Non-standard-model CP violation in $K_{\mu 3}$ decays as a method of probing for new physics

Robert Garisto and Gordon Kane
Department of Physics, University of Michigan, Ann Arbor, Michigan 48109
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The transverse polarization of the muon ($P_\mu^\perp$) in the decay $K^+ \to \pi^0 \mu^+ \nu_\mu$ is a very useful tool for studying CP violation because a detectable nonzero $P_\mu^\perp$ can only arise from physics beyond the standard model. Further, $P_\mu^\perp$ is interesting because it probes a different region of parameter space than many other CP-violating observables. To help justify an experimental search, we present three models which give $P_\mu^\perp \gtrsim 10^{-3}$ and which are not contradicted by other experimental data. We also comment on $K \to \pi\mu\mu$ decays.

I. INTRODUCTION

Since its discovery in 1964 [1], many models have been suggested to explain CP violation in the K^0-$\bar{K}^0$ system. In the standard model the CP-violating parameter ϵ arises entirely from a nontrivial phase in the three-family Cabibbo-Kobayashi-Maskawa (CKM) matrix [2]. But most physics beyond the standard model can lead to phases that provide additional sources for CP violation, often contributing to CP-violating observables in ways entirely different from the CKM contribution. Observing such an effect would be a clear signal of physics beyond the standard model—for example observing a neutron electric dipole moment (d_n) would point to some new physics beyond the standard model since the CKM phase only contributes to d_n in third-loop order [3]. However d_n, like ϵ, usually only involves relative phases between quark-quark couplings, which we term purely hadronic CP violation, and thus probes much of the same parameter space as ϵ. Further, d_n can arise from strong CP violation, so if d_n is observed to be nonzero at the sensitivity achievable in the next few years, the results cannot be uniquely interpreted.

Most people who have thought about the standard model and its foundations expect new physics to arise in some form to extend the standard model. Almost every attempt to construct such theories gives physics that introduces new phases into fermion interactions. It would be surprising if such new phases did not show up in CP-violating effects. Thus we expect that the CKM phase will generate much of the CP-violating parameter ϵ, and that there will be additional CP-violating effects from other phases.

It has been noted [4] that the transverse polarization of the muon ($P_\mu^\perp$) in the decay $K^+ \to \pi^0 \mu^+ \nu_\mu$ is an excellent CP-violating observable to study since $P_\mu^\perp$ is zero in the standard model (up to CP-conserving electromagnetic effects of order 10^{-6}) [5]. Thus, detection of $P_\mu^\perp \neq 0$ would immediately imply physics beyond the standard model, because no model-independent calculations are required to separate standard-model and possible new contributions. Further, $P_\mu^\perp$ involves phases between quark-quark and lepton-lepton couplings (which we term semileptonic CP violation), thus generally probing a different region of parameter space than d_n or ϵ.

The current bound on $P_\mu^\perp$ is [6]

$$P_\mu^\perp = (-1.85 \pm 3.60) \times 10^{-3} , \tag{1.1}$$

with the error coming mostly from statistics. Experimentalists may be interested in searching for an effect or pushing the bound down in the near future. To justify such an experiment, one needs to know that at least some *reasonable* theories exist which could give $P_\mu^\perp$ large enough to be seen. The challenge we set for ourselves in this paper was to come up with models for which $P_\mu^\perp \sim 10^{-3}$, and which were not ruled out by other experimental data. We exhibit three such models: a three-Higgs-doublet model employing ratios of vacuum-expectation values (VEV's) proportional to fermion masses, a model with a non-Higgs scalar doublet which couples to fermions, and a scalar leptoquark model.

Following a section outlining the calculation of $P_\mu^\perp$, we present one section for each of the three models, detailing the steps to the effective-Lagrangian parameters used to calculate the $P_\mu^\perp$ for that model, and then discussing possible constraints from various data. Finally, we summarize and remark on the possibilities of seeing CP violation in $K \to \pi\mu\mu$ decays from model B and note that multiple-Higgs-doublet models (such as model B) are being used in explanations for the baryon asymmetry of the Universe in some recent papers.

II. MUON TRANSVERSE POLARIZATION

For the process

$$K^+(K) \to \pi^0(k)\mu^+(p)\nu_\mu(q) , \tag{2.1}$$

we want to calculate $P_\mu^\perp$, the polarization of the muon perpendicular to the decay plane, which we define, as a function of the two free kinematical variables (over some finite range),

$$P_\mu^\perp = \frac{|\mathcal{M}^+|^2 - |\mathcal{M}^-|^2}{|\mathcal{M}^+|^2 + |\mathcal{M}^-|^2} . \tag{2.2}$$

This function can be averaged over the kinematical variables, or the numerator and denominator can be averaged separately, corresponding to two different experimental methods for averaging. We define w^μ to be the polarization vector of the muon and consider values of $w^\mu = \pm z^\mu \equiv (0; \mathbf{p}\times\mathbf{q}/|\mathbf{p}\times\mathbf{q}|)$, where the superscripts in (2.2) refer to the muon spin aligned to the $\pm z$ axis. To give a contribution to $P_\mu^\perp$, the matrix element must depend on w since otherwise the numerator in (2.2) vanishes. Dotting any momentum in the decay plane with w will clearly give zero, so the polarization must be proportional to

$$\epsilon_{\alpha\beta\gamma\delta} w^\alpha K^\beta k^\gamma q^\delta \ . \tag{2.3}$$

The matrix element for (2.1) is just the standard-model contribution, plus any contributions from new physics. For us the new physics will arise from scalar exchanges. At the tree level,

$$|\mathcal{M}^\pm|^2 = |\mathcal{M}_0^\pm + \mathcal{M}_S^\pm|^2 = |\mathcal{M}_0^\pm|^2 + |\mathcal{M}_S^\pm|^2 + 2\,\mathrm{Re}(\mathcal{M}_0^* \mathcal{M}_S)^\pm \ , \tag{2.4}$$

where $\mathcal{M}_0$ and $\mathcal{M}_S$ are the amplitudes for Figs. 1(a) and 1(b), respectively.

The hadronic matrix element for the *V-A* part of the Lagrangian does not depend on the amount of axial-vector coupling,

$$\begin{aligned} \langle \pi^0|\bar{s}\gamma^\mu(1\pm\gamma^5)u\,|K^+\rangle &= \langle \pi^0|\bar{s}\gamma^\mu u\,|K^+\rangle \\ &= f_+(K+k)^\mu + f_-(K-k)^\mu \ , \end{aligned} \tag{2.5}$$

since there is no way to form an axial vector from $K^\alpha k^\beta$ alone. We also note that f_+ and f_- are relatively real from *CP* invariance of the strong interactions, so that the approximation $f_-/f_+ \simeq 0$ does not change the *CP* physics.

For the scalar hadronic matrix element, one can use the Dirac equation to obtain [7]

$$(K-k)_\mu \langle \pi^0|\bar{s}\gamma^\mu u\,|K^+\rangle = (-m_s + m_u)\langle \pi^0|\bar{s}u\,|K^+\rangle \ , \tag{2.6}$$

which then combined with (2.5) using $m_u \ll m_s$ and $f_-/f_+ \simeq 0$ gives

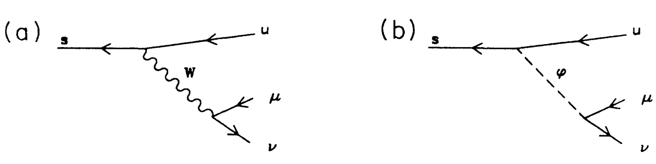

FIG. 1. Quark level standard model contribution to $K^+ \to \pi^0 \mu^+ \nu_\mu$. The arrows indicate particle direction (so this represents a decaying s antiquark). (b) Quark level contribution to $K^+ \to \pi^0 \mu^+ \nu_\mu$ from charged scalar ϕ.

$$\langle \pi^0|\bar{s}u\,|K^+\rangle \simeq -f_+ \frac{M_K^2 - m_\pi^2}{m_s} \simeq -\frac{f_+ M_K^2}{m_s} \ . \tag{2.7}$$

At this point we note that the scalar piece squared $|\mathcal{M}_S|^2$ is independent of K^μ and k^μ by (2.7). Also the hadronic part of the standard-model matrix element squared is just (2.5) squared, which is symmetric in its indices. Thus only the cross term of (2.4) will contribute to $P_\mu^\perp$ because neither $|\mathcal{M}_0|^2$ nor $|\mathcal{M}_S|^2$ can generate a term proportional to (2.3). One sees immediately that any non-standard-model effective vector or axial-vector coupling (such as *LR* model) will yield $P_\mu^\perp = 0$ at the tree level, since the hadronic part of the cross term will be just (2.5) squared again. For an effective scalar coupling, the hadronic part of the cross term in (2.4) will only have one index which can be contracted with a leptonic antisymmetric piece and give a nonzero $P_\mu^\perp$.

Thus we consider scalar Lagrangians which contribute to $P_\mu^\perp$. The most general such effective scalar Lagrangian (which does not involve right-handed neutrinos),

$$\begin{aligned} -\mathcal{L}_{\mathrm{eff}} &= \frac{L_{su} L_{\mu\nu}^*}{M_x^2} (\bar{s}_R u_L)(\bar{\nu}_L \mu_R) \\ &\quad + \frac{R_{su} L_{\mu\nu}^*}{M_x^2} (\bar{s}_L u_R)(\bar{\nu}_L \mu_R) + \mathrm{H.c.} \ , \end{aligned} \tag{2.8}$$

can arise either directly from the Lagrangian for a scalar ϕ_x^- of mass M_x (where x labels the type of scalar):

$$-\mathcal{L} = L_{su}\bar{s}_R u_L \phi_x^- + R_{su}\bar{s}_L u_R \phi_x^- + L_{\mu\nu}\bar{\mu}_R \nu_L \phi_x^- + \mathrm{H.c.} \ , \tag{2.9}$$

or it can also arise from a Fierz transformation, as in a leptoquark model (e.g., model C).

Remembering that only the cross term contributes to $P_\mu^\perp$, we use the standard-model weak vertices and (2.5) for $\mathcal{M}_0$, and the effective Lagrangian (2.8) for $\mathcal{M}_S$, to obtain

$$\begin{aligned} 2\,\mathrm{Re}(\mathcal{M}_0^*\mathcal{M}_S)^\pm &\simeq \frac{1}{2}\frac{G_F}{\sqrt{2}}\sin\theta_C \frac{M_K^2}{m_s}\frac{1}{M_x^2} f_+^2 \\ &\quad \times \mathrm{Re}[(L_{su}+R_{su})L_{\mu\nu}^* \mathcal{T}^\pm] \ , \end{aligned} \tag{2.10}$$

where $\mathcal{T}^\pm$ is the trace,

$$\begin{aligned} \mathcal{T}^\pm &= [\bar{\mu}^\pm(p)(K+k)_\alpha \gamma^\alpha (1-\gamma^5)\nu_\mu(q)] \\ &\quad \times [\bar{\nu}_\mu(q)(1+\gamma^5)\mu^\pm(p)] \ , \end{aligned} \tag{2.11}$$

calculated *without* summing over muon spins (as before the superscripts refer to the μ spin aligned to the $\pm z$ axis). The numerator of (2.2) gets nonzero contributions from the cross term,

$$\begin{aligned} |\mathcal{M}^+|^2 - |\mathcal{M}^-|^2 &= 2\,\mathrm{Re}(\mathcal{M}_0^*\mathcal{M}_S)^+ - 2\,\mathrm{Re}(\mathcal{M}_0^*\mathcal{M}_S)^- \\ &= 4\sqrt{2} G_F \sin\theta_C \frac{M_K^2}{m_s} f_+^2 (\epsilon_{\alpha\beta\gamma\delta} K^\alpha k^\beta q^\gamma z^\delta) \\ &\quad \times \frac{\mathrm{Im}[(L_{su}+R_{su})^* L_{\mu\nu}]}{M_x^2} \ , \end{aligned} \tag{2.12}$$

since the spin-dependent terms of $\mathcal{T}^+$ and $\mathcal{T}^-$ have opposite signs. We take the denominator of (2.2) to be just the spin-averaged standard-model matrix element, neglecting the effect of the scalar contribution. Finally we obtain

$$P_\mu^\perp \simeq \frac{\sqrt{2}}{4}(G_F M_W^2 \sin\theta_C)^{-1}\left[\frac{M_K}{m_s}\right] \times \left[\frac{M_K(\epsilon_{\alpha\beta\gamma\delta}K^\alpha p^\beta q^\gamma z^\delta)}{\Phi}\right]\mathrm{Im}\tilde{\xi} , \tag{2.13}$$

where, in the limit of $f_-/f_+ \simeq 0$,

$$\Phi \simeq 2(K\cdot p)(K\cdot q) - M_K^2(p\cdot q) + m_\mu^2(K\cdot q + \tfrac{1}{4}p\cdot q) , \tag{2.14}$$

and where we define

$$\mathrm{Im}\tilde{\xi} \equiv \frac{\mathrm{Im}[(L_{su}+R_{su})^* L_{\mu\nu}]}{(M_x/M_W)^2} . \tag{2.15}$$

Note that (2.15) implies that $P_\mu^\perp$ for the K^- decay has the opposite sign [8] of $P_\mu^\perp$ for K^+ (since $\mathrm{Im}A^* = -\mathrm{Im}A$).

The only kinematic dependence of $P_\mu^\perp$ in (2.13) lies in the term in braces. It turns out [7] that for averaging $P_\mu^\perp$, the simplest choice is to work in the K^+ rest frame and define the angle between the leptons $\cos\theta \equiv \hat{\mathbf{p}}\cdot\hat{\mathbf{q}}$. Neglecting the order m_μ^2/M_K^2, we have

$$\left[\frac{M_K(\epsilon_{\alpha\beta\gamma\delta}K^\alpha p^\beta q^\gamma z^\delta)}{\Phi}\right] = \frac{|\mathbf{p}||\mathbf{q}|\sin\theta}{E_\mu E_\nu + |\mathbf{p}||\mathbf{q}|\cos\theta + \mathcal{O}(m_\mu^2/M_K^2)} \simeq \frac{\sin\theta}{r(p)+\cos\theta} , \tag{2.16}$$

where

$$r(p) \equiv E_\mu/|\mathbf{p}| . \tag{2.17}$$

Now we average over the parameters $p \equiv |\mathbf{p}|/M_K$ and θ to obtain

$$\overline{P_\mu^\perp} \simeq 44\left[\frac{150\,\mathrm{MeV}}{m_s}\right]\mathrm{Im}\tilde{\xi} . \tag{2.18}$$

It is possible to enhance $\overline{P_\mu^\perp}$ by varying the range over which $P_\mu^\perp$ is averaged. From (2.16), we conclude that the regions of small r (large $|\mathbf{p}|$), and $\theta \gtrsim 90°$ ($\sin\theta$ large and $\cos\theta \le 0$) give larger $\overline{P_\mu^\perp}$ than the total region. Of course the event rate would be lower, since we would be making cuts on phase space, but some experiments may find an enhanced $\overline{P_\mu^\perp}$ worth that cost. In considering our three models, we will work with the most conservative value of $\overline{P_\mu^\perp}$, obtained from averaging $P_\mu^\perp$ over the whole region of p and θ, as given by (2.18). Since the ν is not observed directly, our θ may not be appropriate for some experimental analyses; if so, the averaging can be redone using different kinematical variables, but our basic results will not be affected.

III. MODEL B

A. The Lagrangian

The suggestion that CP violation could be generated via phases induced in a nontrivial Higgs sector was first proposed by Lee [9] and then later refined by Weinberg [10] with the elimination of flavor-changing neutral currents (FCNC's) using natural flavor conservation (NFC). Model B uses three Higgs doublets to generate one nontrivial phase which violates CP in the decay $K^+ \to \pi^0\mu^+\nu_\mu$. As will be explored later, model B contains the reasonable assumption that the ratio of the VEV's is the same as the ratio of the masses which they generate. This makes the ratio of the VEV's far from unity, a departure from the assumption of previous authors [5,7]. Following the notation of Cheng [7], we introduce three Higgs doublets, each allowed a complex VEV:

$$\Phi_j = \begin{bmatrix}\phi_j^+\\ \phi_j^0\end{bmatrix} , \tag{3.1}$$

$$\langle \phi_j^0 \rangle = v_j e^{i\theta_j} . \tag{3.2}$$

To impose NFC we introduce three different discrete symmetries which transform the Φ's and right-handed quarks:

$$\begin{aligned} &\Phi_1, D_R \to -\Phi_1, -D_R ,\\ &\Phi_2, U_R \to -\Phi_2, -U_R ,\\ &\Phi_3, E_R \to -\Phi_3, -E_R , \end{aligned} \tag{3.3}$$

so that $\Phi_1 D_R$, $\Phi_2 U_R$, and $\Phi_3 E_R$ are the only three invariant combinations. The most general $\mathrm{SU}_W(2)\times \mathrm{U}_Y(1)$ coupling of Higgs bosons to fermions which respects these discrete symmetries is

$$-\mathcal{L} = \frac{1}{v_1 e^{i\theta_1}}\bar{Q}'_L \Phi_1 M'_D D'_R + \frac{1}{v_2 e^{i\theta_2}}\bar{U}'_R \tilde{\Phi}_2^\dagger M'_U Q'_L + \frac{1}{v_3 e^{i\theta_3}}\bar{L}'_L \Phi_3 M'_E E'_R + \mathrm{H.c.} , \tag{3.4}$$

where M'_D, M'_U, M'_E are 3×3 matrices in flavor space and where the primed quarks are the $\mathrm{SU}_W(2)$ eigenstates and unprimed ones are the mass eigenstates:

$$Q_L = \begin{bmatrix}U_L\\ D_L\end{bmatrix}, \quad U^T = (u\ c\ t), \quad D^T = (d\ s\ b) ,$$

$$L_L = \begin{bmatrix}N_L\\ E_L\end{bmatrix}, \quad N^T = (\nu_e \nu_\mu \nu_\tau), \quad E^T = (e^-\mu^-\tau^-) .$$

If we define $X'_L = V_L^X X_L$, where X is any fermion flavor triplet, then in order to generate the correct mass Lagrangian

$$-\mathcal{L}_{\text{mass}}=\bar{D}_L M_D D_R+\bar{U}_L M_U U_R+\bar{E}_L M_E E_R+\text{H.c.}\ , \tag{3.5}$$

we require $M_D'=V_L^D M_D V_R^{D\dagger}$, where $M_D=\text{diag}(m_d m_s m_b)$, and similar conditions for M_U' and M_E'. The charged part of the Lagrangian will now be

$$-\mathcal{L}^+=\frac{\phi_1'^+}{v_1}\bar{U}_L V_L M_D D_R-\frac{\phi_2'^+}{v_2}\bar{U}_R M_U V_L D_L+\frac{\phi_3'^+}{v_3}\bar{N}_L M_E E_R+\text{H.c.}\ , \tag{3.6}$$

with $V_L\equiv V_L^{U\dagger}V_L^D$ identified as the CKM matrix, and $\phi_j'^+\equiv\phi_j e^{-i\theta_j}$.

In a model with three complex Higgs doublets there are six charged degrees of freedom and six neutral. One of the neutrals is a Goldstone boson G^0 and is absorbed by the longitudinal part of Z^0, leaving five real neutral fields. Two of the charged degrees of freedom are the charged Goldstone bosons, $G^{\pm}$, which get absorbed by the longitudinal part of $W^{\pm}$, leaving four physical charged degrees of freedom, $H_1^{\pm}$ and $H_2^{\pm}$. We relate the charged partners of the three Higgs doublets ϕ_j^+ to the Goldstone boson and two physical charged Higgs bosons with the unitary matrix U_{H^+},

$$\begin{bmatrix}\phi_1'^+\\ \phi_2'^+\\ \phi_3'^+\end{bmatrix}=U_{H^+}\begin{bmatrix}G^+\\ H_1^+\\ H_2^+\end{bmatrix}\ , \tag{3.7}$$

which can be parametrized in the same way as the CKM matrix [7,11]

$$U_{H^+}=\begin{bmatrix}c_1' & -s_1'c_3' & -s_1's_3'\\ s_1'c_2' & c_1'c_2'c_3'-s_2's_3'e^{i\delta'} & c_1'c_2's_3'+s_2'c_3'e^{i\delta'}\\ s_1's_2' & c_1's_2'c_3'+c_2's_3'e^{i\delta'} & c_1's_2's_3'-c_2'c_3'e^{i\delta'}\end{bmatrix}\ . \tag{3.8}$$

To generate the correct charged Lagrangian we require the Goldstone boson to be

$$G^+=(v_1\phi_1'^+ + v_2\phi_2'^+ + v_3\phi_3'^+)/v\ , \tag{3.9}$$

where $v^2=v_1^2+v_2^2+v_3^2=(4G_F/\sqrt{2})^{-1}$. This condition along with (3.7) and (3.8) provides

$$c_1'=\frac{v_1}{v},\quad s_1'c_2'=\frac{v_2}{v},\quad s_1's_2'=\frac{v_3}{v}\ . \tag{3.10}$$

Keeping in mind Eq. (3.6), we define six complex parameters α_i, β_i, and γ_i ($i=1$ or 2) such that

$$\begin{bmatrix}\dfrac{\phi_1'^+}{v_1}\\ \dfrac{\phi_2'^+}{v_2}\\ \dfrac{\phi_3'^+}{v_3}\end{bmatrix}=\frac{1}{v}\begin{bmatrix}1 & \alpha_1 & \alpha_2\\ 1 & -\beta_1 & -\beta_2\\ 1 & \gamma_1 & \gamma_2\end{bmatrix}\begin{bmatrix}G^+\\ H_1^+\\ H_2^+\end{bmatrix}\ , \tag{3.11}$$

which in the unitary gauge (where G^+ is absorbed as the longitudinal part W^+) yields

$$-\mathcal{L}^+=\frac{1}{v}\sum_{i=1}^{2}(\alpha_i\bar{U}_L V_L M_D D_R H_i^+ +\beta_i\bar{U}_R M_U V_L D_L H_i^+ +\gamma_i\bar{N}_L M_E E_R H_i^+)+\text{H.c.} \tag{3.12}$$

Comparing (3.11) to (3.8), we note that the α_i are real, leaving ten free parameters in (3.11) versus four in (3.8). Thus there are six conditions, three of which are

$$\begin{aligned}\text{Im}(\alpha_1\beta_1^*)&=-\text{Im}(\alpha_2\beta_2^*)\ ,\\ \text{Im}(\alpha_1\gamma_1^*)&=-\text{Im}(\alpha_2\gamma_2^*)\ ,\\ \text{Im}(\beta_1\gamma_1^*)&=-\text{Im}(\beta_2\gamma_2^*)\ ,\end{aligned} \tag{3.13}$$

which implies that any tree-level CP-violating amplitude from (3.12) will be proportional to $(1/M_2^2-1/M_2^2)$. We must have a splitting in the charged-Higgs-boson masses to make $P_\mu^\perp\neq 0$, so we take M_h to be smaller charged-Higgs-boson mass, and if the other charged Higgs boson should prove to be of comparable mass, one would replace $1/M_h^2$ with $(1/M_1^2-1/M_2^2)$. Our final Lagrangian

$$-\mathcal{L}^+\simeq\frac{\alpha}{v}\bar{U}_L V_L M_D D_R h^+ +\frac{\beta}{v}\bar{U}_R M_U V_L D_L h^+ +\frac{\gamma}{v}\bar{N}_L M_E E_R h^+ +\text{H.c.}\ , \tag{3.14}$$

compared to a generalized effective Lagrangian

$$-\mathcal{L}^+=L_{DU}^*\bar{U}_L D_R h^+ + R_{DU}^*\bar{U}_R D_L h^+ + L_{EN}^*\bar{N}_L E_R h^+ +\text{H.c.}\ , \tag{3.15}$$

gives us the parameters

$$\begin{aligned}L_{DU}&=\frac{m_D\alpha^*}{v}V_{UD}^*\ ,\\ R_{DU}&=\frac{m_U\beta^*}{v}V_{UD}^*\ ,\\ L_{EN}&=\frac{m_E\gamma^*}{v}\ ,\end{aligned} \tag{3.16}$$

which for $U=u$, $D=s$, $E=\mu$, and $N=\nu_\mu$, determine the parameters in (2.9),

$$\begin{aligned}L_{su}&=\frac{m_s\alpha^*}{v}\sin\theta_C\ ,\\ R_{su}&=\frac{m_u\beta^*}{v}\sin\theta_C\ ,\\ L_{\mu\nu}&=\frac{m_\mu\gamma^*}{v}\ ,\end{aligned} \tag{3.17}$$

where from Eqs. (3.7), (3.8), and (3.11) one can derive

$$\text{Im}(\alpha\beta^*)=\sin\delta' s_3'c_3'\frac{v}{v_1}\frac{v_3}{v_2}\ , \tag{3.18}$$

and

$$\text{Im}(\alpha\gamma^*)=\sin\delta' s_3'c_3'\frac{v}{v_1}\frac{v_2}{v_3}\ . \tag{3.19}$$

Using the definition of $\mathrm{Im}\tilde{\xi}$, (2.15), and neglecting m_u, we obtain

$$\mathrm{Im}\tilde{\xi}=\frac{m_s m_\mu}{v^2}\sin\theta_C\frac{\mathrm{Im}(\alpha\gamma^*)}{(M_h/M_W)^2}\ . \tag{3.20}$$

If we make the naive assumption that $v_1=v_2=v_3$, then $\mathrm{Im}(\alpha\gamma^*)<1$ and $\overline{P_\mu^\perp}$ is very small. However, another reasonable assumption is that the Yukawa couplings in (3.4), which $\sim m/v$, are all about equal in magnitude. To generate the correct mass Lagrangian this assumption implies *the* VEV's *are proportional to the mass of the fermions to which they couple.* We make the reasonable choice that the third family masses determine the ratio of VEV's

$$v_1:v_2:v_3=m_b:m_t:m_\tau\ , \tag{3.21}$$

which reduces $\mathrm{Im}(\alpha\beta^*)$ by a small amount, but enhances $\mathrm{Im}(\alpha\gamma^*)$ by a lot,

$$\mathrm{Im}(\alpha\beta^*)=\frac{1}{2}\left[\frac{\sin\delta' s_3'c_3'}{1/2}\right]\frac{m_\tau}{m_b}\ , \tag{3.22}$$

$$\mathrm{Im}(\alpha\gamma^*)=\frac{1}{2}\left[\frac{\sin\delta' s_3'c_3'}{1/2}\right]\frac{m_t^2}{m_b m_\tau}\ , \tag{3.23}$$

giving

$$\mathrm{Im}\tilde{\xi}\simeq 1.1\times 10^{-4}\left[\frac{\sin\delta' s_3'c_3'}{1/2}\right]\left[\frac{(m_t/130\ \mathrm{GeV})^2}{(M_h/M_W)^2}\right]\ , \tag{3.24}$$

which yields from (2.18) an averaged polarization of

$$\overline{P_\mu^\perp}\simeq 5.0\times 10^{-3}\left[\frac{\sin\delta' s_3'c_3'}{1/2}\right]\left[\frac{(m_t/130\ \mathrm{GeV})^2}{(M_h/M_W)^2}\right]\ . \tag{3.25}$$

Here $\sin\delta'$, s_3', and c_3' are unknown parameters in (3.8) which have a maximal product of $\frac{1}{2}$. The top-quark dependence results from our assumption (3.21) that the VEV's are proportional to the third-family masses. (The bigger m_t, the better our claim that v_2/v_3 is large.)

Now we must consider constraints to this model.

B. Constraints on model B

1. Electric dipole moments

The main constraint to model B comes from the neutron electric dipole moment d_n. From Eq. (A8) in Appendix A, the down-quark ($D=d$ in subscripts) contribution to d_n is approximately

$$d_d=e\frac{1}{(4\pi)^2}\sum_U^{u,c,t} m_U\frac{\mathrm{Im}(L_{dU}^*R_{dU})}{M_h^2}\times\left[\frac{2}{3}I^a\left[\frac{m_U^2}{M_h^2}\right]+I^b\left[\frac{m_U^2}{M_h^2}\right]\right]\ , \tag{3.26}$$

since the loop fermion must be an up-type quark. We have used $Q_U=\frac{2}{3}$ and $Q_x=-1$. Plugging in (3.16) we obtain

$$d_d=e\frac{1}{(4\pi)^2}\frac{m_d}{M_W^2}\frac{\mathrm{Im}(\alpha\beta^*)}{(M_h/M_W)^2}\sum_U^{u,c,t}\frac{m_U^2}{v^2}|V_{Ud}|^2 I\left[\frac{m_U^2}{M_h^2}\right]\ , \tag{3.27}$$

where, from (A6) and (A7),

$$I(X)=\tfrac{2}{3}I^a(X)+I^b(X)$$
$$=\frac{1}{(1-X)^2}\left[-\frac{1}{2}+\frac{5}{6}X-\frac{2-3X}{3(1-X)}\ln X\right]\ . \tag{3.28}$$

We see that the up-quark electric-dipole-moment contribution to d_n will be negligible in comparison to d_d since $d_d\sim|V_{Ud}|^2 m_d m_U^2$ (where $U=c$ or t) while $d_u\sim|V_{uD}|^2\times m_u m_D^2$ (where $D=s$ or b). Historically one uses the SU(6) wave function to write [3]

$$d_n\simeq\tfrac{4}{3}d_d-\tfrac{1}{3}d_u\simeq\tfrac{4}{3}d_d\ , \tag{3.29}$$

giving

$$d_n\simeq 1.9\times 10^{-25}\ e\ \mathrm{cm}\left[\frac{m_d}{300\ \mathrm{MeV}}\right]\frac{\mathrm{Im}(\alpha\beta^*)}{(M_h/M_W)^2}\left\{\left[\tfrac{2}{3}\ln\left[\frac{M_h^2}{m_c^2}\right]-\frac{1}{2}\right]\frac{m_c^2\sin^2\theta_C}{(1\ \mathrm{GeV})^2}+\left[\frac{\frac{5}{6}X-\frac{1}{2}}{(X-1)^2}-\frac{3X-2}{3(X-1)^3}\ln X\right]\frac{m_t^2|V_{td}|^2}{(1\ \mathrm{GeV})^2}\right\}\ , \tag{3.30}$$

where $X\equiv m_t^2/M_h^2$ is not small. We will assume $M_h\sim M_W$ and $m_t|V_{td}|\simeq 1$ GeV. Then the quantity in brackets $\simeq 0.42+0.20\simeq 0.62$. Employing (3.22) we finally obtain an estimate,

$$d_n\simeq 2.1\times 10^{-26} e\ \mathrm{cm}\left[\frac{m_d}{300\ \mathrm{MeV}}\right]\frac{\frac{\sin\delta' s_3'c_3'}{1/2}}{(M_h/M_W)^2}\ , \tag{3.31}$$

which for $M_h\sim M_W$ is less than the experimental bound [12] of 1.2×10^{-25} e cm. Note that we use the safest value for m_d (the constituent mass) which is often taken to be much smaller. Normalizing d_n to 10^{-26} e cm, we can use (3.31) in (3.25) to express the effect of d_n as a constraint on $\overline{P_\mu^\perp}$,

$$\overline{P_\mu^\perp}\le 2.3\times 10^{-3}\left[\frac{m_t}{130\ \mathrm{GeV}}\right]^2\left[\frac{300\ \mathrm{GeV}}{m_d}\right]\frac{d_n^{\mathrm{expt}}}{10^{-26}\ e\ \mathrm{cm}}\ , \tag{3.32}$$

so that $\overline{P_\mu^\perp}$ is not constrained to be less than 10^{-3} until the experimental bound on d_n is of order 3×10^{-27} e cm, or much less if the current m_d is used to calculate d_n.

The electric dipole moment of the electron probably does not provide much of a constraint on $P_\mu^\perp$ for model B. The charged sector gives $d_e \sim m_\nu^2$ which is negligible. The neutral sector gives a nonzero contribution to d_e via the one-loop diagrams of Appendix A, but it will be proportional [13] to m_e^3. Barr and Zee have argued for a very large d_e at two loops using a heavy top quark [14], but Gunion and Vega find that inclusion of all diagrams gives a rather small d_e [15]. In addition, d_e depends on unknown parameters in the neutral-current Lagrangian and therefore cannot constrain $P_\mu^\perp$.

2. ϵ and ϵ'

For many models of CP violation, $\epsilon_{\rm expt}$ provides a big constraint because it is difficult to generate such a large value for ϵ (2.2×10^{-3}) while keeping other CP-violating observables sufficiently small, such as ϵ' and d_n. But in this work we do not require our models to explain the value of $\epsilon_{\rm expt}$ (i.e., $\epsilon_B \neq \epsilon_{\rm expt}$), and are content to allow $\epsilon_{\rm expt}$ to be explained by the CKM phase. Nevertheless, we must examine ϵ and ϵ' to ensure neither is too large. We being with [16]

$$|\epsilon| \simeq \frac{1}{2\sqrt{2}}(\epsilon_m + 2\xi) , \tag{3.33}$$

where

$$\epsilon_m = \frac{{\rm Im}M_{12}^{\rm SD}}{{\rm Re}M_{12}^{\rm SD}} , \tag{3.34}$$

with $M_{12}^{\rm SD}$ being the short-distance contribution to the major part of the off-diagonal piece of the K^0-$\overline{K}^0$ mass matrix, and where

$$\xi = \frac{{\rm Im}\langle \pi^+\pi^-|H|K^0\rangle}{{\rm Re}\langle \pi^+\pi^-|H|K^0\rangle} . \tag{3.35}$$

From the model B Lagrangian (3.14), one obtains [7] the expression for ϵ_m:

$$\begin{aligned} \epsilon_m &= \frac{M_K^2}{8}\left[1+\frac{3(\tilde{m}_s^2-\tilde{m}_d^2)}{M_K^2}\right]\frac{{\rm Im}(\alpha\beta^*)}{M_h^2} , \\ &\simeq 1.5\times10^{-5}\frac{{\rm Im}(\alpha\beta^*)}{(M_h/M_W)^2} , \end{aligned} \tag{3.36}$$

where $\tilde{m}_q$ are constituent quark masses. By (3.22) we have, for $M_h \geq M_W$,

$$\epsilon_m \lesssim 3\times10^{-6} . \tag{3.37}$$

Sanda and Deshpande separately argued that [17] 2ξ is just proportional to ϵ_m [it also $\sim {\rm Im}(\alpha\beta^*)/M_h^2$], and that $2\xi \sim 30\epsilon_m$. So model B gives ($\epsilon_B \equiv$ the contribution to ϵ from model B)

$$\epsilon_B \simeq \frac{30\epsilon_m}{2\sqrt{2}} \lesssim 3\times10^{-5} \lesssim \frac{\epsilon_{\rm expt}}{75} . \tag{3.38}$$

Next we want to examine ϵ'/ϵ, which is found experimentally to be [18] $(2.1\pm1.2)\times10^{-3}$. In 1979, Gilman and Wise [6] argued

$$\left|\frac{\epsilon'}{\epsilon}\right| = \frac{1}{20}\left[\frac{2\xi}{\epsilon_m+2\xi}\right] \simeq \frac{1}{20} , \tag{3.39}$$

which implies

$$\left|\frac{\epsilon_B'}{\epsilon_{\rm expt}}\right| \lesssim \frac{1}{1500} \lesssim 7\times10^{-4} . \tag{3.40}$$

Actually (3.39) is incomplete and $\epsilon_B'/\epsilon_{\rm expt}$ is probably even smaller because long-distance (LD) effects play a large role [19] in the calculation of ϵ'. Employing a calculation due to Donoghue and Holstein [20], we find

$$\left|\frac{\epsilon_B'}{\epsilon_{\rm expt}}\right|_{\rm DH} \lesssim 8\times10^{-5} . \tag{3.41}$$

From (3.40) it is clear that ϵ'/ϵ cannot currently constrain model B, even if we neglect LD effects. Equation (3.41) implies that the experimental bounds on ϵ'/ϵ would need to come down well over an order of magnitude before beginning to constrain model B.

3. K_L-K_S mass difference

Finally there is the K_L-K_S mass difference Δm_K. Chang [21] has shown that for a three-doublet Weinberg model with all three VEV's equal and $M_h \sim 15$ GeV, that Δm_K is less than the experimental value. Chang's exact expression shows that Δm_K depends only on the up-quark VEV, and in model B $v_{\rm up} \equiv v_1 \simeq v$. Thus Chang's result applies to model B, except that $M_h \sim M_W$. Since Δm_K goes down at least as M_h^{-2}, we see immediately

$$(\Delta m_K)_B \ll (\Delta m_K)_{\rm expt} . \tag{3.42}$$

IV. MODEL A

A. The Lagrangian

We wish to construct another model that gives effective scalar couplings. One way to do this is to introduce a scalar doublet

$$\Phi_A = \begin{bmatrix} \phi_A^+ \\ \phi_A^0 \end{bmatrix} , \tag{4.1}$$

as in model B, but eliminate the requirement that the neutral scalar develop a VEV (later we will allow ϕ_A^0 to develop a very small VEV for renormalizability). The most general coupling of the Φ_A to fermions is of the same form as the model B Lagrangian,

$$\begin{aligned} -\mathcal{L} = {} & G_{QD}(\overline{Q}{}_L'\Phi_A A_D D_R') + G_{UQ}(\overline{U}{}_R'\tilde{\Phi}_A^\dagger A_U Q_L') \\ & + G_{LE}(\overline{L}{}_L'\Phi_A A_E E_R') + \text{H.c.} , \end{aligned} \tag{4.2}$$

with the important differences that the coupling constants G do not depend on VEV's, and the 3×3 flavor matrices A do not necessarily derive from the quark mass matrices. We do get constraints on the A's from FCNC's especially for the G_{QD} term which can generate tree-level Δm. In order to avoid this possibility, we again impose NFC, via a single discrete symmetry,

$$(\Phi_A, D_R, U_R, E_R) \to (-\Phi_A, D_R, -U_R, -E_R) , \quad (4.3)$$

so that $\bar{Q}'_L \Phi_1 A_D D_R$ is not invariant. If the Lagrangian in (4.2) is invariant under this discrete symmetry, $G_{QD}=0$, and $\Delta m_K=0$ at the tree level. We must still look to the neutral Lagrangian for other FCNC terms, though Δm_K was the biggest such constraint. Once again we write the weak eigenstates in terms of the mass eigenstates, $X'_L = V_L^X X_L$, to obtain the neutral Lagrangian

$$-\mathcal{L}^0 = G_{UQ}(\bar{U}_R V^U U_L)\phi_A^0 + G_{LE}(\bar{E}_L V^E E_R)\phi_A^0 + \text{H.c.} , \quad (4.4)$$

where

$$\begin{aligned} V^U &\equiv V_R^{U\dagger} A_U V_L^U , \\ V^E &\equiv V_L^{E\dagger} A_E V_R^E . \end{aligned} \quad (4.5)$$

To avoid FCNC's we need V^U and V^E close to or exactly diagonal. Model B required the analogous matrices to be the quark mass matrices, but since we are not constrained to generate a mass Lagrangian, we choose $V^U = \text{diag}(V_{11}^U V_{22}^U V_{33}^U)$ and $V^E = \text{diag}(V_{11}^E V_{22}^E V_{33}^E)$ for simplicity. Note that we could have required V^D to be diagonal instead of imposing the discrete symmetry (4.3), but to get a small enough Δm_K would then require radiative corrections to the V^D off-diagonal elements to be $\sim 0.5\%$.

Given (4.5), the charged part of (4.2) becomes

$$\begin{aligned} -\mathcal{L}^+ = &-G_{UQ}(\bar{U}_R V^U V_L D_L)\phi_A^+ \\ &+ G_{LE}(\bar{N}_L V^E E_R)\phi_A^+ + \text{H.c.} , \end{aligned} \quad (4.6)$$

where $V_L \equiv V_L^{U\dagger} V_L^D$ is again the CKM matrix. Now our effective-Lagrangian parameters are

$$\begin{aligned} L_{DU} &= 0 , \\ R_{DU} &= -G_{UQ}^* V_{UD}^* V_{UU}^{U*} , \\ L_{EN} &= G_{LE}^* V_{EE}^{E*} , \end{aligned} \quad (4.7)$$

where L_{DU} is zero by our discrete symmetry. Equation (4.7) gives the parameters in (2.9),

$$\begin{aligned} L_{su} &= 0 , \\ R_{su} &= -G_{UQ}^* V_{11}^{U*} \sin\theta_C , \\ L_{\mu\nu} &= G_{LE}^* V_{22}^{E*} , \end{aligned} \quad (4.8)$$

and using (2.15) and (2.18), we obtain an averaged polarization of

$$\overline{P_\mu^\perp} \simeq 10 \frac{\text{Im}(G_{UQ}^* G_{LE})}{(M_A/M_W)^2} , \quad (4.9)$$

where we have absorbed V_{11}^U and V_{22}^E into G_{UQ} and G_{LE}. This means that to achieve a $\overline{P_\mu^\perp}$ of 10^{-3}, we need $\text{Im}(G_{UQ}^* G_{LE})/(M_A/M_W)^2 \sim 10^{-4}$. For reference, if $G_{UQ} = G_{LE} = 1$, and CP is violated maximally, then $M_A \leq 8$ TeV will still give $\overline{P_\mu^\perp} \geq 10^{-3}$.

If we were taking this model seriously as a complete model, we would have to consider its renormalizability and related issues. But we are interested only in the CP-violating aspects of the model which are unlikely to be affected by such questions. As an example, we examine the issue of the renormalizability of the VEV in this model. ϕ_A^0 can get infinite contributions to its VEV via tadpoles with fermion loops (among others), so we need to be able to introduce counterterms in the Lagrangian to cancel these. To ensure renormalizability, we must allow ϕ_A^0 to start with some arbitrarily small VEV, v_A^0, which through corrections becomes v_A, which is in general nonzero. This v_A will then introduce contributions to all fermion masses $\Delta m_f \sim G(f)V(f)v_A$, where $G(f)$ is either G_{UQ} or G_{LE}, and $V(f)$ is the appropriate diagonal element of V^U or V^E. For all fermions except the electron, $V(f)V(t) \sim 1$ will give $\Delta m_f \ll m_f$, but we must be careful about the electron since its mass is so small. The fermion loop tadpole gets a contribution to v_A proportional to $G_{UQ} V_{33}^U m_t^3/M_A^2$ which in turn gives a contribution to the electron mass of

$$\begin{aligned} \Delta m_e &\sim \frac{4}{(4\pi)^2} \frac{G_{LE} G_{UQ}}{(M_A/M_W)^2} \frac{m_t^3}{M_W^2} V_{11}^E V_{33}^U \\ &\simeq (1 \text{ MeV}) V_{11}^E V_{33}^U , \end{aligned} \quad (4.10)$$

for $P_\mu^\perp \sim 10^{-3}$. Since V^E and V^U are arbitrary diagonal matrices, we can easily require $\Delta m_e \ll m_e$ by making the product $V_{11}^E V_{33}^U$ small, without affecting (4.9) (for which we chose our G's such that $V_{22}^E V_{11}^U = 1$). Note that dealing with this issue in no way affected $P_\mu^\perp$.

B. Constraints on model A

1. Electric dipole moments

The neutron electric dipole moment d_n is proportional to $\text{Im}(L_{DU}^* R_{DU})$ [see Eq. (A.8)], which is zero since $L_{DU}^* \sim G_{QD} = 0$. So provided our Lagrangian is invariant under the discrete symmetry (4.3), $d_n = 0$ at the one-loop level. If we instead relax this assumption, we obtain

$$d_n \simeq (10^{-20}\ e\ \text{cm}) \frac{\text{Im}(G_{UQ}^* G_{QD})}{(M_A/M_W)^2} . \quad (4.11)$$

If we arbitrarily take $G_{QD} = G_{UQ} = G_{LE}$ and maximal semileptonic CP violation, we see from (4.9) that

$$d_n \sim (10^{-24}\ e\ \text{cm}) \left[\frac{\overline{P_\mu^\perp}}{10^{-3}} \right] \sin\alpha_{DU} , \quad (4.12)$$

where $\text{Im}(G_{UQ}^* G_{QD}) = |G_{UQ}||G_{QD}|\sin\alpha_{DU}$. Since (4.9) does not depend on $\sin\alpha_{DU}$, even with the removal of the discrete symmetry it is still possible to obtain $\overline{P_\mu^\perp} \sim 10^{-3}$ by requiring $\sin\alpha_{DU} \lesssim 0.1$ (or identically zero). So d_n cannot constrain model A, even without the discrete symmetry.

The electron electric dipole moment is zero (to one-loop order), independent of the value for G_{QD}, and thus independent of our choice to impose the discrete symmetry or not. Once again the charged sector gives $d_e \sim m_\nu$, which is negligible. The neutral sector gives very small d_e because ϕ_A^0 only mixed with ϕ^0 though loops, aside from being proportional to m_e^2.

2. ϵ and ϵ'

If the discrete symmetry (4.3) is imposed, ϵ and ϵ' are identically zero. Simply put, ϵ and ϵ' are measures of *hadronic CP* violation, whereas model A only has *semileptonic CP* violation. This can be expressed by the equation

$$\epsilon,\epsilon' \sim \mathrm{Im}(L_{DU}^* R_{DU})=0 \ . \tag{4.13}$$

3. K_L-K_S mass difference

The discrete symmetry (4.3) removes tree-level contributions to Δm_K, but we still have box-diagram contributions to worry about. In model B, the dominant contribution to Δm_K goes as $(L_{su}^B)^2/(M_h/M_W)^2 \sim 2\times 10^{-5}$, whereas in model A it goes as $(R_{su}^A)^2/(M_A/M_W)^2 \sim 5\times 10^{-6}$. Thus we have

$$(\Delta m_K)_A < (\Delta m_K)_B \ll (\Delta m_K)_{\mathrm{expt}} \ , \tag{4.14}$$

showing that Δm_K is not a constraint to model A either.

V. MODEL C

A. The Lagrangian

As we stated, one can generate $\mathcal{L}_{\mathrm{eff}}$ in (2.8) via a Fierz transformation of, for example, a leptoquark model. We explore this possibility by introducing the scalar leptoquark doublet

$$\Phi_C = \begin{pmatrix} \phi_2 \\ \phi_1 \end{pmatrix} = \begin{pmatrix} \phi^{+5/3} \\ \phi^{+2/3} \end{pmatrix} , \tag{5.1}$$

which couples to fermions analogously to models A and B:

$$-\mathcal{L} = G_{LU}(\bar{U}'_R \tilde{\Phi}_C^\dagger F_{LU} L'_L) + G_{QE}(\bar{Q}'_L \Phi_C F_{QE} E'_R) + \mathrm{H.c.} \tag{5.2}$$

Once again we write the weak eigenstates in terms of the mass eigenstates, $X'_L = V_L^X X_L$, to obtain the Lagrangian for scalars one and two:

$$-\mathcal{L}_1 = G_{LU}(\bar{U}_R V_1^{LU} N_L)\phi_1 + G_{QE}(\bar{D}_L V_1^{QE} E_R)\phi_1 + \mathrm{H.c.} \ , \tag{5.3}$$

$$-\mathcal{L}_2 = -G_{LU}(\bar{U}_R V_2^{LU} E_L)\phi_2 + G_{QE}(\bar{U}_L V_2^{QE} E_R)\phi_2 + \mathrm{H.c.} \ , \tag{5.4}$$

where we have, from left-handed couplings,

$$V_1^{LU} = V_R^{U\dagger} F_{LU} V_L^N = V_R^{U\dagger} F_{LU} V_L^E = V_2^{LU} \ , \tag{5.5}$$

$$V_1^{QE} = V_L^{D\dagger} F_{QE} V_R^E = V_L^\dagger V_L^{U\dagger} F_{QE} V_R^E = V_L^\dagger V_2^{QE} \ , \tag{5.6}$$

where $V_L \equiv V_L^{U\dagger} V_L^D$ is again the CKM matrix. Now the two-family Lagrangians are

$$\begin{aligned} -\mathcal{L}_1 = {} & G_{LU}[c_1^{LU}(\bar{u}_R \nu_{eL}) + c_1^{LU}(\bar{c}_R \nu_{\mu L}) + \underline{s_1^{LU}(\bar{u}_R \nu_{\mu L})} - s_1^{LU}(\bar{c}_R \nu_{eL})]\phi_1 \\ & + G_{QE}[c_1^{QE}(\bar{d}_L e_R) + \underline{c_1^{QE}(\bar{s}_L \mu_R)} + s_1^{QE}(\bar{d}_L \mu_R) - s_1^{QE}(\bar{s}_L e_R)]\phi_1 + \mathrm{H.c.} \ , \end{aligned} \tag{5.7}$$

and

$$\begin{aligned} -\mathcal{L}_2 = {} & -G_{LU}[c_2^{LU}(\bar{u}_R e_L) + c_2^{LU}(\bar{c}_R \mu_L) + s_2^{LU}(\bar{u}_R \mu_L) - s_2^{LU}(\bar{c}_R e_L)]\phi_2 \\ & + G_{QE}[c_2^{QE}(\bar{u}_L e_R) + c_2^{QE}(\bar{c}_L \mu_R) + s_2^{QE}(\bar{u}_L \mu_R) - s_2^{QE}(\bar{c}_L e_R)]\phi_2 + \mathrm{H.c.} \ , \end{aligned} \tag{5.8}$$

where we define the sines and cosines c_1^{LU} and s_1^{LU} by

$$V_1^{LU} = \begin{bmatrix} c_1^{LU} & s_1^{LU} \\ -s_1^{LU} & c_1^{LU} \end{bmatrix} , \tag{5.9}$$

and similarly for c_1^{QE}, s_1^{QE}, and $(1\to 2)$. One can see immediately that $\mathcal{L}_1$ will contribute to $P_\mu^\perp$ via the underlined terms in (5.7), while giving zero for all purely hadronic and purely leptonic CP-violating observables $(\epsilon,\epsilon',d_n,d_e)$. Conversely, $\mathcal{L}_2$ can give finite values to all of these quantities, except $P_\mu^\perp$. As we will see later, this means we can avoid constraints on $\mathcal{L}_2$ by putting conditions on s_2^{LU}, c_2^{QE}, and M_2, without affecting $P_\mu^\perp$.

Using the underlined terms in (5.7), we write

$$\mathcal{L}_{\mathrm{eff}} = \frac{G_{QE} G_{LU}^*}{M_1^2} c_1^{QE} s_1^{LU} (\bar{s}_L \mu_R)(\bar{\nu}_{\mu L} u_R) + \mathrm{H.c.} \ , \tag{5.10}$$

which after a Fierz transformation yields

$$\mathcal{L}_{\mathrm{eff}} = \frac{G_{QE} G_{LU}^*}{M_1^2} c_1^{QE} s_1^{LU} [\tfrac{1}{2}(\bar{s}_L u_R)(\bar{\nu}_{\mu L} \mu_R) + \tfrac{1}{8}(\bar{s}_L \sigma^{\mu\nu} u_R)(\bar{\nu}_{\mu L} \sigma_{\mu\nu} \mu_R)] + \mathrm{H.c.} \tag{5.11}$$

Both the scalar and tensor pieces contribute to $P_\mu^\perp$ on the quark level in a way markedly different from at the hadronic level. For example on the quark level, a helicity flip makes both contributions proportional to the current up-quark mass (m_u), while no such suppression occurs on the hadronic level because the hadronic matrix element contains no axial-vector or pseudoscalar pieces [see (2.5)], and some constituent m_u applies. For simplicity we examine the scalar contribution to $P_\mu^\perp$,

$$\mathrm{Im}\bar{\xi}=\tfrac{1}{2}c_1^{QE}s_1^{LU}\frac{\mathrm{Im}(G_{QE}G_{LU}^*)}{(M_1/M_W)^2}\ , \tag{5.12}$$

which gives an averaged polarization of

$$\overline{P_\mu^\perp}=22c_1^{QE}s_1^{LU}R\frac{|G_{QE}|^2}{(M_1/M_W)^2}s_{\mathrm{Im}}\ , \tag{5.13}$$

with

$$s_{\mathrm{Im}}\equiv\frac{\mathrm{Im}(G_{QE}G_{LU}^*)}{|G_{QE}||G_{LU}|}\ , \tag{5.14}$$

and

$$R\equiv\frac{|G_{LU}|}{|G_{QE}|}\ . \tag{5.15}$$

The biggest constraint to $P_\mu^\perp$ comes from $K_L\to\mu e$, which $\sim|G_{QE}|^2/(M_1/M_W)^2$, but the ratio of the Yukawa-like couplings (R) can still be large, and will indeed have to be in order that $P_\mu^\perp$ not be too small. If the couplings in (5.2) were mass couplings using a single VEV, then each of the G's would be proportional to the mass of the right-handed quarks. Of course G_{QE} and G_{LU} are not mass couplings, but in the same spirit as the choice (3.21) for model B, we can argue that it is *reasonable* that R be of the order of the ratio of the two right-handed fermions (in the third family) to which each G couples:

$$R=\frac{|G_{LU}|}{|G_{QE}|}\sim\frac{m_t}{m_\tau}\ . \tag{5.16}$$

B. Constraints on model C

1. *Rare K decays*

As stated above, we will get a large constraint on G_{QE} from $K_L\to\mu e$, but R can be as large as (5.16) because the rare K decay constraints on G_{LU} are much weaker than those on G_{QE}. This results from the fact that G_{LU} couplings involve only neutrinos and up-type quarks, and not strange quarks. For example, the branching ratio of $K\to\pi\nu\bar{\nu}$ is very small in model C, because one needs at least one W, giving an extra factor of G_F in the amplitude.

The dominant constraints to G_{QE} are from $K_L\to\mu e$ and $K_L\to\pi^-\mu e$ (Figs. 2 and 3). After a Fierz transform, one can write each of them as multiples of $K^+\to\mu^+\nu$ and $K^+\to\pi^0\mu^+\nu$, respectively, giving

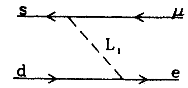

FIG. 2. Contribution to $K^0\to\mu^+e^-$ from scalar leptoquark L_1. (Labels as in Fig. 1.)

$$\frac{B(K_L\to\mu e)}{B(K_L\to\pi^0\mu e)}\simeq\frac{B(K^+\to\mu\nu)}{B(K^+\to\pi^0\mu\nu)}\simeq 20\ . \tag{5.17}$$

Since $K_L\to\mu e$ and $K^+\to\mu^+\nu$ have the same behavior, their ratio is just the ratio of the coupling constants:

$$\frac{\Gamma(K_L\to\mu e)}{\Gamma(K^+\to\mu^+\nu)}\simeq\tfrac{1}{32}(G_F^2M_W^4\sin\theta_c)^{-1}\frac{|G_{QE}|^4}{(M_1/M_W)^4}\ . \tag{5.18}$$

But to get $B(K_L\to\mu e)$, one needs to take into account the difference in the K_L and K^+ lifetimes,

$$B(K_L\to\mu e)=B(K^+\to\mu^+\nu)\frac{\tau(K_L)}{\tau(K^+)}\frac{\Gamma(K_L\to\mu e)}{\Gamma(K^+\to\mu^+\nu)}\ , \tag{5.19}$$

which combined with (5.18) and (5.13) gives

$$B(K_L\to\mu e)=3.0\times10^{-11}(c_1^{QE}s_1^{LU}s_{\mathrm{Im}})^{-2}\left[\frac{R}{72}\right]^{-2}\times\left[\frac{\overline{P_\mu^\perp}}{10^{-3}}\right]^2\ , \tag{5.20}$$

resulting in the constraint

$$\overline{P_\mu^\perp}\le2.6\times10^{-3}(c_1^{QE}s_1^{LU}s_{\mathrm{Im}})\left[\frac{R}{72}\right]\left[\frac{B^{\mathrm{expt}}(K_L\to\mu e)}{2\times10^{-10}}\right]^{1/2}. \tag{5.21}$$

We have normalized R to the same value as v_2/v_3 in model B, and $B^{\mathrm{expt}}(K_L\to\mu e)$ is normalized to the current experimental bound [22]. If $c_1^{QE}s_1^{LU}s_{\mathrm{Im}}$ is not too small (since they are independent sines and cosines their product can be as large as unity), $\overline{P_\mu^\perp}\sim10^{-3}$ is still allowed by $K_L\to\mu e$.

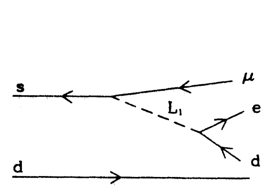

FIG. 3. Contribution to $K^0\to\pi^0\mu^+e^-$ from scalar leptoquark L_1. (Labels as in Fig. 1.)

2. $\mu \to e\gamma$ and $\mu N \to eN$

One does get a constraint on $P_\mu^\perp$ from $\mu \to e\gamma$, and a lesser constraint from $\mu N \to eN$, but both are smaller constraints on $\mathcal{L}_1$ (and thus $P_\mu^\perp$) than $K_L \to \mu e$. The main concern is how these two processes constrain $\mathcal{L}_2$. Since the matrix elements for these processes have the same form

$$|\mathcal{M}_2|^2 = Cm_\mu^2 (s_2^{LU})^2 (c_2^{LU})^2 \frac{|G_{LU}|^4}{(M_2/M_W)^4} , \qquad (5.22)$$

we will concentrate on the larger process, $\mu \to e\gamma$ (see Fig. 4), for which the constant $C \sim 10^{-13} \ln^2(m_c^2/M_2^2)$. Using (5.13) and (5.15), we can relate (5.22) to $P_\mu^\perp$:

$$\frac{|G_{LU}|^4}{(M_2/M_W)^4} = \frac{R^2}{(22c_1^{QE} s_1^{LU} s_{\mathrm{Im}})^2} \left[\frac{M_1}{M_2} \right]^4 (\overline{P_\mu^\perp})^2 . \qquad (5.23)$$

Now $s_1^{LU} = s_2^{LU}$ by (5.5), so we can write

$$B(\mu \to e\gamma) = 0.015 \ln^2 \left[\frac{m_c^2}{M_2^2} \right] (c_2^{LU})^2 \left[\frac{M_1}{M_2} \right]^4 \left[\frac{\overline{P_\mu^\perp}}{10^{-3}} \right]^2 , \qquad (5.24)$$

which means to satisfy the bound [23] $B(\mu \to e\gamma) \leq 4 \times 10^{-11}$ for a $P_\mu^\perp \sim 10^{-3}$, we require

$$M_2 \gtrsim \sqrt{c_2^{LU}}\, 10^3 M_1 . \qquad (5.25)$$

Thus we must set $s_2^{LU} \to 1$ and $M_2 \gg M_1$ in some combination to satisfy (5.25). We have complete freedom to specify c_2^{LU} because of the flavor matrix F_{LU} in (5.2). However $c_1^{LU} \lesssim 10^{-6}$ is unnatural, so some mass splitting is presumably required. This could be achieved by allowing the leptoquark mass matrix to have a zero eigenvalue and (say) letting M_1 be generated through radiative corrections. As stated before, $\mathcal{L}_2$ does not directly affect $P_\mu^\perp$ and therefore cannot constrain it, though (5.25) is needed to be consistent with $B(\mu \to e\gamma)$.

3. ϵ', d_n, and d_e

Since $\mathcal{L}_1$ only has semileptonic CP violation, and no purely hadronic or purely leptonic CP violation, ϵ, ϵ', d_n, and d_e are all zero. The other part of the Lagrangian, $\mathcal{L}_2$, can contribute to any of these quantities but because of (5.25) their magnitudes will be negligible.

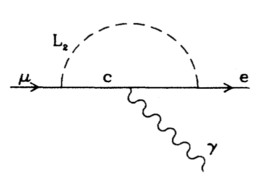

FIG. 4. Contribution to $\mu \to e\gamma$ from scalar leptoquark L_2. [There is also a diagram with the topology of 5(b). Labels as in Fig. 1.]

VI. DISCUSSION OF $K \to \pi\mu\mu$

We now briefly discuss model-B contributions to the rare K decays $K \to \pi\mu\mu$ because they have the interesting property that all three independent CP-violating coefficients arise and can contribute to the transverse polarization of a muon, $P_\mu^\perp(\pi\mu\mu)$. There are contributions from the h penguin diagrams (where a W in the standard-model penguin diagram is replaced by an h) and hW box diagrams (where one W in a WW box replaced by an h) that depend on the three CP-violating coefficients

$$d_n \text{ and } P_\mu^\perp(\pi\mu\mu)_{h\,\text{penguin}} \sim \mathrm{Im}(\alpha\beta^*) \sim \frac{vv_3}{v_1 v_2} ,$$

$$P_\mu^\perp(\pi\mu\nu) \text{ and } P_\mu^\perp(\pi\mu\mu)_{hW\,\text{box}} \sim \mathrm{Im}(\alpha\gamma^*) \sim \frac{vv_2}{v_1 v_3} , \qquad (6.1)$$

$$P_\mu^\perp(\pi\mu\mu)_{hW\,\text{box}} \sim \mathrm{Im}(\beta\gamma^*) \sim \frac{vv_1}{v_2 v_3} ,$$

where for reference we have noted the behavior of d_n and $P_\mu^\perp(\pi\mu\nu)$. Recent calculations [24] show that these diagrams get a large enhancement proportional to m_t^2, but unfortunately the ratio of the couplings of the Higgs boson diagrams to those of the W diagrams is still small. A preliminary estimate shows that for the decay $K^+ \to \pi^+\mu^+\mu^-$ [which should have [25] a branching ratio $\gtrsim 10^{-8}$ since $B^{\text{expt}}(K^+ \to \pi^+ e^+ e^-) \sim 3 \times 10^{-7}$], $P_\mu^\perp$ is probably small even for large ratios of VEV's.

The decay $K_L \to \pi^0\mu^+\mu^-$ is quite different because it is essentially a CP-violating decay (there is a CP-conserving two-photon contribution which is expected to be small) [24], so its contribution from the standard model is expected to be quite small [a rough estimate of the rate is $\epsilon^2\Gamma(K^+ \to \pi^+\mu^+\mu^-)$]. Model B will suffer no such suppression, and in fact is helped because there are no longer any CP-conserving terms contributing to the denominator of $P_\mu^\perp$. A $P_\mu^\perp$ of order unity is not unreasonable. The problem is that the standard-model contribution to the CP-violating $P_\mu^\perp$ can now, in principle, be large, so $P_\mu^\perp$ in $K_L \to \pi^0\mu^+\mu^-$ might still not be a good signal for non-standard-model CP violation. This point needs further analysis. However, it is still possible to see the effect of a CP-violating multi-Higgs-boson model in the branching ratio itself. The key point is that since the hW box diagram is proportional to the outgoing lepton mass, one can get a large contribution to $B(K_L \to \pi^0\mu^+\mu^-)$ but a negligible contribution to $B(K_L \to \pi^0 e^+ e^-)$, so that, for some range of parameters,

$$B(K_L \to \pi^0\mu^+\mu^-) \gtrsim B(K_L \to \pi^0 e^+ e^-) , \qquad (6.2)$$

which would clearly point to physics beyond the standard model.

VII. CONCLUDING REMARKS

We have presented three models which achieve the desired goal of generating $P_\mu^\perp \sim 10^{-3}$ such that each model satisfies existing experimental constraints. Most models of CP violation get constrained by d_n, ϵ, or ϵ', but models A and C do not because they only generate semileptonic CP violation whereas the other observables come from hadronic CP violation. There are other types of constraints on these models (for example rare K decays constrain model C), but we find it intriguing that CP violation from models A and C could be seen *only* in a semileptonic process. Model B (which uses three Higgs doublets) can potentially be constrained by d_n, ϵ, or ϵ' because it generates hadronic as well as semileptonic CP violation. As we have seen, these observables do not bound $P_\mu^\perp$ to be below 10^{-3} because having VEV's proportional to fermion masses enhances the coefficient $\mathrm{Im}(\alpha\gamma^*)$ for $P_\mu^\perp$ without substantially affecting $\mathrm{Im}(\alpha\beta^*)$, the coefficient of the constraints. We also show that there is some hope of seeing CP violation in the decay $K_L \to \pi^0\mu^+\mu^-$, but that further investigation is needed.

Finally we note that there have been several recent attempts [26] to explain the observed baryon asymmetry of the Universe using B violation that may occur at high temperatures in the standard model, and CP violation in the Higgs potential of multiple-Higgs-doublet models. The results are consistent (within large theoretical uncertainties) with the value of the baryon asymmetry of the Universe deduced from primordial nucleosynthesis. A wide class of multiple-Higgs-doublet models can generate the necessary CP violation since the parameters in the Higgs potential are rather unconstrained. One constraint is that the lightest Higgs boson should not have a mass much greater than M_W, but in models such as model B, this condition can still be met even if the bounds on d_n drop significantly because there are large regions of allowed parameter space where the CP-violating coefficient $\mathrm{Im}(\alpha\beta^*)$ is small. Since multiple-Higgs-doublet models can also potentially generate a detectable $P_\mu^\perp$, perhaps CP violation in $K \to \pi\mu\nu$ could be related to explaining the observed baryon-number asymmetry.

We reiterate that our purpose in proposing these three models is to help justify an experimental search for $P_\mu^\perp$ by showing that there is plenty of room for seeing CP violation in $K \to \pi\mu\nu$, and not necessarily to tout the models as good approximations of reality. We assume there are a number of additional models which could be written which give a large $P_\mu^\perp$ while satisfying relevant experimental constraints. Should an effect be detected, we would welcome the challenge of theoretically untangling the results.

We conclude by emphasizing that even after further examination, CP-violating observables in semileptonic K decays, particularly the transverse polarization of the muon, remain an underexploited and potentially valuable tool for studying CP violation and for detecting physics beyond the standard model.

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APPENDIX A: ELECTRIC DIPOLE MOMENTS

Since we use constraints from electric dipole moments several times, in this appendix we write the one-loop con-

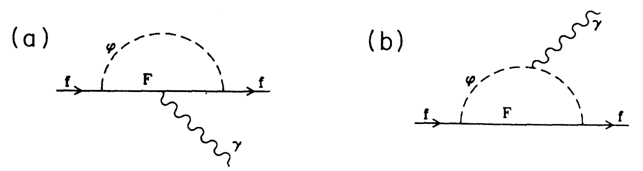

FIG. 5. One-loop quark level contributions to d_{fF} (fermion F's contribution to the electric dipole moment of fermion f) from scalar ϕ. Diagram 5(b) is only present for charged scalars. [Labels as in Fig. 1.]

tribution to the electric dipole moment d_f of an arbitrary fermion f arising from the scalar Lagrangian:

$$\mathcal{L} = L_{fF}\bar{f}_R F_L \phi_x + R_{fF}\bar{f}_L F_R \phi_x + \mathrm{H.c.} \tag{A1}$$

We label the contribution of F to the electric dipole as d_{fF}. In general there will be several F's contributing to d_f, each with a different L_{fF}, R_{fF}, and m_F, so that

$$d_f = \sum_F d_{fF} \; . \tag{A2}$$

There are two diagrams which contribute to d_{fF} [Figs. 5(a) and 5(b)], which we label (a) and (b), giving electric dipole moments $d_{fF}^{(\mathrm{a})}$ and $d_{fF}^{(\mathrm{b})}$, respectively. We obtain

$$d_{fF}^{(\mathrm{a})} = Q_F e m_F \frac{\mathrm{Im}(L_{fF}^* R_{fF})}{M_x^2} \frac{I^{(\mathrm{a})}(X)}{(4\pi)^2} \; , \tag{A3}$$

$$d_{fF}^{(\mathrm{b})} = -Q_x e m_F \frac{\mathrm{Im}(L_{fF}^* R_{fF})}{M_x^2} \frac{I^{(\mathrm{b})}(X)}{(4\pi)^2} \; , \tag{A4}$$

where

$$X \equiv \frac{m_F^2}{M_x^2} \; , \tag{A5}$$

and where $Q_F e$ and $Q_x e$ are the charges of fermion F and scalar ϕ_x, respectively. The integrals $I^{(\mathrm{a})}$ and $I^{(\mathrm{b})}$ are over Feynman parameters, and have values (assuming $m_f \ll m_F$)

$$\begin{aligned} I^{(\mathrm{a})}(X) &= \int_0^1 \frac{y^2}{1-y(1-X)} \\ &= \frac{1}{(1-X)^2}\left[-\frac{3}{2}+\frac{1}{2}X-\frac{1}{1-X}\ln X\right] \; , \end{aligned} \tag{A6}$$

$$\begin{aligned} I^{(\mathrm{b})}(X) &= \int_0^1 dy \frac{y-y^2}{1-y(1-X)} \\ &= \frac{1}{(1-X)^2}\left[\frac{1}{2}(1+X)+\frac{X}{1-X}\ln X\right] \; . \end{aligned} \tag{A7}$$

Using (A5), (A3), and (A4) in (A2), we obtain the fermion electric dipole moment

$$d_f = e\frac{1}{(4\pi)^2}\sum_F m_F \frac{\mathrm{Im}(L_{fF}^* R_{fF})}{M_x^2} \times \left[Q_F I^{(\mathrm{a})}\left[\frac{m_F^2}{M_x^2}\right] - Q_x I^{(\mathrm{b})}\left[\frac{m_F^2}{M_x^2}\right]\right] \; . \tag{A8}$$

[1] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turley, Phys. Rev. Lett. **13**, 138 (1964).

[2] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[3] For a review, see the article by S. M. Barr and W. Marciano, in *CP Violation,* edited by C. Jarlskog (World Scientific, Singapore, 1989).

[4] P. Castoldi, J. M. Frère, and G. Kane, Phys. Rev. D **39**, 2633 (1989); see also M. Leurer, Phys. Rev. Lett. **62**, 1967 (1989).

[5] A. R. Zhitnitskii, Yad. Fiz. **31**, 1024 (1980) [Sov. J. Nucl. Phys. **31**, 529 (1980)].

[6] M. Campbell *et al.*, Phys. Rev. D **27**, 1056 (1983).

[7] H. Y. Cheng, Phys. Rev. D **26**, 143 (1982).

[8] I. B. Okun and I. B. Khriplovich, Yad. Fiz. **6**, 821 (1967) [Sov. J. Nucl. Phys. **6**, 598 (1967)].

[9] T. D. Lee, Phys. Rev. D **8**, 1226 (1973).

[10] S. Weinberg, Phys. Rev. Lett. **37**, 657 (1976).

[11] C. Albright, J. Smith, and S. Tye, Phys. Rev. D **21**, 711 (1980).

[12] K. Smith *et al.*, Phys. Lett. B **234**, 191 (1990).

[13] H. Y. Cheng, Phys. Rev. D **28**, 150 (1983).

[14] S. M. Barr and A. Zee, Phys. Rev. Lett. **65**, 21 (1990).

[15] J. F. Gunion and R. Vega, Phys. Lett. B **251**, 157 (1990).

[16] F. Gilman and M. Wise, Phys. Lett. **83B**, 83 (1979).

[17] A. Sanda, Phys. Rev. D **23**, 2647 (1981); N. Deshpande, *ibid.* **23**, 2654 (1981).

[18] There are two experiments, Fermilab E711 and CERN NA31, which give conflicting results on ϵ'/ϵ. We follow the Particle Data Group, J. J. Hernández *et al.*, Phys. Lett. B **239**, 1 (1990), in simply averaging them.

[19] See, for example, the article by I. Bigi and A. Sanda, in *CP Violation* (Ref. [3]).

[20] J. Donoghue and B. Holstein, Phys. Rev. D **32**, 1152 (1985).

[21] D. Chang, Phys. Rev. D **25**, 1318 (1982).

[22] C. Mathiazhagan *et al.*, Phys. Rev. Lett. **63**, 2177 (1989).

[23] R. Bolton *et al.*, Phys. Rev. D **38**, 2077 (1988).

[24] C. Dib, I. Dunietz, and F. Gilman, Phys. Rev. D **39**, 2639 (1989); M. Savage and M. Wise, Phys. Lett. B **250**, 151 (1990).

[25] From a talk given by M. Schmidt on rare *K* decays (unpublished).

[26] L. Mclerran, M. Shaposhnikov, N. Turok, and M. Voloshin, Phys. Lett. B **256**, 451 (1991); N. Turok and J. Zadrozny, Nucl. Phys. B **358**, 471 (1991); M. Dine, P. Huet, R. Singleton, and L. Susskind, Phys. Lett. B **257**, 351 (1991).