now calculate C within the framework of some field-theoretic model

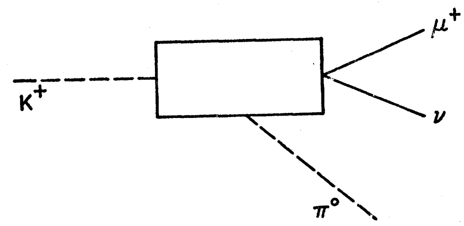

FIG. 1. Diagram for the decay of $K_{\mu 3}^+ \rightarrow \mu^+ + \pi^0 + \nu$. The rectangular box represents virtual strong interactions such as the creation and annihilation of a baryon pair and other more complicated processes.

as previously considered by Pais and Treiman,[7] by Furuichi *et al.* (Hiroshima group),[8] by Feinberg,[9] by Werle,[10] and by MacDowell[11] in other connections. The basic assumptions underlying those field-theoretic calculations are:

(1) The muon and the neutrino are created at a single vertex in the sense of Feynman diagrams. (Radiative corrections are ignored.)

(2) The fundamental lepton interaction does not involve a derivative coupling.

(3) The spin of the K is zero.[12]

The appropriate Feynman diagram is shown in Fig. 1.

With these assumptions they have arrived at the "effective" Hamiltonian

$$\begin{aligned} R = &\left(f_S - g_V \frac{m_\mu}{m_K} \right) \bar{\psi}_\mu \psi_\nu - f_V \bar{\psi}_\nu \gamma_4 \psi_\nu + i \frac{f_T}{m_K} \bar{\psi}_\mu \gamma_4 \boldsymbol{\gamma} \cdot \mathbf{p}_\pi \psi_\nu \\ &+ \left(f_S' - g_V' \frac{m_\mu}{m_K} \right) \bar{\psi}_\mu \gamma_5 \psi_\nu - f_V' \bar{\psi}_\mu \gamma_4 \gamma_5 \psi_\nu \\ &+ \frac{i f_T'}{m_K} \bar{\psi}_\mu \gamma_4 \boldsymbol{\gamma} \cdot \mathbf{p}_\pi \gamma_5 \psi_\nu + \text{H.c.} \qquad (4) \end{aligned}$$

$f_i (= f_S, g_V, \text{etc.})$ and $f_i' (= f_S', g_V', \text{etc.})$ are in general, functions of $|\mathbf{p}_\pi|$, but if we assume the simplest momentum dependence, which may or may not be reasonable, they are just constant.[13] We use Hermitian γ matrices with $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$ and the phases of the "effective" coupling constants are chosen in such a way that they are all relatively real if invariance under time reversal holds.[14] g_V and f_V are relatively in phase, if strong interactions are invariant under T. A similar statement holds for g_V' and f_V', but nothing can be

[5] The author is indebted to Professor L. M. Lederman for this remark.

[6] T. Kinoshita and A. Sirlin, Phys. Rev. **107**, 638 (1957). In fact the study of the π^+–μ^+–e^+ sequence for low-energy positrons is consistent with essentially an isotropic distribution. Pless, Brenner, Williams, Bizzarri, Hildebrand, Milburn, Shapiro, Stanch, Street, and Young, Phys. Rev. **108**, 159 (1957).

[7] A. Pais and S. B. Treiman, Phys. Rev. **105**, 1616 (1957).

[8] Furuichi, Kodama, Ogawa, Sugawara, Wakasa, and Yonezawa, Progr. Theoret. Phys. Japan **17**, 89 (1957).

[9] G. Feinberg (unpublished).

[10] J. Werle, Nuclear Phys. **4**, 171 (1957).

[11] S. W. MacDowell (to be published).

[12] The Hiroshima group[8] considers the spin-1 case separately.

[13] The energy spectrum in this case has been studied by the Hiroshima group, and some comparison has been made with experiment. However, we keep in mind the possibility that f_i, f_i' may be energy-dependent.

[14] Our definition of f_T differs from that of Pais and Treiman[7] by a factor of i.

said about the relative phase of f_i and f_i' until we assume the validity of some specific parity-nonconserving theories such as the two-component theory or a "twin" neutrino theory, which requires $f_i = \mp f_i'$.

From (4) we can compute I_μ and C.

$$I_\mu(p_\pi,\theta)dp_\pi d(\cos\theta) \propto (1-x^2-y^2)(m_K-E_\pi)^2 p_\pi^2 \times\xi(x,\theta)(1+x\cos\theta)^{-4}E_\pi^{-1}dp_\pi d(\cos\theta), \quad (5)$$

$$C(x,\theta)\xi(x,\theta) = \pm 2\sin\theta\Bigg\{ -\mathrm{Im}[f_S f_V^* + f_S' f_V'^*] \times x(1+x\cos\theta) + \mathrm{Im}\Bigg[\left(f_S - g_V\frac{m_\mu}{m_K}\right)f_T^* + \left(f_S' - g_V'\frac{m_\mu}{m_K}\right)f_T'^*\Bigg]\frac{p_\pi}{m_K}y(1+x\cos\theta) - \mathrm{Im}[f_V f_T^* + f_V' f_T'^*]\frac{p_\pi}{m_K}[x(x+\cos\theta)+y^2]\Bigg\}, \quad (6)$$

where

$$\xi(x,\theta) = \left[\left|f_S - g_V\frac{m_\mu}{m_K}\right|^2 + \left|f_S' - g_V'\frac{m_\mu}{m_K}\right|^2\right](1+x\cos\theta)^2 + [|f_V|^2+|f_V'|^2](x^2\sin^2\theta+y^2) + [|f_T|^2+|f_T'|^2]\frac{p_\pi^2}{m_K^2}[(x+\cos\theta)^2+y^2\sin^2\theta] - 2\,\mathrm{Re}\Bigg[\left(f_S - g_V\frac{m_\mu}{m_K}\right)f_V^* + \left(f_S' - g_V'\frac{m_\mu}{m_K}\right)f_V'^*\Bigg]\frac{p_\pi}{m_K}(x+\cos\theta)(1+x\cos\theta) - 2\,\mathrm{Re}[f_V f_T^* + f_V' f_T'^*]\frac{m_\mu}{m_K}x\cos\theta(1+x\cos\theta),$$

$$x \equiv p_\pi/(m_K-E_\pi), \quad y\equiv m_\mu/(m_K-E_\pi), \quad (7)$$

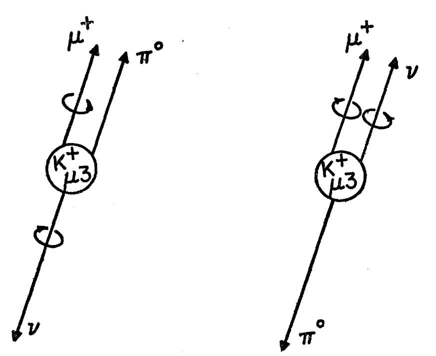

FIG. 2. For the decay configurations in which the three decay products are collinear, the muons must be 100% polarized if the two-component neutrino theory holds.

and θ is the angle between the pion momentum and the *neutrino* momentum. In (6) the upper sign is to be used for the $K_{\mu3}^-$ decay, and the lower sign for the $K_{\mu3}^+$ decay if the field operator $\bar{\psi}_\mu$ in (4) annihilates μ^+ and creates μ^-. That the polarization is equal and opposite for the μ^+ and μ^- is a direct consequence of *TCP* invariance, which our Hamiltonian (4) naturally satisfies. It appears that an "up-down" asymmetry in the subsequent muon decay, if present at all, is likely to attain its maximum value when $\theta = 60°–90°$ unless there are fortuitous cancellations.

IV.

The expressions for A and B are somewhat involved if they are expressed in terms of x and θ. We simply remark that a large longitudinal polarization is possible for relativistic muons if the two-component theory is valid. This situation has been previously discussed by Werle[10,15] and MacDowell[11] independently in the case when the pion is not observed.

When the pion is observed, it is particularly interesting to look into the configurations in which all three decay products are collinear. The decay kinematics are completely specified by the muon energy and the relative direction of the muon and the neutrino (i.e., parallel or antiparallel). If the two-component theory with lepton conservation is valid, the μ^+ from the $K_{\mu3}^+$ decay is completely polarized in the direction of motion whenever the muon momentum and the neutrino momentum are antiparallel *independently of the muon energy*. Similarly, whenever the two momenta are parallel, the muon is completely polarized in the direction opposite to its motion. (See Fig. 2.) This follows trivially from angular momentum conservation, but can be checked explicitly for our Hamiltonian. (See appendix.)

The essential point worth noting is that this requirement might be used to distinguish the original two-component theory of the neutrino,[16] in which only neutrinos of one kind of "helicity" are emitted, from a "twin" neutrino theory, in which the "helicity" of the neutrino depends on the type of interaction.[17] The usual π–μ–e sequence and the $K_{\mu2}$–μ–e sequence are not too sensitive to the choice between the two theories for the following reasons.

(1) Although the two-component theory offers the simplest and most attractive explanation of the π–μ–e sequence including the energy dependence of the asymmetry, which is in rough agreement with experiment,

[15] Unfortunately Werle's paper[10] contains a few errors which have subsequently been corrected. J. Werle, Nuclear Phys. 4, 693 (1957).

[16] T. D. Lee and C. N. Yang, Phys. Rev. **106**, 1671 (1957); L. Landau ,Nuclear Phys. **3**, 127 (1957); A. Salam, Nuovo cimento **5**, 299 (1957).

[17] M. G. Mayer and V. L. Telegdi, Phys. Rev. **107**, 1445 (1957). An interesting theory which starts from a very different point of view but leads to results somewhat similar to the Mayer-Telegdi theory has been proposed by R. P. Feynman. *Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics* (Interscience Publishers, Inc., New York, 1957).

it is extremely difficult to show conclusively that muons from pion decay are completely polarized, since the observed asymmetry in the electron distribution (aside from the depolarization effect) depends on the product of the muon polarization produced in pion decay and the angular asymmetry inherent in the decay of 100% polarized muons. Even if the observed forward-backward asymmetry in muon decay turns out to be the same for muons from the $\pi^\pm$ decay and $K_{\mu 2}{}^\pm$ decay, we can still attribute this effect to some "universal" property of the $(\mu\nu)$ interaction.

(2) A "twin" neutrino theory may give a complete or nearly complete polarization for the muon from pion decay provided that a single covariant is dominant in the decay interaction.[18] Note in this connection that some of the covariants (such as tensor) cannot contribute to the $\pi^\pm$ (spin-zero $K_{\mu 2}$) decay to start with.

In contrast, that the muon polarization be complete in the collinear configurations for the $K_{\mu 3}$ decay is a far more stringent condition imposed by the two-component theory. If $f_i = -f_i' \neq 0$ and $f_j = +f_j' \neq 0$ for some pairs (i,j) as a "twin" neutrino theory suggests, the muon polarization necessarily varies with energy and cannot stay complete as can be readily seen from Eq. (8) in Appendix. This effect should be significant in the low-energy region ($p_\mu/E_\mu \ll 1$) where all covariants present can contribute.

So even if time-reversal invariance is shown to hold in the $K_{\mu 3}$ decay, the study of the $K_{\mu 3}$–μ–e sequence in a heavy-liquid bubble chamber is not without interest. In addition, the angular correlation effect discussed by Pais and Treiman[7] for the $K_{e3}{}^0$ decay works just as well for the $K_{e3}{}^+$ decay, which can be studied simultaneously.

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APPENDIX

The longitudinal polarization of the *positive* muon from the $K_{\mu 3}{}^+$ when the three decay products are collinear, can be calculated from (4). We have

$$
\begin{aligned}
\langle \boldsymbol{\sigma}_\mu\rangle\cdot\hat{p}_\mu = \Big\{ & \pm 2\,\mathrm{Re}\Big[\Big(f_S - g_V\frac{m_\mu}{m_K}\Big)\Big(f_S'^* - g_V'^*\frac{m_\mu}{m_K}\Big)\Big]\Big(1\mp\frac{p_\mu}{E_\mu}\Big) \pm 2\,\mathrm{Re}[f_V f_V'^*]\Big(1\pm\frac{p_\mu}{E_\mu}\Big) \\
& \pm 2\,\mathrm{Re}[f_T f_T'^*]\Big(1\mp\frac{p_\mu}{E_\mu}\Big)\frac{p_\pi^2}{m_K^2} \mp 2\,\mathrm{Re}\Big[\Big(f_S'-g_V'\frac{m_\mu}{m_K}\Big)f_V^* + \Big(f_S-g_V\frac{m_\mu}{m_K}\Big)f_V'^*\Big]\frac{m_\mu}{E_\mu} \\
& -2\,\mathrm{Re}\Big[\Big(f_S'-g_V'\frac{m_\mu}{m_K}\Big)f_T^* + \Big(f_S - g_V\frac{m_\mu}{m_K}\Big)f_T'^*\Big]\Big(1\mp\frac{p_\mu}{E_\mu}\Big)\frac{p_\pi}{m_K} + 2\,\mathrm{Re}[f_V f_T'^* + f_V' f_T^*]\frac{m_\mu}{E_\mu}\frac{p_\pi}{m_K}\Big\}\Big/ \\
& \Big\{\Big[\Big|f_S - g_V\frac{m_\mu}{m_K}\Big| + \Big|f_S'-g_V'\frac{m_\mu}{m_K}\Big|^2\Big]\Big(1\mp\frac{p_\mu}{E_\mu}\Big) + [|f_V|^2+|f_V'|^2]\Big(1\pm\frac{p_\mu}{E_\mu}\Big) + [|f_T|^2+|f_T'|^2] \\
& \times\Big(1\mp\frac{p_\mu}{E_\mu}\Big)\frac{p_\pi^2}{m_K^2} - 2\,\mathrm{Re}\Big[\Big(f_S-g_V\frac{m_\mu}{m_K}\Big)f_V^* + \Big(f_S'-g_V'\frac{m_\mu}{m_K}\Big)f_V'^*\Big]\frac{m_\mu}{E_\mu} \mp 2\,\mathrm{Re}\Big[\Big(f_S-g_V\frac{m_\mu}{m_K}\Big)f_T^* \\
& + \Big(f_S'-g_V'\frac{m_\mu}{m_K}\Big)f_T'^*\Big]\Big(1\mp\frac{p_\mu}{E_\mu}\Big)\frac{p_\pi}{m_K} \pm 2\,\mathrm{Re}[f_V f_T^* + f_V' f_T'^*]\frac{m_\mu}{E_\mu}\frac{p_\pi}{m_K}\Big], \qquad (8)
\end{aligned}
$$

where the upper (lower) sign is to be chosen when the neutrino and muon are emitted in the same (opposite) direction(s). If $f_i = -f_i'$ as the two-component neutrino theory with lepton conservation requires

$$\langle \boldsymbol{\sigma}_\mu\rangle\cdot\hat{p}_\mu = \mp 1,$$

which is obvious from angular momentum conservation. A "twin" neutrino theory with $f_i = -f_i'$ and $f_j = +f_j$ for some pair (i,j) gives a reduced polarization that varies with energy.‡

[18] For instance, if the Feynman theory[17] is accepted, this polarization is still complete, but is in the opposite direction to that expected from the two-component theory, when the vector (axial vector) interaction is dominant.

‡ *Note added in proof.*—In discussing the two-component theory of the neutrino we have used the helicity assignment that follows from the He^6 recoil experiment [B. M. Rustad and S. L. Ruby, Phys. Rev. **97**, 991 (1955)] taken together with various parity experiments.