

## *T*-violating muon polarization in $K_{\mu 3}$ decays

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We update the analysis on the muon polarization from the  $K_{\mu 3}$  decays normal to the decay plane due to *CP* violation in various models. We find that the muon polarization could reach a level of  $10^{-3}$  in multi-Higgs-boson and leptoquark models without conflicting with experimental constraints.

### I. INTRODUCTION

It is well known that measuring a component of muon polarization normal to the decay plane in  $K_{\mu 3}$  decays would signal *T* violation [1]. This muon polarization called transverse polarization ( $P_{\perp}$ ) is related to the *T*-odd triple correlation:

$$\mathbf{s}_{\mu} \cdot (\mathbf{p}_{\mu} \times \mathbf{p}_{\pi}), \quad (1.1)$$

where  $\mathbf{s}_{\mu}$  and  $\mathbf{p}_{\mu(\pi)}$  are the muon spin vector and the muon (pion) momentum respectively. A *T*-odd operator can arise in *T*-invariant theories since *T* invariance also requires an exchange of initial and final states. However, it is possible to extract signals from genuine *T* violation (or *CP* violation) in  $K_{\mu 3}$  decays. A small  $P_{\perp}$  can be induced by electromagnetic final-state interactions [2,3] even in the absence of *CP* violation. It has been estimated that such effects lead to  $P_{\perp} \sim 10^{-3}$  in  $K_{\mu 3}^0$  ( $K^0 \rightarrow \pi^- \mu^+ \nu$  and  $\bar{K}^0 \rightarrow \pi^+ \mu^- \bar{\nu}$ ) [2,3] and  $P_{\perp} \leq 10^{-6}$  in  $K_{\mu 3}^{\pm}$  ( $K^+ \rightarrow \pi^0 \mu^+ \nu$  and  $K^- \rightarrow \pi^0 \mu^- \bar{\nu}$ ) [4] decays. At the level of  $10^{-3}$ , an observation of  $P_{\perp}$  for  $K_{\mu 3}^{\pm}$  decay will indicate *T* violation, whereas for  $K_{\mu 3}^0$  decays, one needs to measure the difference between the muon polarization in  $K^0 \rightarrow \pi^- \mu^+ \nu$  and  $\bar{K}^0 \rightarrow \pi^+ \mu^- \bar{\nu}$  decays to distinguish the real *CP*-violating effects from the final-state interactions. These interactions give the same  $P_{\perp}$  for the two neutral *K* decays whereas *CP* violation gives a different sign for  $P_{\perp}$  [3]. Due to the electromagnetic corrections and the difficulty of measuring  $\mu^-$  polarization, the decay

$$K^+ \rightarrow \pi^0 \mu^+ \nu \quad (1.2)$$

is the most promising decay mode among the  $K_{\mu 3}$  decays to study *CP* violation [5]. Thus, we will concentrate on this particular  $K^+$  decay in (1.2) and study the possibility of having a sizable  $P_{\perp}$  in various models of *CP* violation; the electromagnetic final-state interaction will be ignored. In the standard model of electroweak interactions, *CP* violation comes from the complex Kobayashi-Maskawa (KM) [6] matrix in the quark sector where only one physical phase exists, and there is no mixing and therefore no *CP* violation in the lepton sector. Since only one diagram induces the process  $K^+ \rightarrow \pi^0 \mu^+ \nu$  occurs at the tree level, there cannot be interference effects or *CP* violation. Hence,  $P_{\perp}$  is zero in the standard KM model. A nonzero

signal of  $P_{\perp}$  must result from new *CP*-violation mechanisms beyond the standard KM model. Recently, a general examination of the nonstandard effects for  $P_{\perp}$  in  $K_{\mu 3}$  was done [7,8]. It has been shown that a nonzero value of  $P_{\perp}$  can be achieved with an effective scalar or leptoquark interaction. A transverse polarization cannot arise from effective vector interactions such as the ones in the standard model or in the left-right symmetric models. We will derive the result on effective vector interactions in a more direct way. Among the models with scalar interactions, the most popular one which would lead to a large transverse polarization of the muon [4,9] in the decay (1.2) is Weinberg's three-Higgs-doublet model [10] in which the *CP* violation arises dominantly from the exchange of charged Higgs bosons. The multi-Higgs-boson models have received renewed interest because of the recent theoretical developments as well as new experimental constraints [11]. However, most of the efforts were on the effects of the neutron electric dipole moment (NEDM)  $d_n$  (Ref. [12]). Furthermore, as the minimum charged-Higgs-bosons mass is raised to be  $> 45$  GeV by experiments at the CERN  $e^+e^-$  collider LEP, the estimated value of  $P_{\perp}$  in Refs. [4] and [9] clearly will change. It should be interesting to improve the prediction on  $P_{\perp}$  in this class of models. While for leptoquark models definite predictions are still lacking, it is important to study these effects as well.

In this report, we will systematically study the transverse muon polarization in various specific *CP*-violation models incorporating phenomenological constraints. In particular, we will reexamine the *CP*-violating effects in the multi-Higgs-boson and leptoquark models and give updated predictions on  $P_{\perp}$ .

The paper is organized as follows. In Sec. II we give a general analysis for muon polarization. We then study the *T*-violating transverse polarization of the muon in various models in Sec. III. Our conclusions are summarized in Sec. IV.

### II. GENERAL ANALYSIS FOR MUON POLARIZATION

We first carry out a general analysis of the decay  $K^+ \rightarrow \pi^0 \mu^+ \nu$  based on Lorentz invariance. The most general invariant amplitude can be written in the form

$$\begin{aligned} \mathcal{M} = & F_S \bar{v}(p_\nu) \mu(p_\mu, s_\mu) + F_P \bar{v}(p_\nu) i \gamma_5 \mu(p_\mu, s_\mu) \\ & + F_V p_k^\alpha \bar{v}(p_\nu) \gamma_\alpha \mu(p_\mu, s_\mu) + F_A p_k^\alpha \bar{v}(p_\nu) \gamma_\alpha \gamma_5 \mu(p_\mu, s_\mu) \end{aligned} \quad (2.1)$$

where  $F_S$ ,  $F_P$ ,  $F_V$ , and  $F_A$  are scalar, pseudoscalar, vector, and axial-vector form factors, respectively. These form factors are complex functions of Lorentz invariant quantities. The  $p_k$ ,  $p_\pi$ ,  $p_\mu$ , and  $p_\nu$  are the four-momenta of  $K^+$ ,  $\pi^0$ ,  $\mu^+$ , and  $\nu$ , respectively, while  $s_\mu$  is the polarization vector of the muon. To compute a physical quantity for such a process, we have to estimate the contribution of various diagrams to these form factors.

The probability of the decay (1.2) as a function of the four-momenta of the particles and the polarization four-

vector  $s_\mu$  of the muon is easily calculated by the standard techniques and is given by

$$dw = (1 + s_\mu \cdot P) \Phi' \rho / (2E_k), \quad (2.2)$$

where

$$\rho = \frac{d\mathbf{p}_\pi d\mathbf{p}_\mu d\mathbf{p}_\nu}{2E_\pi 2E_\mu 2E_\nu} \delta^4(p_k - p_\pi - p_\mu - p_\nu) (2\pi)^{-5}, \quad (2.3)$$

$$\begin{aligned} \Phi' = & [ |F_S|^2 + |F_P|^2 - (|F_V|^2 + |F_A|^2) m_K^2 ] 2p_\mu \cdot p_\nu \\ & + (|F_V|^2 + |F_A|^2) 4p_\mu \cdot p_k p_k \cdot p_\nu \\ & - [ \text{Re}(F_S F_V^*) + \text{Im}(F_P F_A^*) ] 4m_\mu p_k \cdot p_\nu, \end{aligned} \quad (2.4)$$

and the four-vector  $P$  is defined as follows:

$$P = P_1 + P_2 + P_\perp, \quad (2.5)$$

$$P_1^\alpha = 8m_\mu \left[ -\text{Re}(F_V F_A^*) p_k \cdot p_\nu + \frac{1}{2m_\mu} [ \text{Re}(F_S F_A^*) + \text{Im}(F_P F_V^*) ] p_\mu \cdot p_\nu \right] p_k^\alpha / \Phi', \quad (2.6)$$

$$P_2^\alpha = -4m_\mu \left[ \text{Im}(F_S F_P^*) + \frac{1}{m_\mu} [ \text{Re}(F_S F_A^*) + \text{Im}(F_P F_V^*) ] p_k \cdot p_\mu - \text{Re}(F_V F_A^*) m_K^2 \right] p_\nu^\alpha / \Phi', \quad (2.7)$$

$$P_\perp^\alpha = -4 [ -\text{Im}(F_S F_V^*) + \text{Re}(F_P F_A^*) ] \varepsilon^{\alpha\beta\gamma\delta} p_{\mu\beta} p_{\nu\gamma} p_{k\delta} / \Phi'. \quad (2.8)$$

It is easy to see that the term  $s_\mu \cdot P_\perp$  is odd under the time-reversal transformation and it is proportional to the  $T$ -odd triple correlation  $\mathbf{s}_\mu \cdot (\mathbf{p}_\mu \times \mathbf{p}_\nu)$  or  $\mathbf{s}_\mu \cdot (\mathbf{p}_\mu \times \mathbf{p}_\pi)$  in the kaon rest system.  $P_1$  and  $P_2$  are related to the polarization in the decay plane of the muon and the pion, these are not  $CP$  violating. The components of the four-vector muon polarization  $s_\mu$  can be written in terms of  $\xi$ , a unit vector along the muon spin in its rest frame, as

$$s_0 = \frac{\mathbf{p}_\mu \cdot \xi}{m_\mu}, \quad \mathbf{s}_\mu = \xi + \frac{s_0}{E_\mu + m_\mu} \mathbf{p}_\mu. \quad (2.9)$$

In the rest frame of the kaon, the transverse polarization in (2.8) can be rewritten in three-dimensional form

$$\mathbf{P}_\perp = -\mathbf{n}_\mu \times \mathbf{n}_\nu 4m_K E_\nu |\mathbf{p}_\mu| [ -\text{Im}(F_S F_V^*) + \text{Re}(F_P F_A^*) ] / \Phi' \quad (2.10)$$

with

$$\begin{aligned} \Phi' = & 4m_K^2 E_\nu \left[ \left[ \frac{1}{m_K^2} (|F_S|^2 + |F_P|^2) - (|F_V|^2 + |F_A|^2) \right] \frac{1}{2} (E_\mu - \mathbf{n}_\mu \cdot \mathbf{n}_\nu |\mathbf{p}_\mu|) \right. \\ & \left. + E_\mu (|F_V|^2 + |F_A|^2) - \frac{m_\mu}{m_K} [ \text{Re}(F_S F_V^*) + \text{Im}(F_P F_A^*) ] \right], \end{aligned} \quad (2.11)$$

where  $\mathbf{n}_\mu$  and  $\mathbf{n}_\nu$  are the unit vectors along  $\mathbf{p}_\mu$  and  $\mathbf{p}_\nu$ , respectively.

### III. TRANSVERSE MUON POLARIZATION

The various models of  $CP$  violation can be classified according to the type of intermediate boson exchanges that could give a tree-level contribution to the decay  $K^+ \rightarrow \pi^0 \mu^+ \nu$ . There are only three possible types (a)–(c) of tree diagrams shown in Figs. 1(a)–1(c), corresponding

to intermediate bosons of electric charges 1,  $\frac{2}{3}$ , and  $-\frac{1}{3}$  respectively. Most of the existing models belong to type (a) since they have either a charged vector or scalar boson responsible for this decay. For example, the standard, the left-right-symmetric, and the horizontal-symmetry models can have gauge boson exchanges and the multi-Higgs-boson models can have scalar-charged-Higgs-boson exchanges. Figures (b) and (c) arise in leptoquark models with intermediate leptoquark exchanges. In this section we will study the transverse muon polarization,

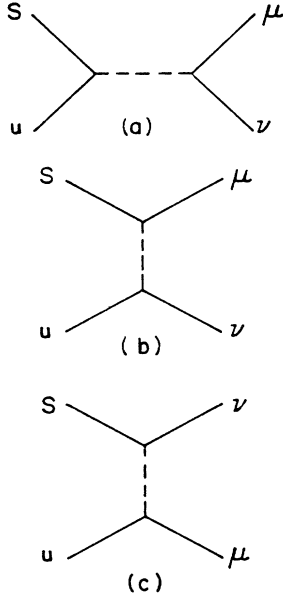


FIG. 1. Tree diagrams contributing to  $K^+ \rightarrow \pi^0 \mu^+ \nu$  decay.

including the phenomenological constraints, in each specific model in those three classes.

#### A. The standard model: An overview

From Fig. 1(a) the amplitude of the decay  $K^+ \rightarrow \pi^0 \mu^+ \nu$  in the standard model with  $V-A$  interactions is given by

$$\mathcal{M}^0 = \frac{G_F}{2} \sin\theta_C [f_+(q^2)(p_k + p_\pi)^\alpha + f_-(q^2)(p_k - p_\pi)^\alpha] \bar{\nu} \gamma_\alpha (1 - \gamma_5) \mu, \quad (3.1)$$

where  $G_F$  is the Fermi coupling constant,  $\theta_C$  is the Cabibbo angle, and  $f_+(q^2)$  and  $f_-(q^2)$  are the form factors from the hadronic matrix element:

$$\begin{aligned} & \langle \pi^0 | \bar{\nu} \gamma_\alpha (1 - \gamma_5) u | K^+ \rangle \\ &= \frac{1}{\sqrt{2}} [f_+(q^2)(p_k + p_\pi)_\alpha + f_-(q^2)(p_k - p_\pi)_\alpha] \end{aligned} \quad (3.2)$$

with  $q = (p_k - p_\pi) = (p_\mu + p_\nu)$ . Comparing Eqs. (2.1) and

(3.1), we find that

$$\begin{aligned} F_S^0 &= iF_P^0 = -G_F \sin\theta_C m_\mu f_+ + \chi, \\ F_V^0 &= -F_A^0 = G_F \sin\theta_C f_+, \end{aligned} \quad (3.3)$$

where  $\chi = \frac{1}{2}[(f_-/f_+) - 1]$ . Equations (2.6)–(2.8) and (3.3) lead to

$$P_1^{0\alpha} = 2m_\mu \text{Re}(p_\nu \cdot R) p_k^\alpha / \Phi^0, \quad (3.4)$$

$$P_2^{0\alpha} = -m_\mu |R|^2 p_\nu^\alpha / \Phi_0, \quad (3.5)$$

$$P_\perp^{0\alpha} = -2m_\mu \text{Im}(\chi) \epsilon^{\alpha\beta\gamma\delta} p_{\mu\beta} p_{\nu\gamma} p_{k\delta} / \Phi^0, \quad (3.6)$$

where  $R = p_k + \chi p_\mu$  and

$$\begin{aligned} \Phi^0 &= (4G_F^2 \sin^2\theta_C |f_+|^2)^{-1} \Phi^{0'} \\ &= 2p_\mu \cdot p_k p_\nu \cdot p_k - m_K^2 p_\mu \cdot p_\nu \\ &\quad + 2\text{Re}\chi m_\mu^2 p_\nu \cdot p_k + m_\mu^2 |\chi|^2 p_\mu \cdot p_\nu. \end{aligned} \quad (3.7)$$

We note that the results in Eqs. (3.4)–(3.6) agree with that given in Ref. [3]. Since the ratio of the two form factors  $f_+(q^2)$  and  $f_-(q^2)$  is real, i.e.,  $\text{Im}\chi = 0$ , we have  $P_\perp = 0$  in the standard model as mentioned in the Introduction. In the leading order of chiral perturbation theory  $f_+ = 1$  and  $f_- = 0$ , then  $\chi \simeq -\frac{1}{2}$ , and we thus have

$$\begin{aligned} \Phi^0 &= m_K^2 E_\nu \left[ \left[ 1 + \frac{1}{4} \frac{m_\mu^2}{m_K^2} \right] E_\mu \right. \\ &\quad \left. + \left[ 1 - \frac{1}{4} \frac{m_\mu^2}{m_K^2} \right] |\mathbf{p}_\mu| \mathbf{n}_\mu \cdot \mathbf{n}_\nu - \frac{m_\mu^2}{m_K} \right] \end{aligned} \quad (3.8)$$

in the rest frame of the kaon.

#### B. Models with only $V$ and $A$ interactions

We start by discussing the left-right-symmetric models [13]. For these models, it has been pointed out [7,8,14] that there is no transverse polarization of the muon. We now show this result by using our general expression given by Eq. (2.10). We will discuss the most general left-right-symmetric models in which the neutrinos are massive and therefore  $CP$  violation comes from not only the quark mixings but also the lepton mixings. The model involves both  $V-A$  and  $V+A$  interactions. There are four tree diagrams of type (a) contributing to the decay as shown in the Fig. 1 of Ref. [8]. Without loss of generality the amplitude can be written as

$$\begin{aligned} \mathcal{M} &= \frac{G_F}{\sqrt{2}} \sin\theta_C \langle \pi^0 | X \bar{\nu} \gamma_\alpha (1 - \gamma_5) u \bar{\nu} \gamma^\alpha (1 - \gamma_5) \mu + Y \bar{\nu} \gamma_\alpha (1 - \gamma_5) u \bar{\nu} \gamma^\alpha (1 + \gamma_5) \mu \\ &\quad + Z \bar{\nu} \gamma_\alpha (1 + \gamma_5) u \bar{\nu} \gamma^\alpha (1 - \gamma_5) \mu + W \bar{\nu} \gamma_\alpha (1 + \gamma_5) u \bar{\nu} \gamma^\alpha (1 + \gamma_5) \mu | K^+ \rangle, \end{aligned} \quad (3.9)$$

where  $X$ ,  $Y$ ,  $Z$ , and  $W$  are arbitrary complex factors arising from the diagrams of Figs. 1(a)–1(d) of Ref. [8], respectively. The matrix elements

$$\begin{aligned} \langle \pi^0 | \bar{s} \gamma_\alpha (1 - \gamma_5) u | K^+ \rangle &= \langle \pi^0 | \bar{s} \gamma_\alpha (1 + \gamma_5) u | K^+ \rangle \\ &= \langle \pi^0 | \bar{s} \gamma_\alpha u | K^+ \rangle \end{aligned} \quad (3.10)$$

since the axial-vector piece vanishes due to parity. Using this and Eq. (3.2), we obtain, from the amplitude (3.9) the following form factors defined in Eq. (2.1):

$$\begin{aligned} F_S &= -G_F \sin \theta_C (m_\mu - m_\nu) f_+ \chi (X + Y + Z + W), \\ F_P &= i G_F \sin \theta_C (m_\mu + m_\nu) f_+ \chi (X - Y + Z - W), \\ F_V &= G_F \sin \theta_C f_+ (X + Y + Z + W), \\ F_A &= -G_F \sin \theta_C f_+ (X - Y + Z - W). \end{aligned} \quad (3.11)$$

It is easy to see from the above equations that

$$\text{Im}(F_S F_V^*) = \text{Re}(F_P F_A^*) = 0 \quad (3.12)$$

since  $\text{Im} \chi = 0$ . From Eq. (2.10), we conclude that  $P_\perp = 0$  in left-right-symmetric models [15]. This result is independent of the masses of neutrinos or on the presence of neutrino mixing. For models with horizontal symmetries where the gauge bosons have vector and axial-vector couplings [14], the amplitude can always be put in the form (3.9) resulting in a zero value for the muon transverse polarization. In fact, it is straightforward to see that the result of  $P_\perp = 0$  can be extended to arbitrary models with effective  $V$  or  $A$  interactions as already been concluded by many authors [7,8,14].

### C. Multi-Higgs-boson models

We study the charged-Higgs-boson effects on  $P_\perp$  in multi-Higgs-boson models. As originally pointed out by Weinberg [10] to have spontaneous  $CP$  violation due to different relative phases of the vacuum expectation values (VEV's) of the Higgs-doublet fields and natural flavor conservation (NFC), [16] at least three Higgs doublets ( $\phi_i, i = 1, \dots, 3$ ) are needed [17]. The NFC is guaranteed if, for example,  $\phi_1$  only couples to  $D_R$ ,  $\phi_2$  to  $U_R$ , and  $\phi_3$  to  $E_R$ , respectively. However, to achieve it, a symmetry such as a discrete or Peccei-Quinn (PQ) [18] symmetry has to be introduced [19]. The model contains three possible sources of  $CP$  violation: (i) the complex KM matrix; (ii) a phase in the charged Higgs boson mixing, and (iii) neutral scalar-pseudoscalar mixing [20,21]. In the original Weinberg three-Higgs-doublet model,  $CP$

is broken spontaneously and the KM matrix is real. The observed  $CP$  violation in the neutral kaon system comes solely from charged-Higgs-boson exchange. However, both charged- and neutral-Higgs-boson exchanges [11] in this model would give rise to sizable  $d_n$  whereas only the charged-Higgs-boson exchanges [4] yield contributions to  $P_\perp$  in  $K^+ \rightarrow \pi^0 \mu^+ \nu$  through Fig. 1(a). In order to isolate the  $CP$ -violating transverse muon polarization arising from (ii), in this report we will assume that  $CP$ -violating effects by the neutral-Higgs-boson exchanges are much smaller than that by the charged ones. We will first study the original Weinberg model and then the models in which  $CP$  violation occurs both in (i) and (ii). For the latter case, the observed  $CP$  violation in  $K \rightarrow \pi\pi$  decays [22] will be accounted dominantly by the standard KM mechanisms and thus the main constraint on the model is from the experimental limits on the neutron EDM.

The Yukawa interactions of quarks and leptons with the charged-Higgs-boson in the mass eigenstates are given by

$$\begin{aligned} \mathcal{L}_Y &= (2\sqrt{2}G_F)^{1/2} \sum_{i=1}^2 (\alpha_i \bar{U}_L K M_D D_R + \beta_i \bar{U}_R M_u K D_L \\ &\quad + \gamma_i \bar{N}_L M_E E_R) H_i^+ + \text{H.c.}, \end{aligned} \quad (3.13)$$

where  $K$  is the KM matrix,  $M_U$ ,  $M_D$ , and  $M_E$  are the fermion mass matrices for  $d$ - and  $u$ -type quarks and charged leptons, respectively, and  $\alpha_i$ ,  $\beta_i$ , and  $\gamma_i$  are complex coupling constants which are related to each other by the following conditions [23]:

$$\frac{\text{Im}(\alpha_2 \beta_2^*)}{\text{Im}(\alpha_1 \beta_1^*)} = \frac{\text{Im}(\beta_2 \gamma_2^*)}{\text{Im}(\beta_1 \gamma_1^*)} = \frac{\text{Im}(\alpha_2 \gamma_2^*)}{\text{Im}(\alpha_1 \gamma_1^*)} = -1 \quad (3.14a)$$

and

$$\frac{1}{v_2^2} \text{Im}(\alpha_1 \gamma_1^*) = -\frac{1}{v_1^2} \text{Im}(\beta_1 \gamma_1^*) = -\frac{1}{v_3^2} \text{Im}(\alpha_1 \beta_1^*), \quad (3.14b)$$

where  $v_i$  ( $i = 1, 2, 3$ ) are the VEV's of the Higgs doublets  $\phi_i$  and

$$v \equiv (v_1^2 + v_2^2 + v_3^2)^{1/2} = (2\sqrt{2}G_F)^{-1/2}.$$

The amplitude of the decay  $K^+ \rightarrow \pi^0 \mu^+ \nu$  in the multi-Higgs-boson models can be written as

$$\mathcal{M} = \mathcal{M}^0 + \mathcal{M}^1, \quad (3.15)$$

where  $\mathcal{M}^0$  is the standard piece given in (3.1) and

$$\mathcal{M}^1 = -\frac{G_F}{\sqrt{2}} \sin \theta_C m_\mu \sum_{i=1}^2 \left\langle \pi^0 \left| m_s \frac{\alpha_i^* \gamma_i}{M_{H_i}^2} \bar{s} (1 - \gamma_5) u \bar{\nu} (1 + \gamma_5) \mu + m_u \frac{\beta_i^* \gamma_i}{M_{H_i}^2} \bar{s} (1 + \gamma_5) u \bar{\nu} (1 + \gamma_5) \mu \right| K^+ \right\rangle \quad (3.16)$$

as calculated from Fig. 1(a) with the intermediate charged-Higgs-boson exchanges. For simplicity we will assume that  $M_{H_2} \gg M_{H_1} = M_H$  and neglect the  $m_u$ -dependent terms in Eq. (3.15). Using the matrix elements

$$\begin{aligned} \langle \pi^0 | \bar{s}(1-\gamma_5)u | K^+ \rangle &= \langle \pi^0 | \bar{s}(1+\gamma_5)u | K^+ \rangle \\ &\simeq \frac{1}{\sqrt{2}} \frac{m_K^2}{m_s} f_+ , \end{aligned} \quad (3.17)$$

we find that

$$\begin{aligned} F_S &= F_S^0 + F_S^1, \quad F_P = F_P^0 + F_P^1, \\ F_V &= F_V^0, \quad F_A = F_A^0, \end{aligned} \quad (3.18)$$

where  $F_\alpha$  ( $\alpha = S, P, V$ , and  $A$ ) are given in Eq. (3.3) and

$$F_S^1 = iF_P^1 = -\frac{G_F}{2} \sin\theta_C m_\mu m_K^2 f_+ \frac{\alpha_1^* \gamma_1}{M_H^2}. \quad (3.19)$$

From Eqs. (2.10), (3.8), (3.14), (3.18), and (3.19) we get [4,9]

$$\begin{aligned} \mathbf{P}_\perp &\simeq -\mathbf{n}_\mu \times \mathbf{n}_\nu \frac{\text{Im}(\alpha_1 \beta_1^*)}{M_H^2} \frac{v_2^2}{v_3^2} m_\mu m_K \\ &\times \frac{|\mathbf{p}_\mu|}{[E_\mu + |\mathbf{p}_\mu| \mathbf{n}_\mu \cdot \mathbf{n}_\nu - (m_\mu^2/m_K)]}. \end{aligned} \quad (3.20)$$

In order to estimate  $P_\perp$  one has to know the constraints on both the parameters  $\text{Im}(\alpha_1 \beta_1^*)/M_H^2$  and  $v_2^2/v_3^2$ , in various multi-Higgs-boson models. We start by considering the Weinberg three-Higgs-doublet model. The constraints on this model were updated recently by Cheng [12]. He finds that

$$\frac{\text{Im}(\alpha_1 \beta_1^*)}{M_H^2} \left[ \ln \frac{M_H^2}{m_c^2} - \frac{3}{2} \right] = (0.024 - 0.027) \text{ GeV}^{-2} \quad (3.21)$$

with the use of the experimental value [24]  $|\epsilon| = 2.26 \times 10^{-3}$  and the consideration of not too large  $\epsilon'$ . The NEDM due to the charged-Higgs-boson exchange in the one-loop diagrams, given by [25]

$$\begin{aligned} d_n &\simeq \frac{4}{3} d_d = -\frac{\sqrt{2} G_F}{9\pi^2} m_d \text{Im}(\alpha_1 \beta_1^*) \\ &\times \sum_{i=c,t} \frac{x_i}{(1-x_i)^2} \\ &\times \left[ \frac{5}{4} x_i - \frac{3}{4} - \frac{1-\frac{3}{2}x_i}{1-x_i} \ln x_i \right] K_{id}^2 \end{aligned} \quad (3.22)$$

with  $x_i = m_i^2/M_H^2$ , is expected to be  $\sim -9 \times 10^{-26} e \text{ cm}$  with Eq. (3.21) and  $K_{cd} \sim 0.22$ ,  $K_{td} \sim 0.01$ ,  $M_H \sim 45 \text{ GeV}$ , and  $m_t \sim 100 \text{ GeV}$ . Thus, the value of  $d_n$  at one-loop level is not far from the experimental limit [26]  $d_n^{\text{expt}} < 1.2 \times 10^{-25} e \text{ cm}$ . As pointed out by Cheng [12], the original Weinberg model could be ruled out by higher

loop contributions to  $d_n$ . The naive-dimensional-analysis (NDA) estimate of  $d_n$  on the Weinberg three-gluon operator [27,28] arising from two-loop diagrams with charged-Higgs-boson exchanges [29] yields [12]

$$\begin{aligned} d_n^{3g} &\simeq 1 \times 10^{-24} \frac{\text{Im}(\alpha_1 \beta_1^*)}{M_H^2} \frac{m_t^2}{[1 - (m_t^2/M_H^2)]^3} \\ &\times \left[ -\ln \frac{m_t^2}{M_H^2} - \frac{3}{2} + 2 \frac{m_t^2}{M_H^2} - \frac{1}{2} \frac{m_t^4}{M_H^4} \right] e \text{ cm}, \end{aligned} \quad (3.23)$$

and  $d_n^{3g} \sim 4 \times 10^{-24} e \text{ cm}$  by using Eq. (3.21) and  $M_H \sim 45 \text{ GeV}$  and  $m_t \sim 100 \text{ GeV}$ . However, there are large uncertainties [30,31] such as the choice of the hadronic scale [31] in calculating the contribution to  $d_n$  from the gluon operator and the value in (3.23) can be easily wrong by an order of magnitude. Moreover, in a model with a PQ symmetry, it has been shown by Bander [32] that a large part of the contribution of the three-gluon operator will be eliminated.

Without considering the Weinberg three-gluon operator, we find, using Eq. (3.20) and the constraint in (3.21), that

$$P_\perp \equiv |\mathbf{P}_\perp|_{\mathbf{n}_\mu \cdot \mathbf{n}_\nu = 0} \sim 2.6 \times 10^{-4} \frac{v_2^2}{v_3^2} \quad (3.24)$$

for an outgoing muon and neutrino at right angle, i.e.,  $\mathbf{n}_\mu \cdot \mathbf{n}_\nu = 0$  and  $M_H \sim 45 \text{ GeV}$ . The prediction of  $P_\perp$  in (3.24) is not very sensitive to the mass of the charged Higgs boson because of its logarithmic dependence, the transverse polarization could reach  $10^{-3}$  provided  $v_2/v_3 \geq 2$ . To see if this is possible, we study the phenomenological constraint on the factor  $v_2/v_3$ . We recall the analog of the KM matrix for the charged-Higgs-boson mixings [33] and we have

$$\text{Im}(\alpha_1 \beta_1^*) = \frac{v v_3}{2v_1 v_2} \sin 2\bar{\theta}_3 \sin \delta_H. \quad (3.25)$$

For  $M_H \geq 45 \text{ GeV}$ , from Eqs. (3.21) and (3.25) we obtain

$$\frac{v_3 \sqrt{v_1^2 + v_2^2 + v_3^2}}{2v_1 v_2} \geq 9. \quad (3.26)$$

To satisfy (3.26) for a large ratio of  $v_2/v_3$ , the only possibility is to have large ratios of  $v_2/v_1$  and  $v_3/v_1$ . Experimentally, the ratio  $v_2/v_1$  is constrained by  $D^0 - \bar{D}^0$  mixing. The dominant contribution to the  $D$  mass difference  $\Delta M_D$  due to the  $D^0 - \bar{D}^0$  mixing from the box diagrams involving the charged Higgs boson is given by

$$\Delta M_D = \frac{G_F^2}{24\pi^2} \sin^2\theta_C m_D f_D^2 B_D \frac{m_s^4}{M_H^2} \left[ \frac{v_2}{v_1} \right]^4. \quad (3.27)$$

Using the experimental upper limit [24]  $\Delta M_D < 1.3 \times 10^{-13} \text{ GeV}$  and  $m_D = 1.87 \text{ GeV}$ ,  $m_s = 0.2 \text{ GeV}$  and  $f_D B_D^{1/2} = 170 \text{ MeV}$ , we find

$$\frac{v_2}{v_1} \leq 15 \left( \frac{M_H}{\text{GeV}} \right)^{1/2}, \quad (3.28)$$

which implies for the most conservative bound corresponding to the lightest allowed charged Higgs boson,  $M_H \sim 45$  GeV,

$$\frac{v_2}{v_1} \leq 102. \quad (3.29)$$

Combining this result with (3.26) leads to

$$\frac{v_2}{v_3} \leq 5.7. \quad (3.30)$$

The upper limit of the transverse muon polarization is thus given by

$$P_\perp \leq 8.4 \times 10^{-3} \quad (3.31)$$

in the Weinberg three-Higgs-doublet model.

We now examine the phenomenological constraints on the multi-Higgs-boson models in which the KM matrix is complex. In these models we assume that  $\epsilon$  is dominated by the short-distance contribution and thus it will arise mainly from the standard KM  $CP$ -violation mechanism since the short-distance contribution from the charged-Higgs-boson exchange is very small [34]. In this case, the constraint on the parameter  $\text{Im}(\alpha_1 \beta_1^*) / M_H^2$  comes directly from  $d_n^{\text{expt}}$ .

From Eq. (3.22) we obtain

$$\frac{\text{Im}(\alpha_1 \beta_1^*)}{M_H^2} < 6 \times 10^{-3} \quad (3.32a)$$

and

$$\frac{\text{Im}(\alpha_1 \beta_1^*)}{M_H^2} < 2 \times 10^{-3} \quad (3.32b)$$

for  $m_t \sim 100$  GeV and (1)  $M_H \sim 45$  GeV and (2)  $M_H \sim 200$  GeV, respectively, which lead to

$$P_\perp^{(1)} < 3.2 \times 10^{-4} \frac{v_2^2}{v_3^2}, \quad P_\perp^{(2)} < 1.1 \times 10^{-4} \frac{v_2^2}{v_3^2}, \quad (3.33)$$

from Eq. (3.20). Unlike the previous Weinberg type of multi-Higgs-boson models, there are no experimental

constraints on the ratio of  $v_2/v_3$ . Therefore, the polarization  $P_\perp$  in (3.33) could very well reach a measurable level of  $10^{-3}$ . On the other hand, if we take the result of NDA on the three-gluon operator in Eq. (3.23) seriously, it is much less likely to have a transverse muon polarization of  $10^{-3}$ , since it requires an unnaturally large ratio of  $v_2/v_3$ . Although there is no constraint on this ratio, we have, instead of (3.33),

$$P_\perp^{(1)} < 7 \times 10^{-6} \frac{v_2^2}{v_3^2}, \quad P_\perp^{(2)} < 7 \times 10^{-7} \frac{v_2^2}{v_3^2}. \quad (3.34)$$

#### D. Leptoquarks

There are four scalar-leptoquark models which give contributions to the decay  $K^+ \rightarrow \pi^0 \mu^+ \nu$  through the tree diagrams in Figs. 1(b) and 1(c). The quantum numbers of the leptoquarks under the standard group  $SU(3)_C \times SU(2)_L \times U(1)_Y$  are [35,36]

$$\phi_1 = (3, 2, \frac{7}{3}) \quad (\text{model I}), \quad (3.35a)$$

$$\phi_2 = (3, 2, \frac{1}{3}) \quad (\text{model II}), \quad (3.35b)$$

$$\phi_3 = (3, 1, -\frac{2}{3}) \quad (\text{model III}), \quad (3.35c)$$

$$\phi_4 = (3, 3, -\frac{2}{3}) \quad (\text{model IV}), \quad (3.35d)$$

respectively. The general couplings involving these leptoquarks are given by [36]

$$\mathcal{L}^I = (\lambda_1 \bar{Q}_L e_R + \lambda'_1 \bar{u}_R L_L) \phi_1 + \text{H.c.}, \quad (3.36a)$$

$$\mathcal{L}^{II} = (\lambda_2 \bar{d}_R L_L + \lambda'_2 \bar{Q}_L \nu_R) \phi_2 + \text{H.c.}, \quad (3.36b)$$

$$\mathcal{L}^{III} = (\lambda_3 \bar{d}_R \nu_R^c + \lambda'_3 \bar{Q}_L L_L^c + \lambda'_3 \bar{u}_R e_R^c) \phi_3 + \text{H.c.}, \quad (3.36c)$$

$$\mathcal{L}^{IV} = \lambda_4 \bar{Q}_L L_L^c \phi_4 + \text{H.c.}, \quad (3.36d)$$

where  $Q = \begin{pmatrix} u \\ d \end{pmatrix}$  and  $L = \begin{pmatrix} \nu \\ e \end{pmatrix}$ . Here the coupling constants  $\lambda_k$  ( $k=1, \dots, 4$ ) are complex and thus  $CP$  violation could arise from either the Yukawa interactions in (3.36) or from the standard phase in the KM matrix. We assume that  $CP$  violation in  $K \rightarrow \pi\pi$  decays can be accounted for by the nonvanishing KM phase, and investigate the effect on the muon polarization of adding another  $CP$ -violation mechanism in (3.36).

In terms of each charge components of the leptoquarks, we rewrite Eq. (3.36) as

$$\mathcal{L}^I = \sum_{i,j} \{ [\lambda_1^{ij} \bar{u}_{i\frac{1}{2}} (1 + \gamma_5) e_j + \lambda'_1{}^{ij} \bar{u}_{i\frac{1}{2}} (1 - \gamma_5) e_j] \phi_1^{(5/3)} + [\lambda_1^{ij} \bar{d}_{i\frac{1}{2}} (1 + \gamma_5) e_j + \lambda'_1{}^{ij} \bar{u}_{i\frac{1}{2}} (1 - \gamma_5) \nu_j] \phi_1^{(2/3)} \} + \text{H.c.} \quad (3.37a)$$

$$\mathcal{L}^{II} = \sum_{i,j} \{ [\lambda_2^{ij} \bar{d}_{i\frac{1}{2}} (1 - \gamma_5) e_j + \lambda'_2{}^{ij} \bar{u}_{i\frac{1}{2}} (1 + \gamma_5) \nu_j] \phi_2^{(2/3)} + [\lambda_2^{ij} \bar{d}_{i\frac{1}{2}} (1 + \gamma_5) \nu_j + \lambda'_2{}^{ij} \bar{d}_{i\frac{1}{2}} (1 - \gamma_5) \nu_j] \phi_1^{(-1/3)} \} + \text{H.c.}, \quad (3.37b)$$

$$\mathcal{L}^{\text{III}} = \sum_{i,j} \{ \lambda_3^{ij} \bar{d}_i \frac{1}{2} (1 - \gamma_5) \nu_j^c + \lambda_3'^{ij} [ -\bar{u}_i \frac{1}{2} (1 + \gamma_5) e_j^c + \bar{d}_i \frac{1}{2} (1 + \gamma_5) \nu_j^c ] + \lambda_3''^{ij} \bar{u}_i \frac{1}{2} (1 - \gamma_5) e_j^c \} \phi_3^{(-1/3)} + \text{H.c.} , \quad (3.37c)$$

$$\mathcal{L}^{\text{IV}} = \sum_{i,j} \lambda_4^{ij} \{ \bar{u}_i \frac{1}{2} (1 + \gamma_5) \nu_j^c \phi_4^{(2/3)} + [ \bar{u}_i \frac{1}{2} (1 + \gamma_5) e_j^c + \bar{d}_i \frac{1}{2} (1 + \gamma_5) \nu_j^c ] \phi_4^{(-1/3)} + \bar{d}_i \frac{1}{2} (1 + \gamma_5) e_j^c \phi_4^{(-4/3)} \} + \text{H.c.} \quad (3.37d)$$

where  $i, j$  are family indices and  $Q_e$  in  $\phi_k^{(Q_e)}$  are the electric charges. From (3.37) we see that the relevant terms in the process  $K^+ \rightarrow \pi^0 \mu^+ \nu$  we are considering are the ones involving  $\phi_1^{(2/3)}$ ,  $\phi_2^{(2/3)}$ ,  $\phi_3^{(-1/3)}$ , and  $\phi_4^{(-1/3)}$  couplings, respectively. We will concentrate on these terms in our discussions. The effective interactions from these leptoquark exchanges shown in Figs. 1(b) and 1(c) are

$$\mathcal{L}_{\text{eff}}^{\text{I}} = \frac{\lambda_1^{22} (\lambda_1'^{1i})^*}{4M_{\phi_1}^2} \bar{s} (1 + \gamma_5) \mu \bar{\nu}_i (1 + \gamma_5) u + \text{H.c.} , \quad (3.38a)$$

$$\mathcal{L}_{\text{eff}}^{\text{II}} = \frac{\lambda_2^{22} (\lambda_2'^{1i})^*}{4M_{\phi_2}^2} \bar{s} (1 - \gamma_5) \mu \bar{\nu}_i (1 - \gamma_5) u + \text{H.c.} , \quad (3.38b)$$

$$\begin{aligned} \mathcal{L}_{\text{eff}}^{\text{III}} = & \frac{1}{4M_{\phi_3}^2} [ -\lambda_3^{2i} (\lambda_3'^{12})^* \bar{s} (1 - \gamma_5) \nu_i^c \bar{\mu}^c (1 - \gamma_5) u + \lambda_3^{2i} (\lambda_3''^{12})^* \bar{s} (1 + \gamma_5) \nu_i^c \bar{\mu}^c (1 - \gamma_5) u \\ & - \lambda_3'^{2i} (\lambda_3'^{12})^* \bar{s} (1 + \gamma_5) \nu_i^c \bar{\mu}^c (1 - \gamma_5) u + \lambda_3'^{2i} (\lambda_3''^{12})^* \bar{s} (1 + \gamma_5) \nu_i^c \bar{\mu}^c (1 + \gamma_5) u ] + \text{H.c.} , \end{aligned} \quad (3.38c)$$

$$\mathcal{L}_{\text{eff}}^{\text{IV}} = \frac{\lambda_4^{2i} (\lambda_4^{12})^*}{4M_{\phi_4}^2} \bar{s} (1 - \gamma_5) \nu_i^c \bar{\mu}^c (1 + \gamma_5) u + \text{H.c.} , \quad (3.38d)$$

where  $M_{\phi_k}$  ( $k=1, \dots, 4$ ) are the masses of  $\phi_1^{(2/3)}$ ,  $\phi_2^{(2/3)}$ ,  $\phi_3^{(-1/3)}$ , and  $\phi_4^{(-1/3)}$ , respectively.

Using the Fierz transformations

$$2\bar{s}(1 \pm \gamma_5) a \bar{b}(1 \mp \gamma_5) u = -\bar{s} \gamma_\mu (1 \mp \gamma_5) u \bar{b} \gamma^\mu (1 \pm \gamma_5) a \quad (3.39a)$$

and

$$\begin{aligned} 8\bar{s}(1 \pm \gamma_5) a \bar{b}(1 \pm \gamma_5) u = & -4\bar{s}(1 \pm \gamma_5) u \bar{b}(1 \pm \gamma_5) a \\ & -\bar{s} \sigma_{\mu\nu} (1 \pm \gamma_5) u \bar{b} \sigma^{\mu\nu} (1 \pm \gamma_5) a , \end{aligned} \quad (3.39b)$$

we can obtain the effective couplings

$$F_{\alpha k} = F_\alpha^0 + F_{\alpha k}^1 , \quad (3.40)$$

where  $\alpha = S, P, V$ , and  $A$  and  $k = 1, \dots, 4$ . From Fig. 1 we get

$$\begin{aligned} F_{S k}^1 = & (-1)^{k-1} i F_{P k}^1 \\ \simeq & (-1)^k \frac{\lambda_k^{22} (\lambda_k'^{1i})^*}{8\sqrt{2} M_{\phi_k}^2} \frac{m_K^2}{m_s} \quad (k=1, 2) , \\ F_{V k}^1 = & F_{A k}^1 \simeq 0 \quad (k=1, 2) , \\ F_{S 3}^1 \simeq & \frac{1}{8\sqrt{2} M_{\phi_3}^2} \frac{m_K^2}{m_s} \left[ -\lambda_3^{2i} (\lambda_3'^{12})^* + \lambda_3'^{2i} (\lambda_3''^{12})^* \right. \\ & \left. + \frac{m_s m_\mu}{m_K^2} [ -\lambda_3'^{2i} (\lambda_3''^{12})^* \right. \\ & \left. + \lambda_3'^{2i} (\lambda_3'^{12})^* ] \right] , \\ F_{P 3}^1 \simeq & -i \frac{1}{8\sqrt{2} M_{\phi_3}^2} \frac{m_K^2}{m_s} \left[ \lambda_3^{2i} (\lambda_3'^{12})^* + \lambda_3'^{2i} (\lambda_3''^{12})^* \right. \\ & \left. + \frac{m_s m_\mu}{m_K^2} [ \lambda_3'^{2i} (\lambda_3''^{12})^* \right. \\ & \left. + \lambda_3'^{2i} (\lambda_3'^{12})^* ] \right] , \end{aligned} \quad (3.41)$$

$$\begin{aligned}
F_{V3}^1 &\simeq \frac{1}{4\sqrt{2}M_{\phi_3}^2} [-\lambda_3^{2i}(\lambda_3''^{12})^* + \lambda_3'^{2i}(\lambda_3^{12})^*], & \frac{\text{Im}[\lambda_3'^{2i}(\lambda_3''^{12})^*]}{M_{\phi_3}^2}, \\
F_{A3}^1 &\simeq \frac{-1}{4\sqrt{2}M_{\phi_3}^2} [\lambda_3^{2i}(\lambda_3''^{12})^* + \lambda_3'^{2i}(\lambda_3^{12})^*], \\
F_{S4}^1 &= iF_{P4}^1 = \frac{m_\mu}{2} F_{V4}^1 \\
&= -\frac{m_\mu}{2} F_{A4}^1 \simeq -\frac{m_\mu}{8\sqrt{2}M_{\phi_4}^2} \lambda_4^{2i}(\lambda_4^{12})^*,
\end{aligned} \tag{3.43b}$$

where we have ignored the tensor interactions because the form factor  $f_T$  is much smaller than  $f_+ \simeq 1$  in  $K_{\mu 3}^+$  decays [24]. From Eqs. (2.10), (3.3), (3.7), (3.8), and (3.41) it follows that the transverse polarization is given by

$$\begin{aligned}
\mathbf{P}_1^I &\simeq -\mathbf{n}_\mu \times \mathbf{n}_\nu \frac{\sum_i \text{Im}[\lambda_1^{22}(\lambda_1'^{1i})^*]}{4\sqrt{2}G_F \sin\theta_C M_{\phi_1}^2} \frac{m_K}{m_s} \\
&\quad \times \frac{|\mathbf{p}_\mu|}{[E_\mu + |\mathbf{p}_\mu| \mathbf{n}_\mu \cdot \mathbf{n}_\nu - (m_\mu^2/m_K)]}, \tag{3.42a}
\end{aligned}$$

$$\mathbf{P}_1^{II} \simeq 0, \tag{3.42b}$$

$$\begin{aligned}
\mathbf{P}_1^{III} &\simeq \mathbf{n}_\mu \times \mathbf{n}_\nu \frac{\sum_i \text{Im}[\lambda_3'^{2i}(\lambda_3''^{12})^*]}{4\sqrt{2}G_F \sin\theta_C M_{\phi_3}^2} \frac{m_K}{m_s} \\
&\quad \times \frac{|\mathbf{p}_\mu|}{[E_\mu + |\mathbf{p}_\mu| \mathbf{n}_\mu \cdot \mathbf{n}_\nu - (m_\mu^2/m_K)]}, \tag{3.42c}
\end{aligned}$$

$$\mathbf{P}_1^{IV} \simeq 0, \tag{3.42d}$$

where the terms proportional to  $1/M_{\phi_k}^4$  were neglected. The result that  $P_1=0$  in model IV is expected as the effective interaction to the decay  $K^+ \rightarrow \pi^0 \mu^+ \nu$  is  $V \pm A$  as discussed in Sec. II. However, the zero transverse polarization in model II is based on the cancellation between  $\text{Im}(F_S F_V^*)$  and  $\text{Re}(F_P F_A^*)$  in Eq. (2.10). This is a general feature for nonstandard models with a coupling of the type  $F_S^1 \bar{\nu}(1-\gamma_5)\mu$ . Such a coupling is also present in model III together with  $V \pm A$  couplings, leaving only one term to contribute to the transverse polarization as obtained above.

To estimate the transverse muon polarization in models I and III we need to find out the bounds on the parameters

$$\frac{\text{Im}[\lambda_1^{22}(\lambda_1'^{1i})^*]}{M_{\phi_1}^2} \tag{3.43a}$$

and

model I:

$$\left[ \lambda_1^{22} \bar{c}_2^{\frac{1}{2}}(1+\gamma_5)\mu + \sum_i \lambda_1'^{1i} \bar{u}_2^{\frac{1}{2}}(1-\gamma_5)e_i \right] \phi_1^{(5/3)} + \left[ \lambda_1^{22} \bar{s}_2^{\frac{1}{2}}(1+\gamma_5)\mu + \sum_i \lambda_1'^{1i} \bar{u}_2^{\frac{1}{2}}(1-\gamma_5)\nu_i \right] \phi_1^{(2/3)} + \text{H.c.}, \tag{3.46a}$$

model III:

$$\left[ \sum_i \lambda_3'^{12} \bar{u}_2^{\frac{1}{2}}(1-\gamma_5)\mu^c + \lambda_3'^{2i} [-\bar{c}_2^{\frac{1}{2}}(1+\gamma_5)e_i^c + \bar{s}_2^{\frac{1}{2}}(1+\gamma_5)\nu_i^c] \right] \phi_3^{(-1/3)} + \text{H.c.} \tag{3.46b}$$

respectively.

The leptoquark couplings in (3.37) will induce several rare processes which could all put constraints on the parameters (3.43). In model I one expects contributions to either  $CP$ -conserving processes such as  $(g-2)_\mu$ ,  $\mu \rightarrow e\gamma$ ,  $\mu N \rightarrow eN$ ,  $P^0 \rightarrow l_i \bar{l}_j$  ( $P^0 = \pi^0, \eta, D^0, K_L, B^0$ , and  $l_{i,j} = e, \mu$ ),  $\pi^0 \rightarrow \nu \bar{\nu}$ ,  $K \rightarrow \pi \mu e$ , and  $b \rightarrow s\gamma$  or  $CP$ -violating ones, such as  $\epsilon$  in  $K^0 - \bar{K}^0$  mixing,  $K_L \rightarrow \pi^0 e \bar{e}$ ,  $d_n$ , and the electric dipole moments of leptons  $d_l$  ( $l = e, \mu, \tau, \nu_i$ ). From the Lagrangian (3.37c) of model III one expects a contribution to  $(g-2)_\mu$ ,  $\mu \rightarrow e\gamma$ ,  $\mu N \rightarrow eN$ ,  $P^0 \rightarrow l_i \bar{l}_j$  ( $P^0 = \pi^0, \eta, D^0$ ),  $\pi^0 \rightarrow \nu \bar{\nu}$ ,  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ ,  $K_L \rightarrow \pi^0 \nu \bar{\nu}$ ,  $\epsilon$  in  $K^0 - \bar{K}^0$  mixing,  $d_n$  and  $d_l$ . Very tight constraints on the parameters (3.43) are obtained by ignoring the generation index, i.e.,  $\lambda_k \sim |\lambda_k^{ij}| \sim |\lambda_k'^{ij}| \sim |\lambda_k''^{ij}|$  ( $i = 1, 3$ ) and  $M_{\phi_k} = M_{\phi_k}^{(Q_c)}$  in Eq. (3.37). For example, from the experimental limit [24] of  $B(K_L \rightarrow \mu e)_{\text{expt}} < 2.2 \times 10^{-10}$ , one finds that in model I [37]

$$\frac{\lambda_1^2}{M_{\phi_1}^2} < 10^{-10} \text{ GeV}^{-2} \tag{3.44a}$$

and in model III,

$$\frac{\lambda_3^2}{M_{\phi_3}^2} < 10^{-11} \text{ GeV}^{-2} \tag{3.44b}$$

from  $\Gamma(\mu Ti \rightarrow e Ti) / \Gamma(\mu Ti \rightarrow \text{all})_{\text{expt}} < 4.6 \times 10^{-12}$  (Ref. [24]). These two constraints, in turn, imply very small polarizations,

$$P_1^I < 5 \times 10^{-5}$$

and

$$P_1^{III} < 5 \times 10^{-6} \tag{3.45}$$

from Eqs. (3.42a) and (3.42c), respectively. Of course, the parameters in Eq. (3.43) could escape some experimental constraints since *a priori* there are no relations among the couplings corresponding to either flavor diagonal or flavor changing interactions. For instance, the limit in (3.44a) depends on

$$\frac{|\lambda_1^{21}(\lambda_1^{12})^* + \lambda_1^{11}(\lambda_1^{22})^*|}{M_{\phi_1}^2}.$$

To get direct constraints on  $\lambda_1^{22}(\lambda_1'^{1i})^*$  and  $\lambda_3'^{2i}(\lambda_3''^{12})^*$ , we write only the relevant couplings in Eq. (3.37) as



It is interesting to see that the most stringent constraints on models I and III arise from the rare decays of  $D^0 \rightarrow \mu \bar{l}$  ( $l = e, \mu$ ) [see Figs. 2(a) and 2(b)], respectively. To study these constraints we assume  $M_{\phi_1}^{(5/3)} \sim M_{\phi_1}^{(2/3)}$ ,  $\lambda_1^{i1} = \lambda_1^{ij}$ , and  $\lambda_3^{2i} = \lambda_3^{2j}$  for  $i, j = 1, 2, 3$  and neglect the tensor interactions. The decay rate for  $D^0 \rightarrow \mu \bar{l}$  ( $l = e, \mu$ ) can be calculated from the diagram in Fig. 2(a),

$$\Gamma(D^0 \rightarrow \mu \bar{l}) \simeq \frac{1}{256\pi} \frac{f_D^2 m_D^5}{m_c^2} \frac{|\lambda_1^{22}(\lambda_1^{i1})^*|^2}{M_{\phi_1}^4}. \quad (3.47)$$

Using  $f_D \sim 0.2$  GeV and the experimental limits [24]  $B(D^0 \rightarrow \mu \bar{\mu})_{\text{expt}} < 1.1 \times 10^{-5}$  and  $B(D^0 \rightarrow \mu \bar{e})_{\text{expt}} < 1.0 \times 10^{-4}$ , we find

$$\frac{|\lambda_1^{22}(\lambda_1^{i1})^*|}{M_{\phi_1}^2} < 1.4 \times 10^{-7} \text{ GeV}^{-2}. \quad (3.48)$$

This leads to

$$P_1^I < 7.2 \times 10^{-2} \quad (3.49)$$

in model I. Such a large polarization is already ruled out [24], but the possibility of measuring a transverse polarization in  $K^+ \rightarrow \pi^0 \mu^+ \nu$  remains until there are significant improvements in  $D^0$  decays. Similar results are obtained in model III.

#### IV. CONCLUSIONS

We have studied the transverse muon polarization in  $K^+ \rightarrow \pi^0 \mu^+ \nu$  decay in various specific extensions of the standard KM model of  $CP$  violation. We have rederived the result that there is no muon transverse polarization for models with purely effective  $V$  and  $A$  interactions such as the standard, left-right-symmetric, and horizontal-symmetry models. We have also shown that such a result holds even with nonzero neutrino masses. The charged-Higgs-boson exchange effect on  $P_\perp$  in multi-Higgs-boson models has been reexamined. In particular, we estimated that in the Weinberg three-Higgs-doublet model  $P_\perp \sim 2.6 \times 10^{-4} v_2^2 / v_3^2$ , which yields an upper bound around  $8.4 \times 10^{-3}$ . For the models in which the KM matrix has a nonzero phase, we find that the transverse muon polarization can easily reach the  $10^{-3}$  level without conflicting with experimental constraints. We have considered all possible leptoquark models (I–IV) that give contributions to  $K_{\mu 3}$  decays through the

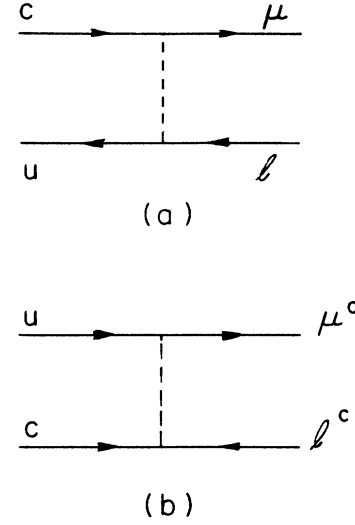


FIG. 2. Contributions to  $D^0 \rightarrow \mu \bar{l}$  decays where  $l = e$  and  $\mu$  (a) in model I and (b) in model III.

scalar leptoquark exchanges at tree level and we find that models I and III could lead to a large  $P_\perp$ , there are basically no direct experimental constraints on these models except for the polarization measurement itself. The transverse polarization was shown to be zero in models II and IV.

In conclusion, the measurement of the transverse muon polarization in  $K^+ \rightarrow \pi^0 \mu^+ \nu$  is a clean signature of  $CP$  violation beyond the standard model. This  $T$ -odd muon polarization in the multi-Higgs-boson and leptoquark models could reach a level of  $10^{-3}$  which is accessible to future experiments at KEK [38] or a kaon factory [5].

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