

シリコン飛跡検出器における Neural Networkによる飛跡再構成

東京大学 M1

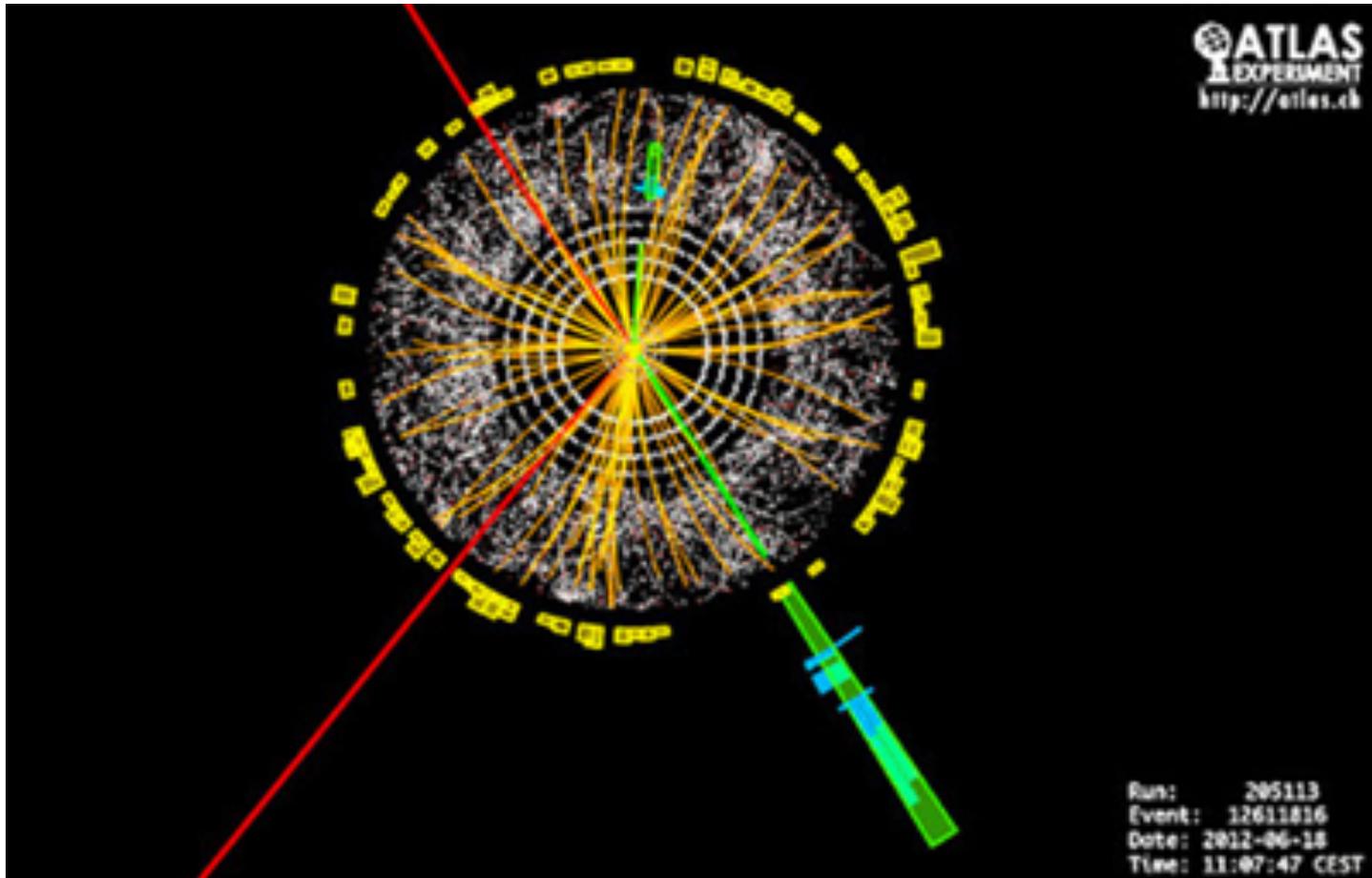
恩田理奈

2017/5/18

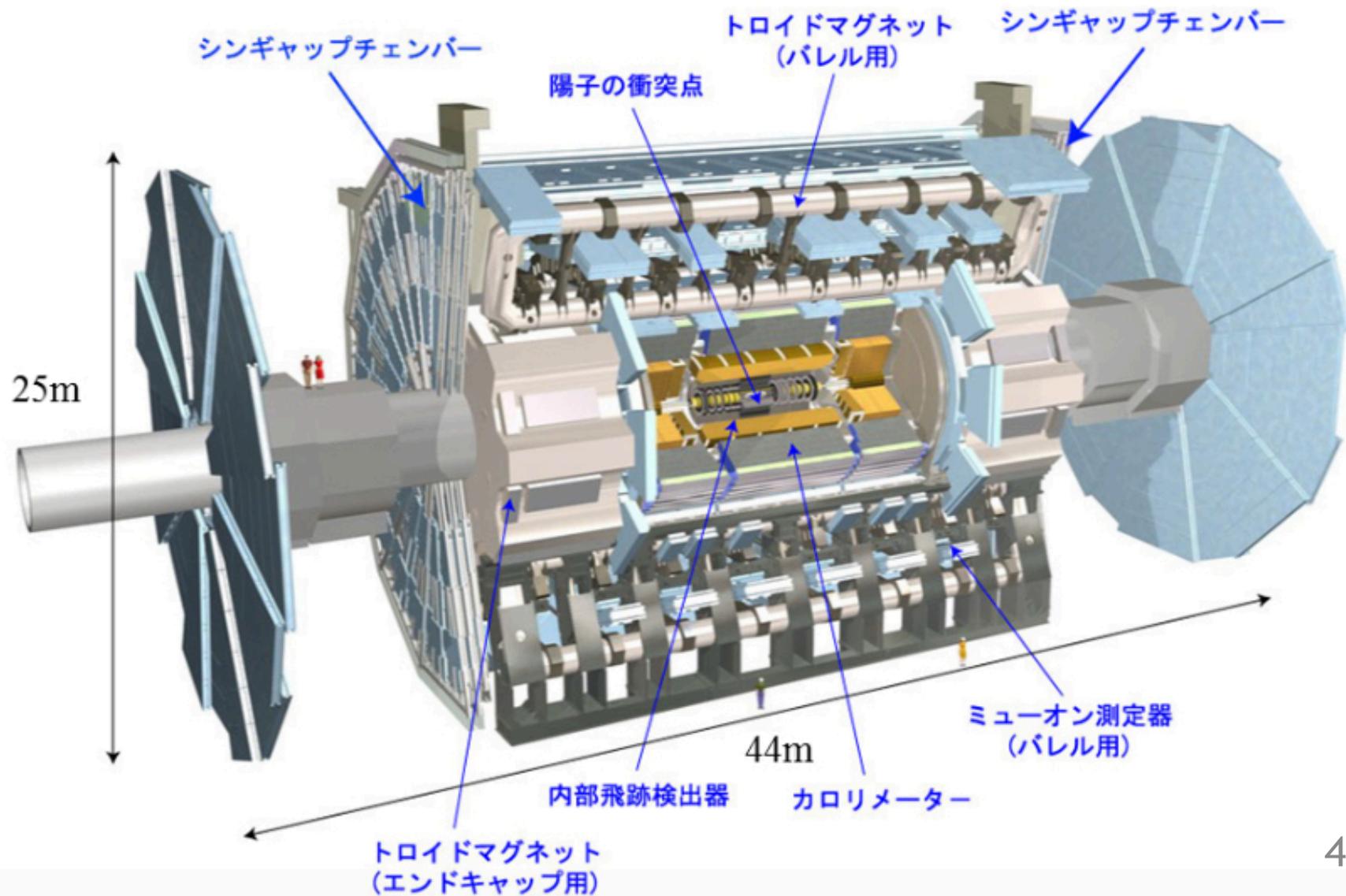
Outline

1. 背景
2. 目的
3. 結果
4. 結論

素粒子実験



検出器



運動量の計算

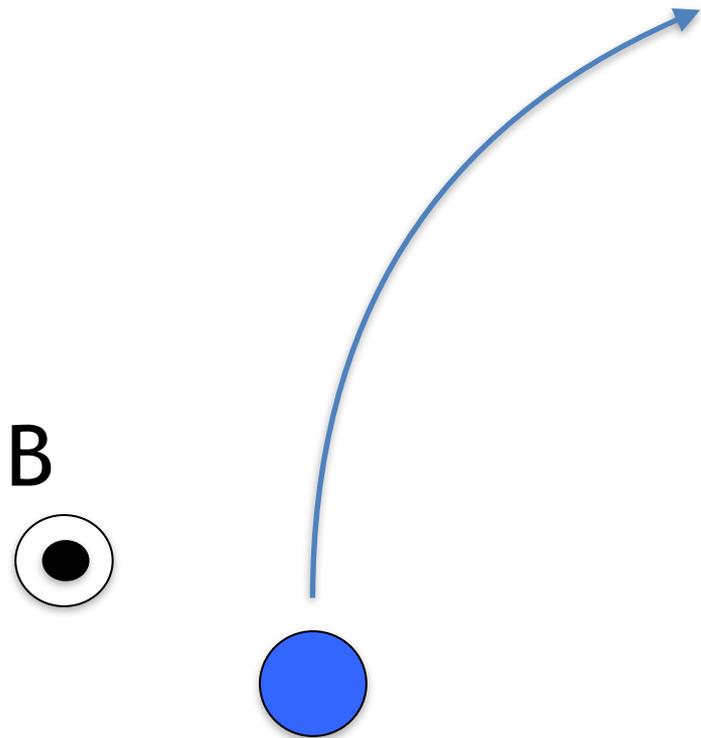


運動量の計算

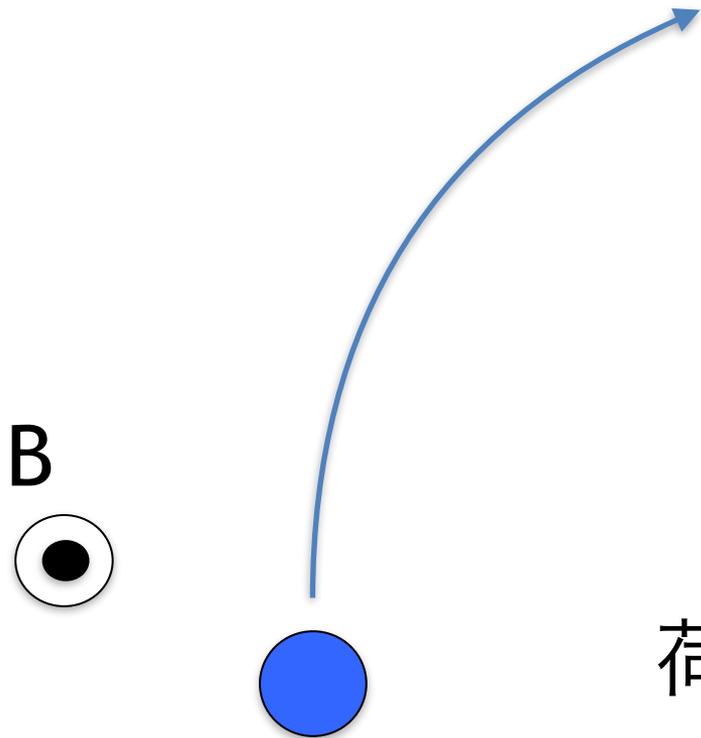
B



運動量の計算

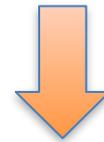


運動量の計算



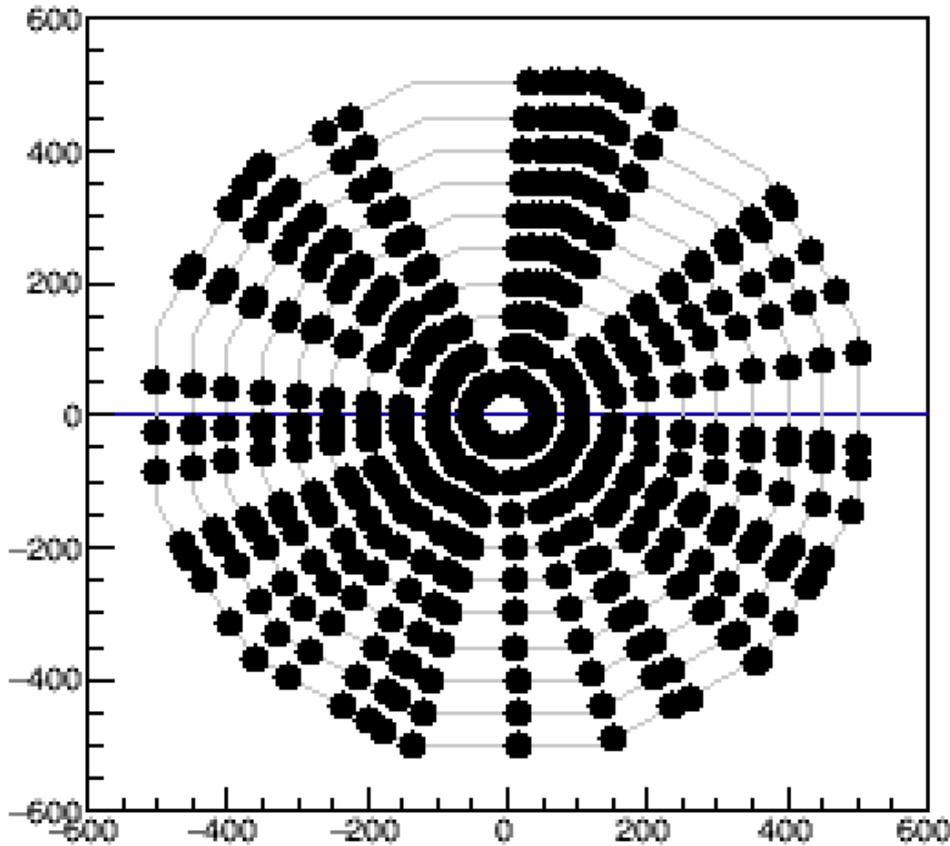
$$m \frac{v^2}{r} = qvB$$

$$\Leftrightarrow p = mv = qBr$$



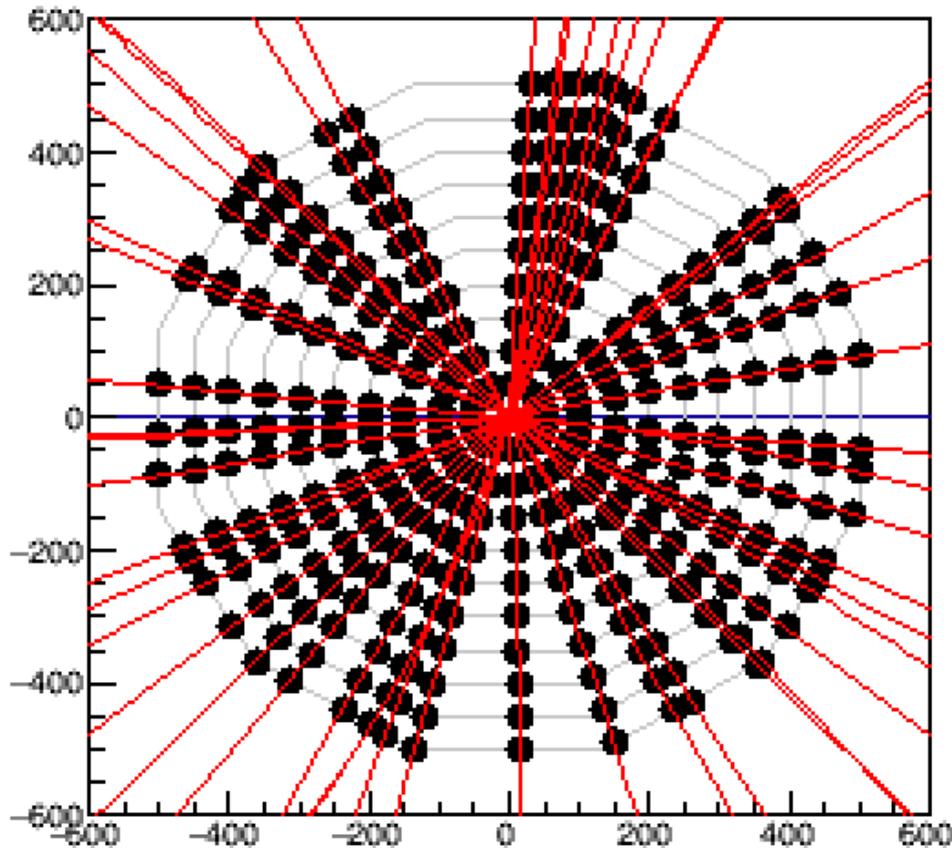
荷電粒子の円軌道の半径より
運動量を求めることができる

飛跡再構成



選んだhitの組み合わせを円軌道でfitし、適切なものを選ぶ

飛跡再構成



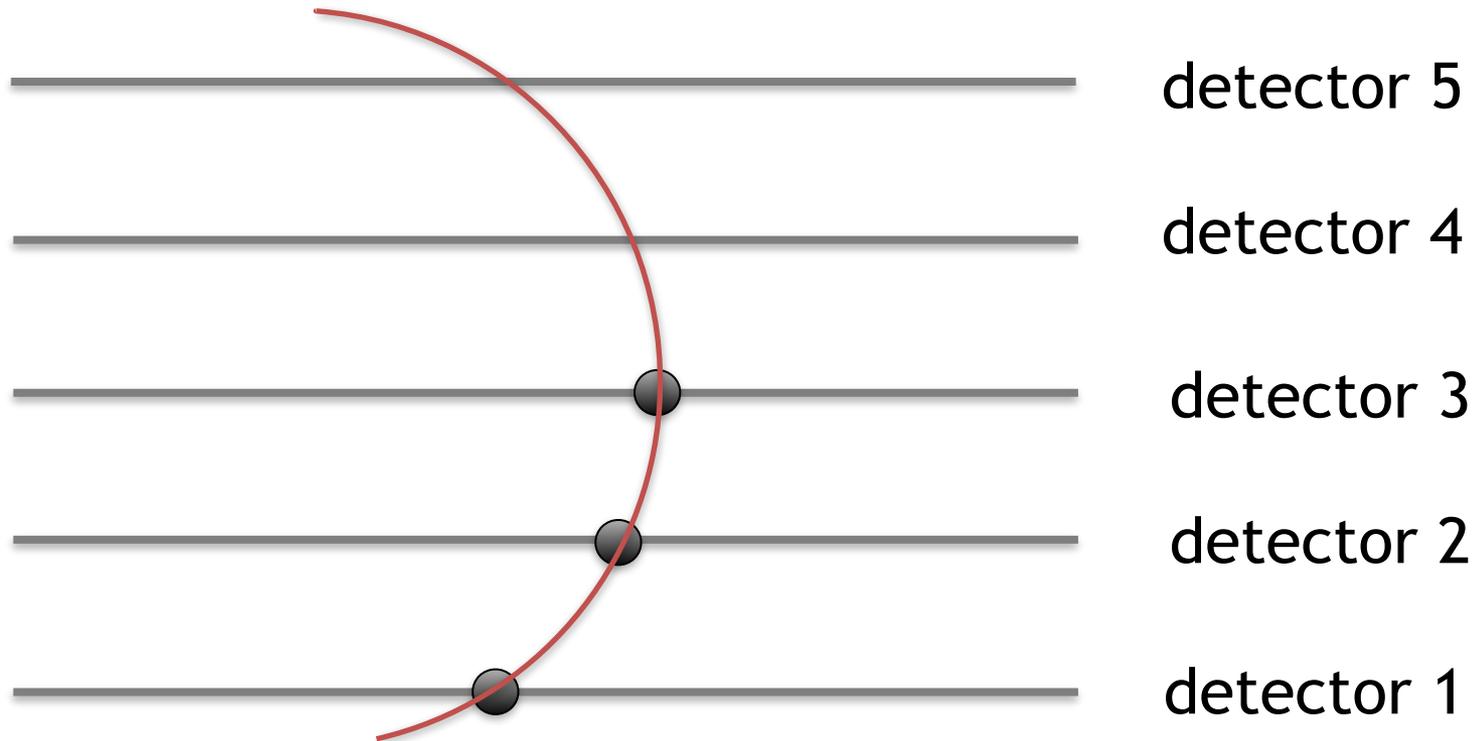
選んだhitの組み合わせを円軌道でfitし、適切なものを選ぶ

Kalman Filter



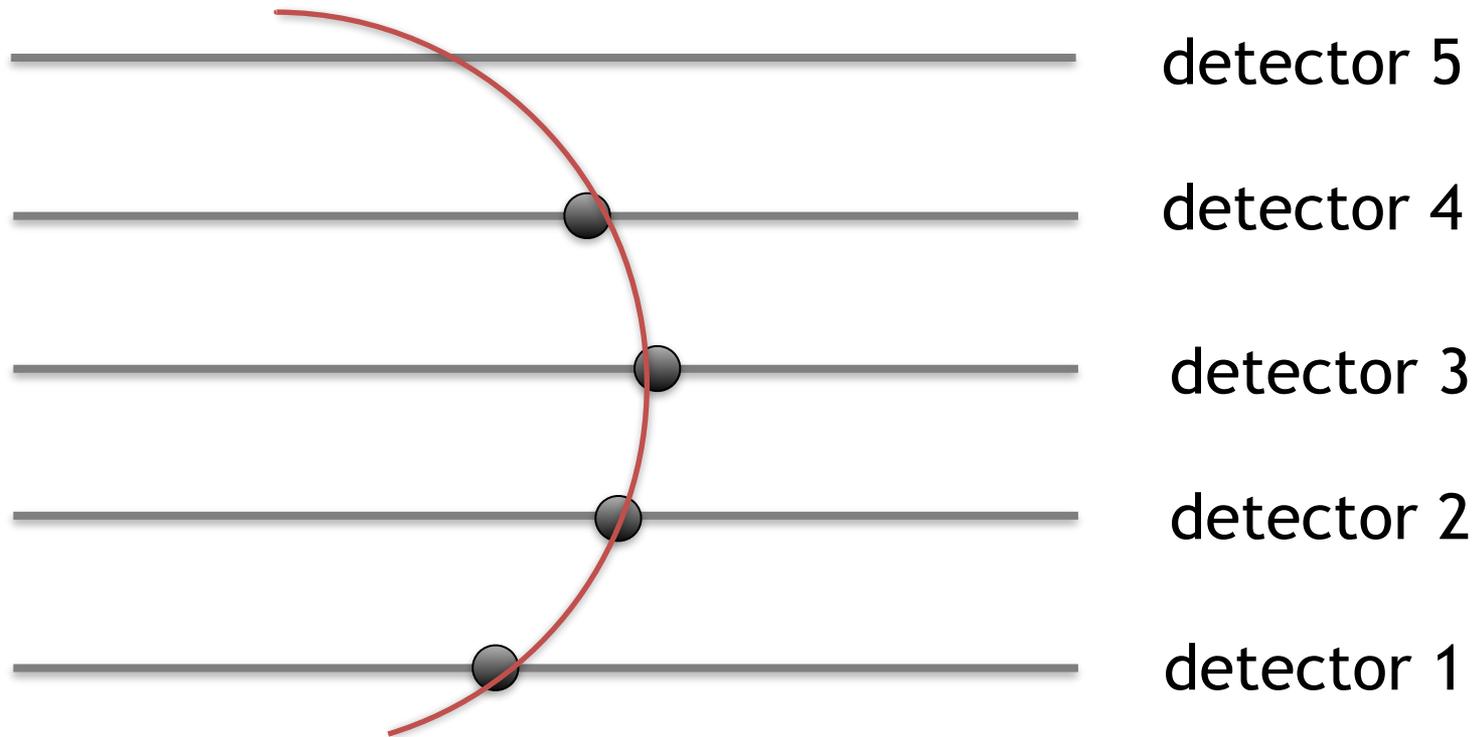
detectorのhitを加えてparameterを補正していく

Kalman Filter



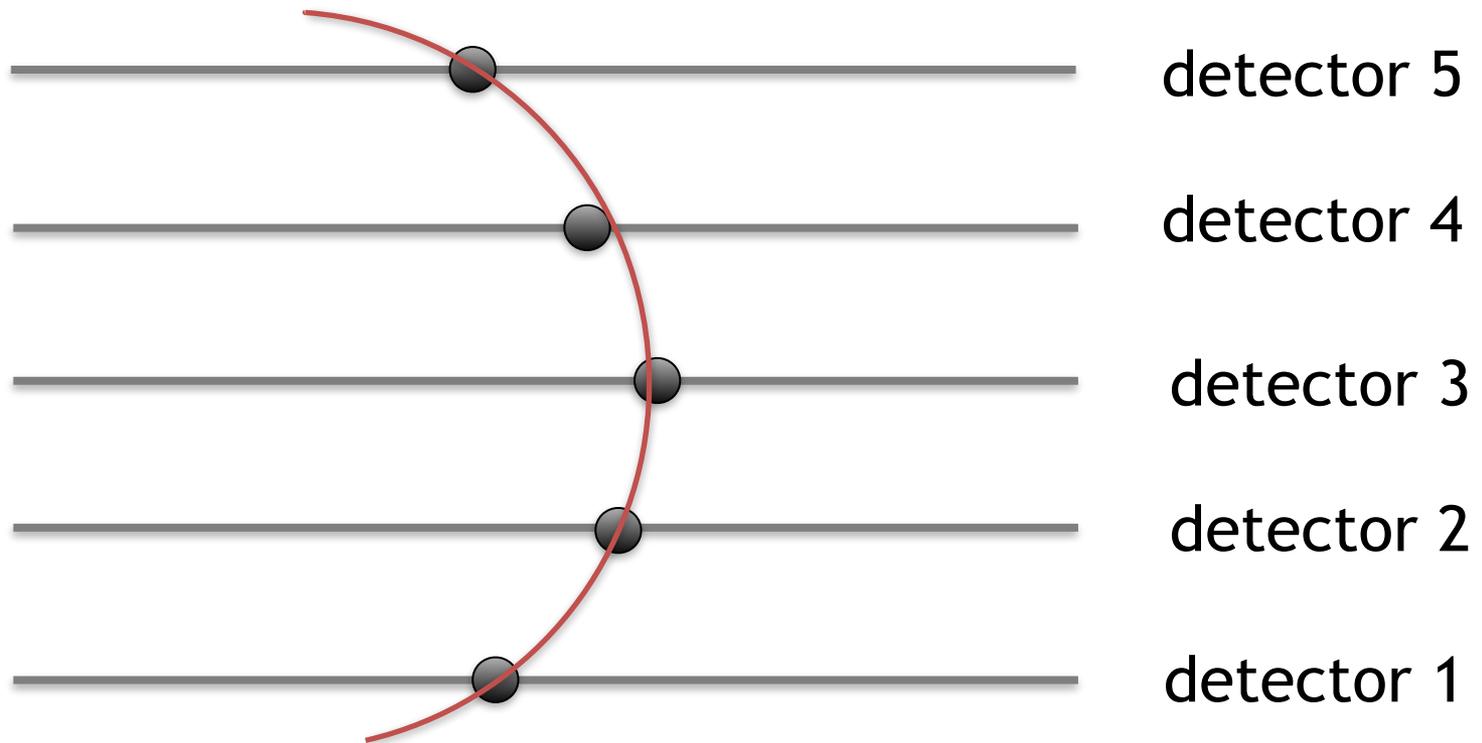
detectorのhitを加えてparameterを補正していく

Kalman Filter



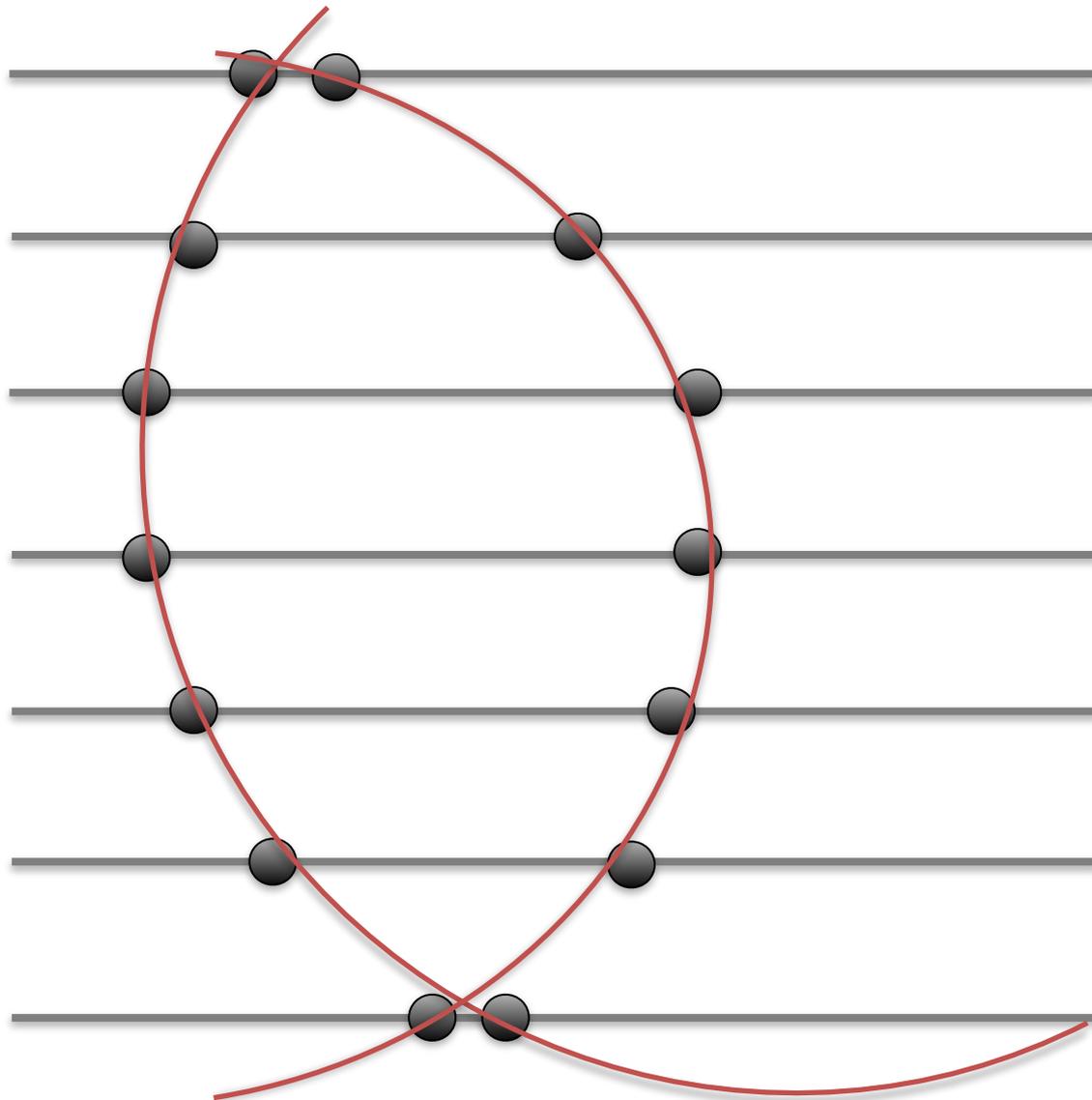
detectorのhitを加えてparameterを補正していく

Kalman Filter

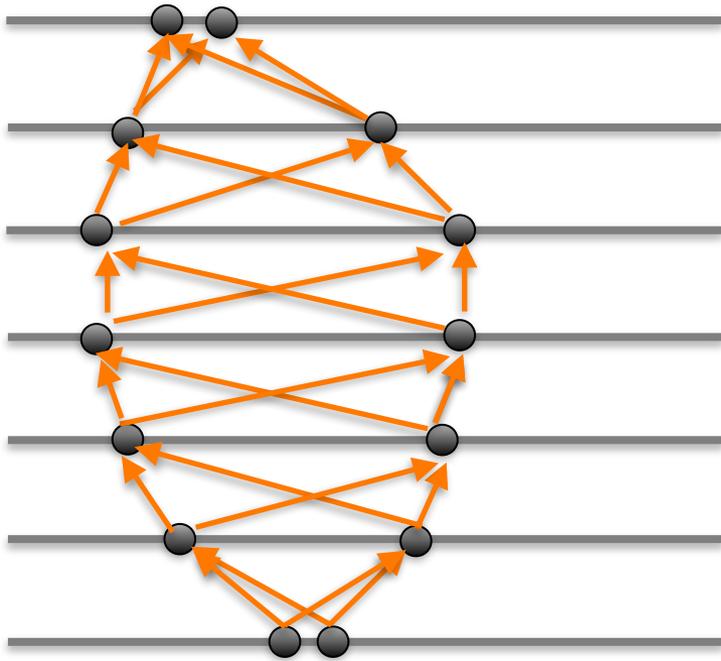


detectorのhitを加えてparameterを補正していく

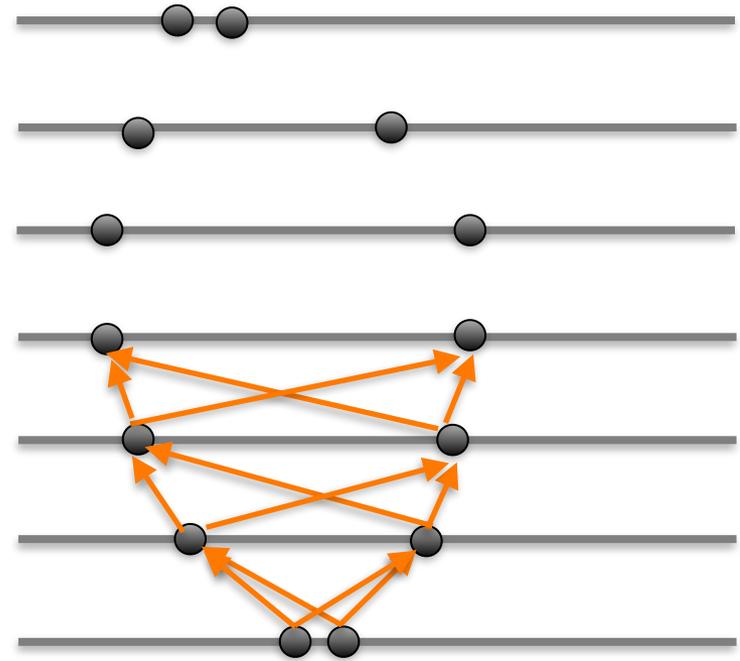
Least Squares vs Kalman Filter



Least Squares vs Kalman Filter

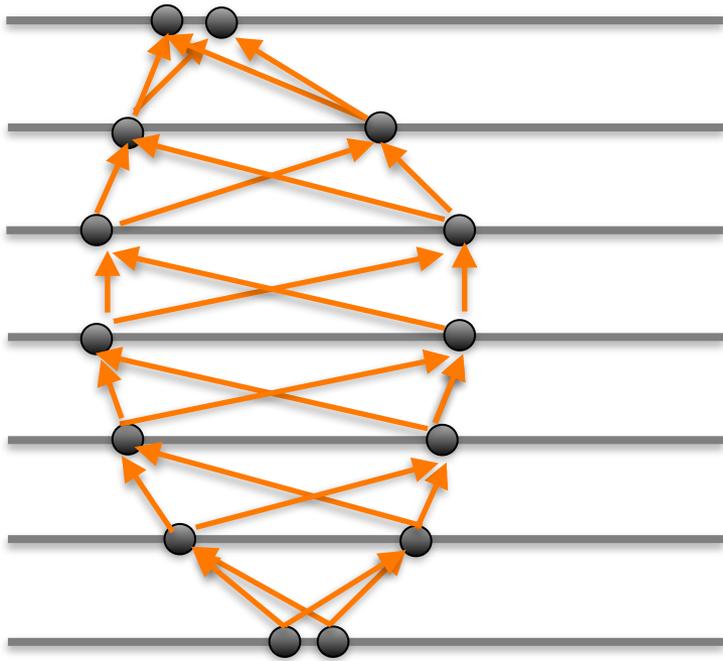


$$2^7 = 128$$

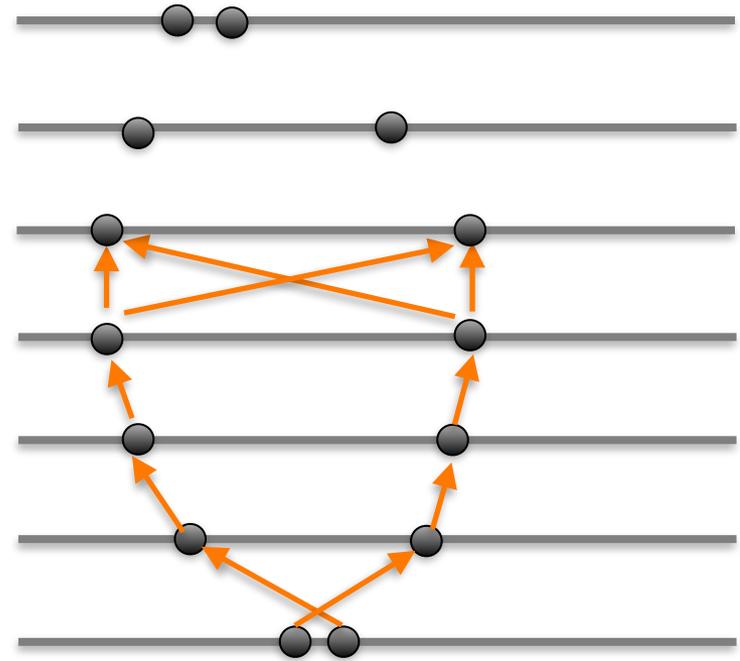


$$2^4 = 16$$

Least Squares vs Kalman Filter

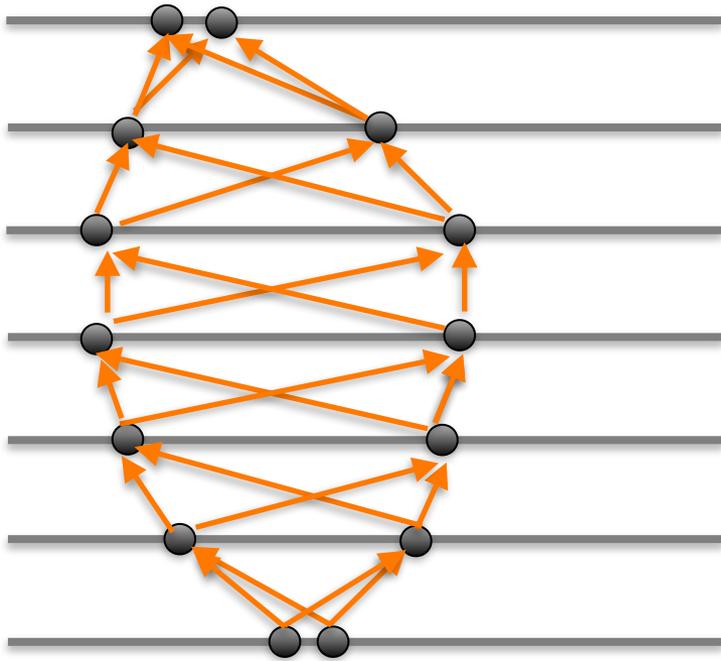


$$2^7 = 128$$

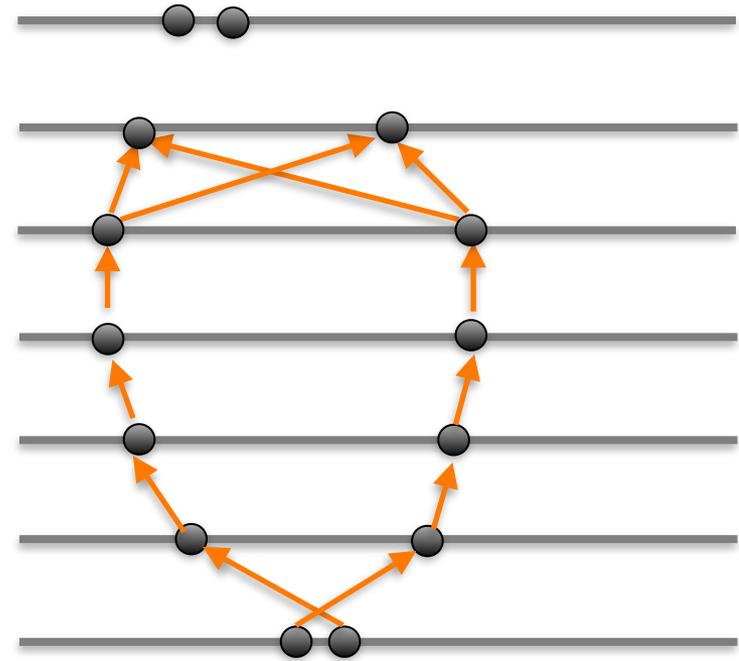


$$2^4 + 2 \times 2 = 20$$

Least Squares vs Kalman Filter

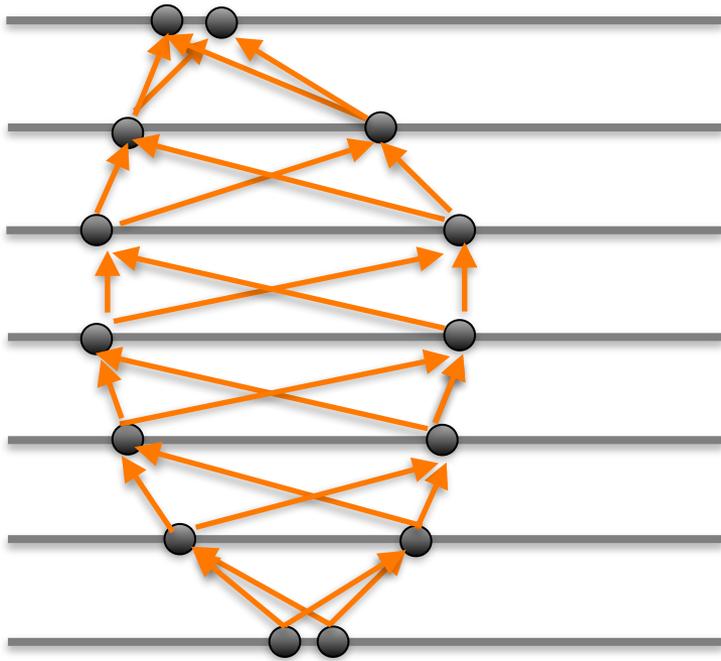


$$2^7 = 128$$



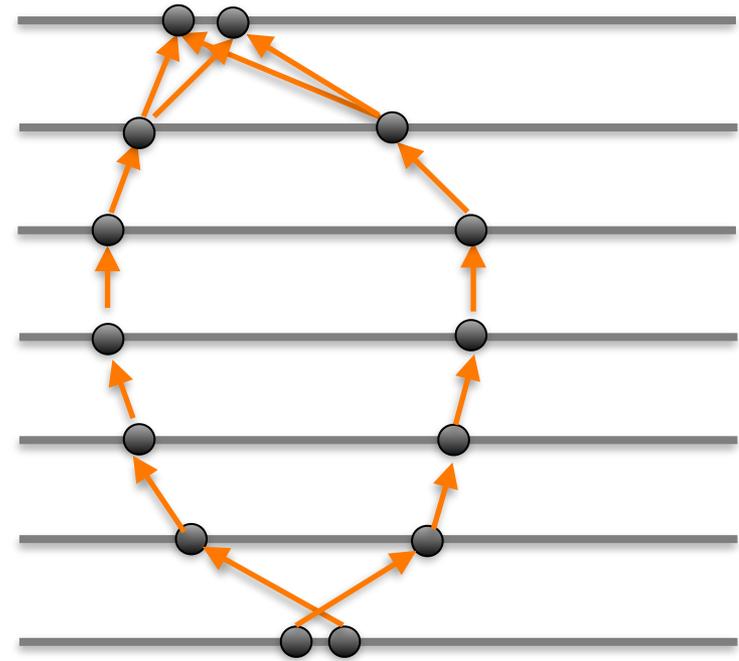
$$2^4 + 2 \times 2 + 2 \times 2 = 24$$

Least Squares vs Kalman Filter



$$2^7 = 128$$

$\sim (\#track)^{\#detector}$



$$2^4 + 2 \times 2 + 2 \times 2 + 2 \times 2 = 28$$

$\sim (\#track)^4$

計算が速い！

目的

Neural Networkを用いて、飛跡再構成をより効率よく、短時間で行う

Neural Network

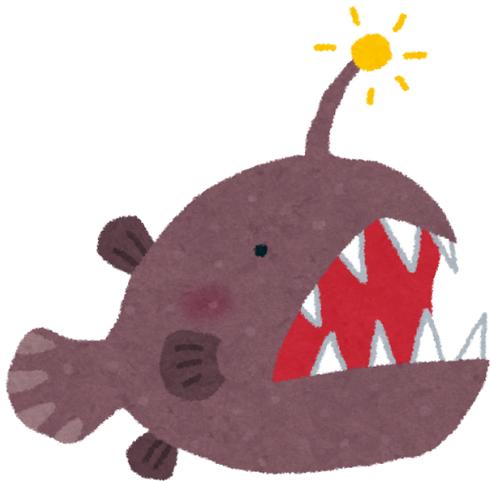
脳機能に見られるいくつかの特性を計算機上のシミュレーションによって表現することを目指した数学モデル

利用例： 画像認識、音声認識

Neural Network

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利用例： 画像認識、音声認識

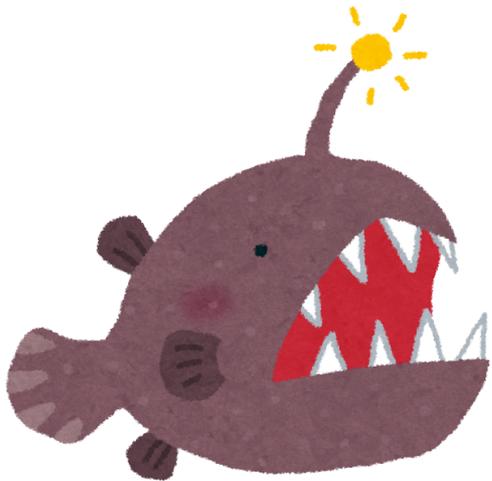


Neural Network

脳機能に見られるいくつかの特性を計算機で模倣し、
レーシジョンによって表現することを目的とする。

利用例： 画像認識、音声認識

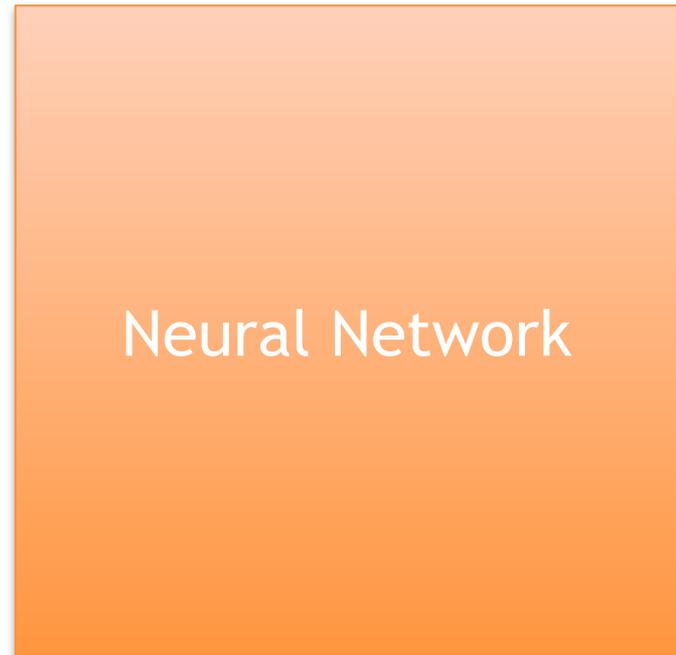
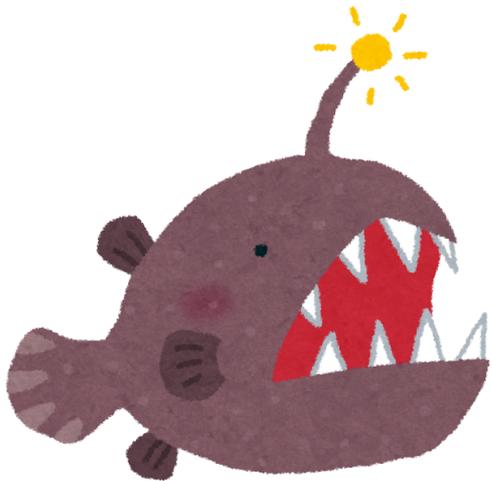
チョウチンアンコウ



Neural Network

脳機能に見られるいくつかの特性を計算機上のシミュレーションによって表現することを目指した数学モデル

利用例： 画像認識、音声認識

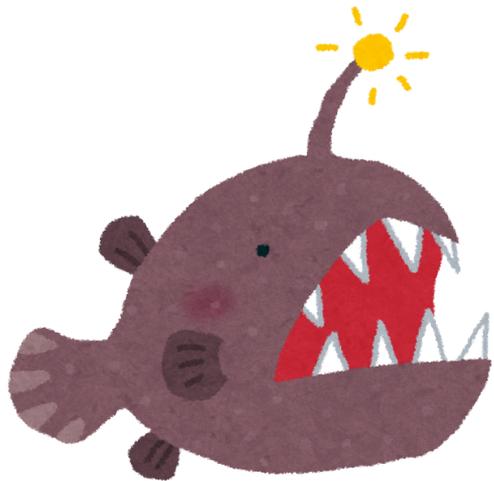


Neural Network

脳機能に見られるいくつかの特性を計算機で模倣し、
レーシジョンによって表現することを目的とする。
モデル

利用例： 画像認識、音声認識

チョウチンアンコウ



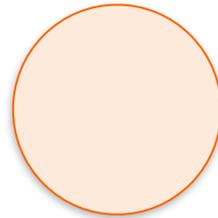
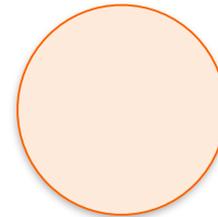
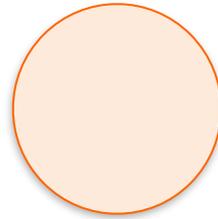
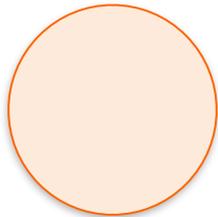
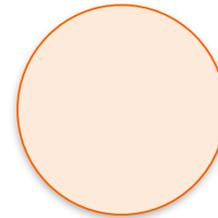
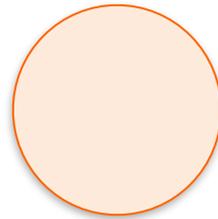
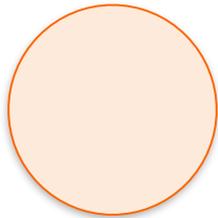
Neural Network

Neural Network

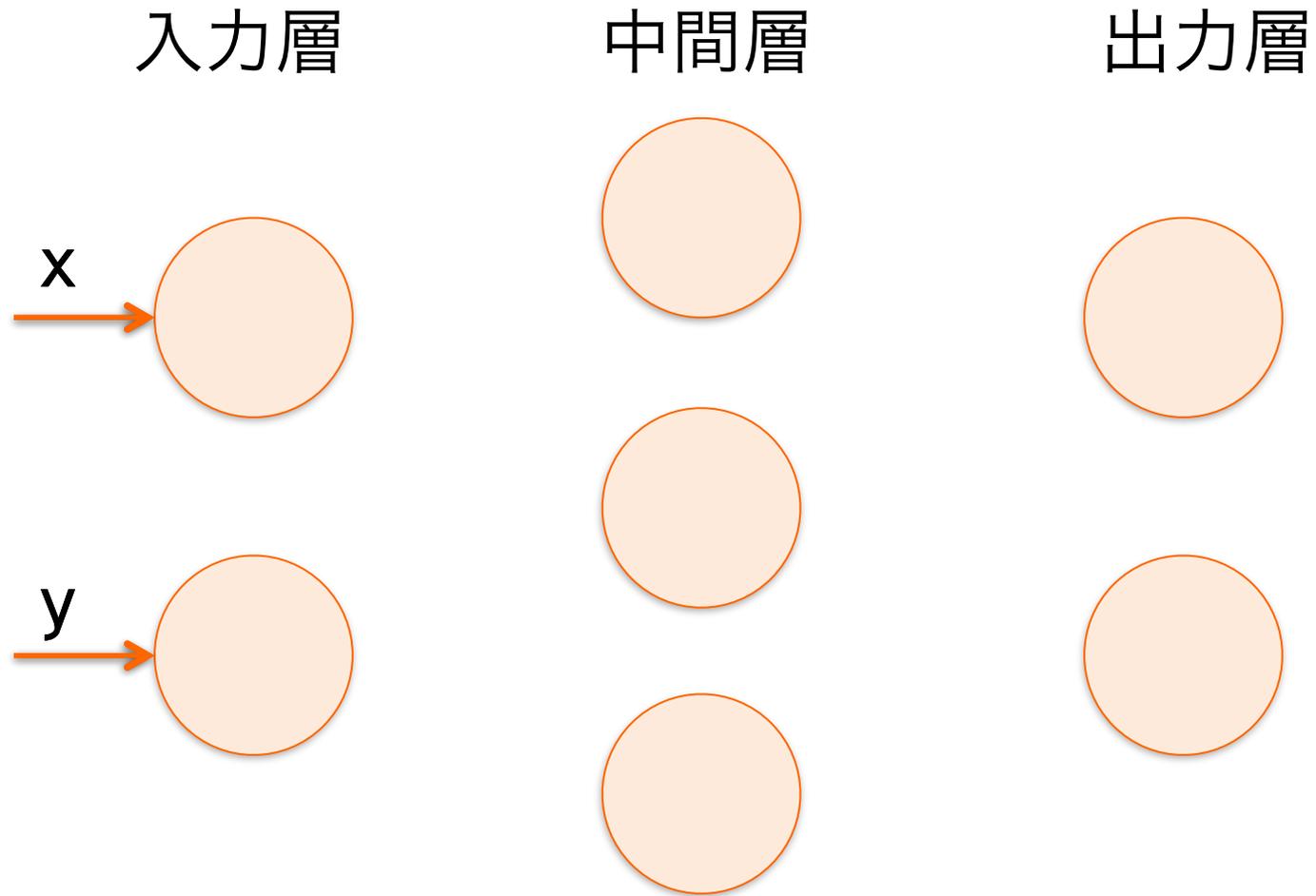
入力層

中間層

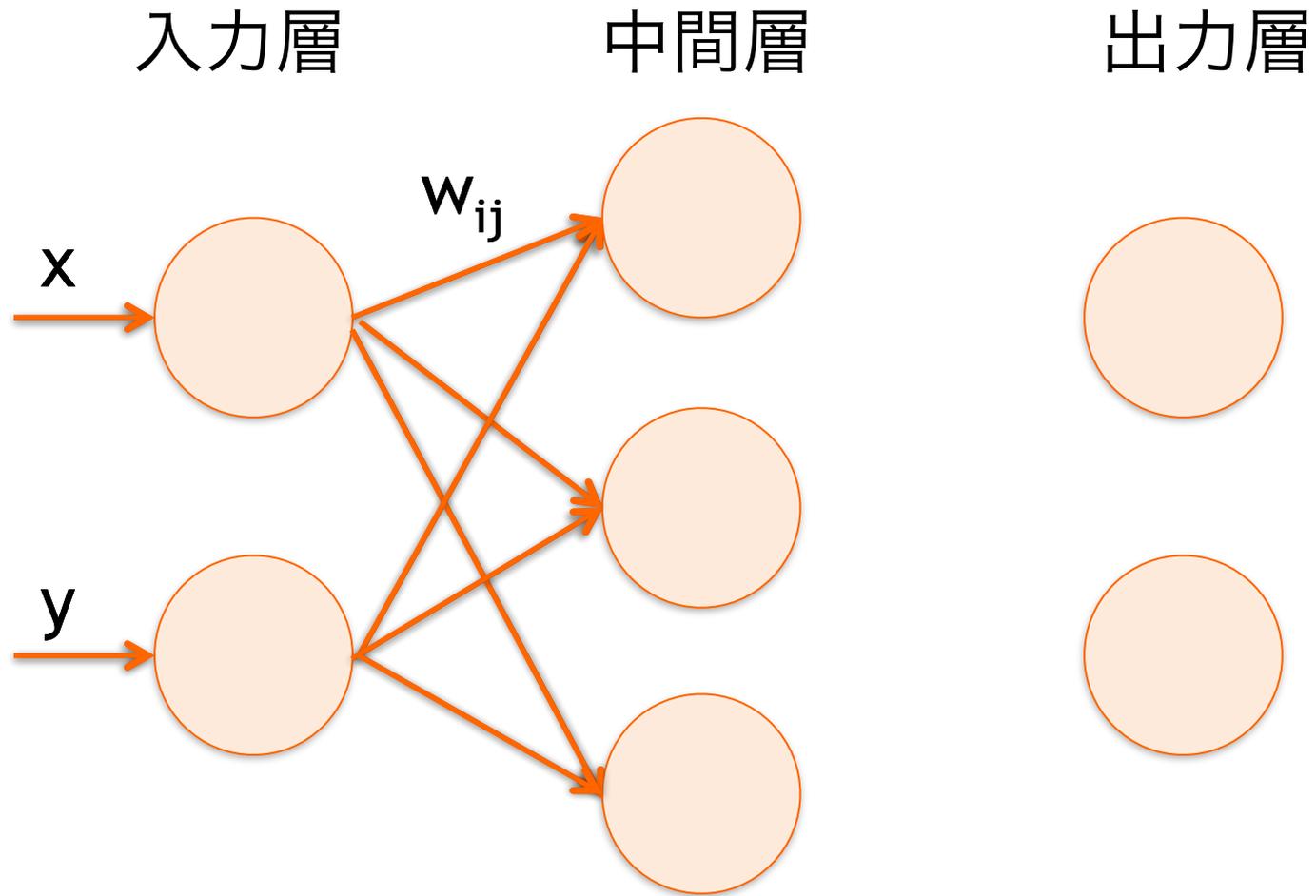
出力層



Neural Network

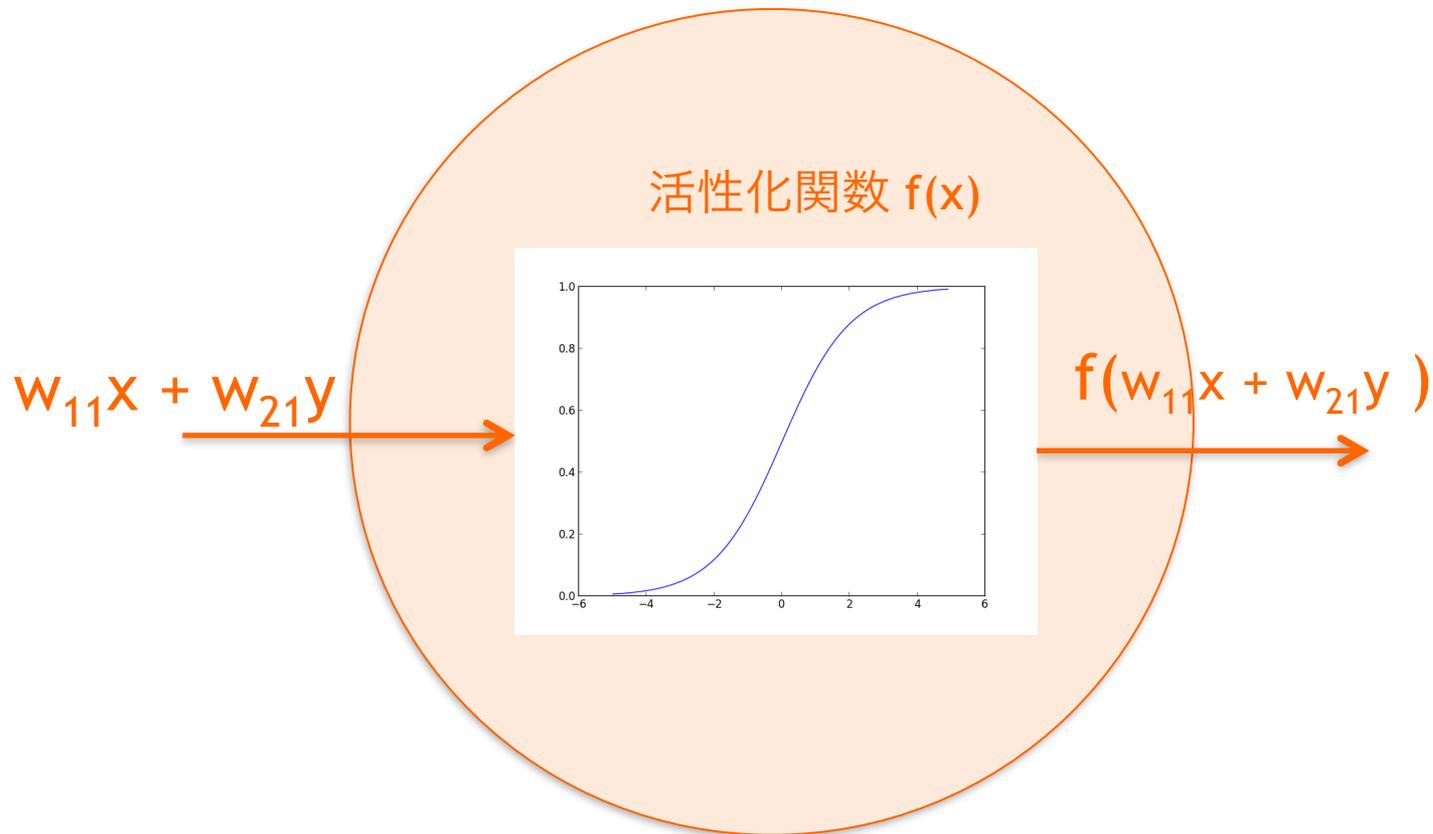


Neural Network

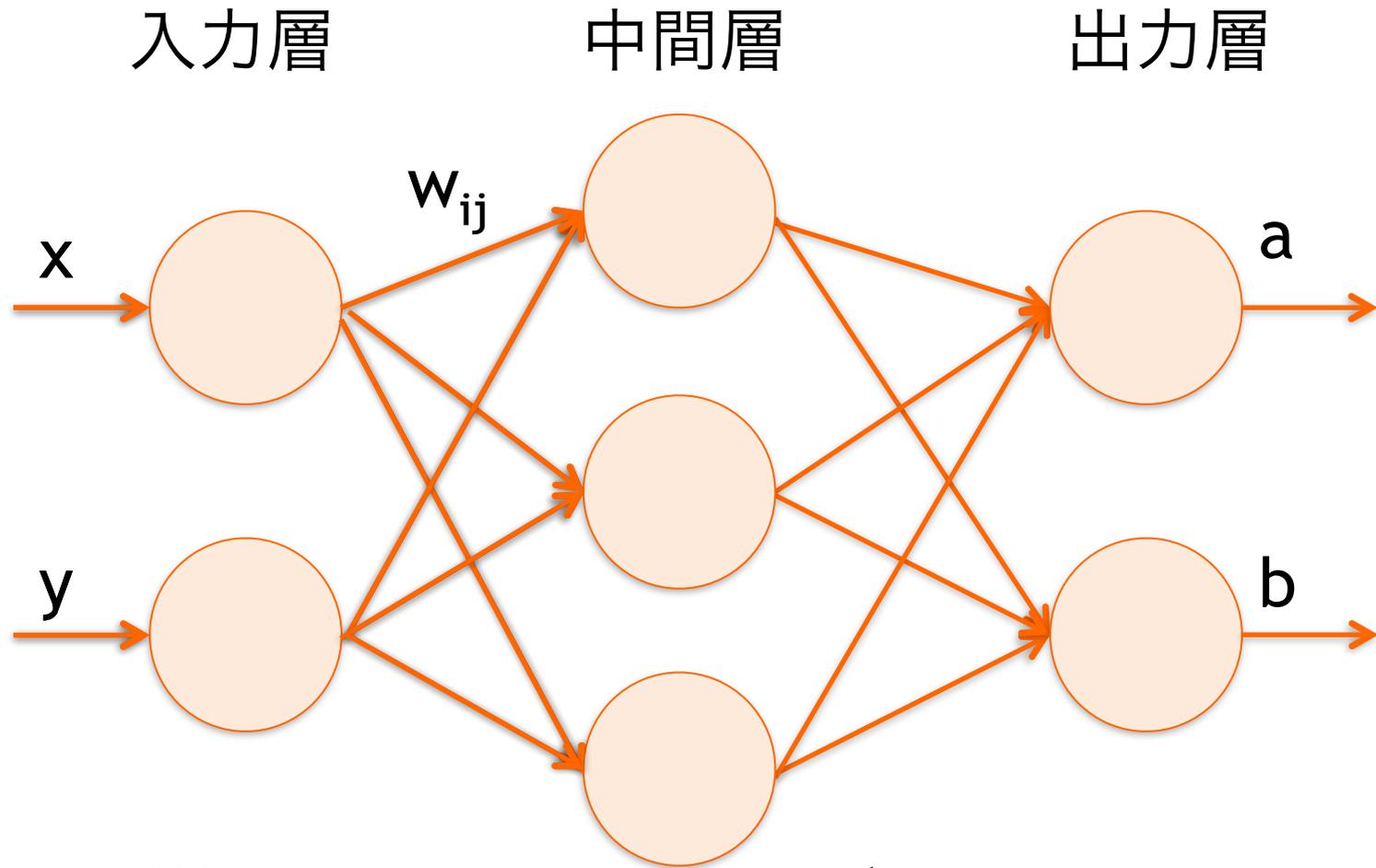


Neural Network

中間層

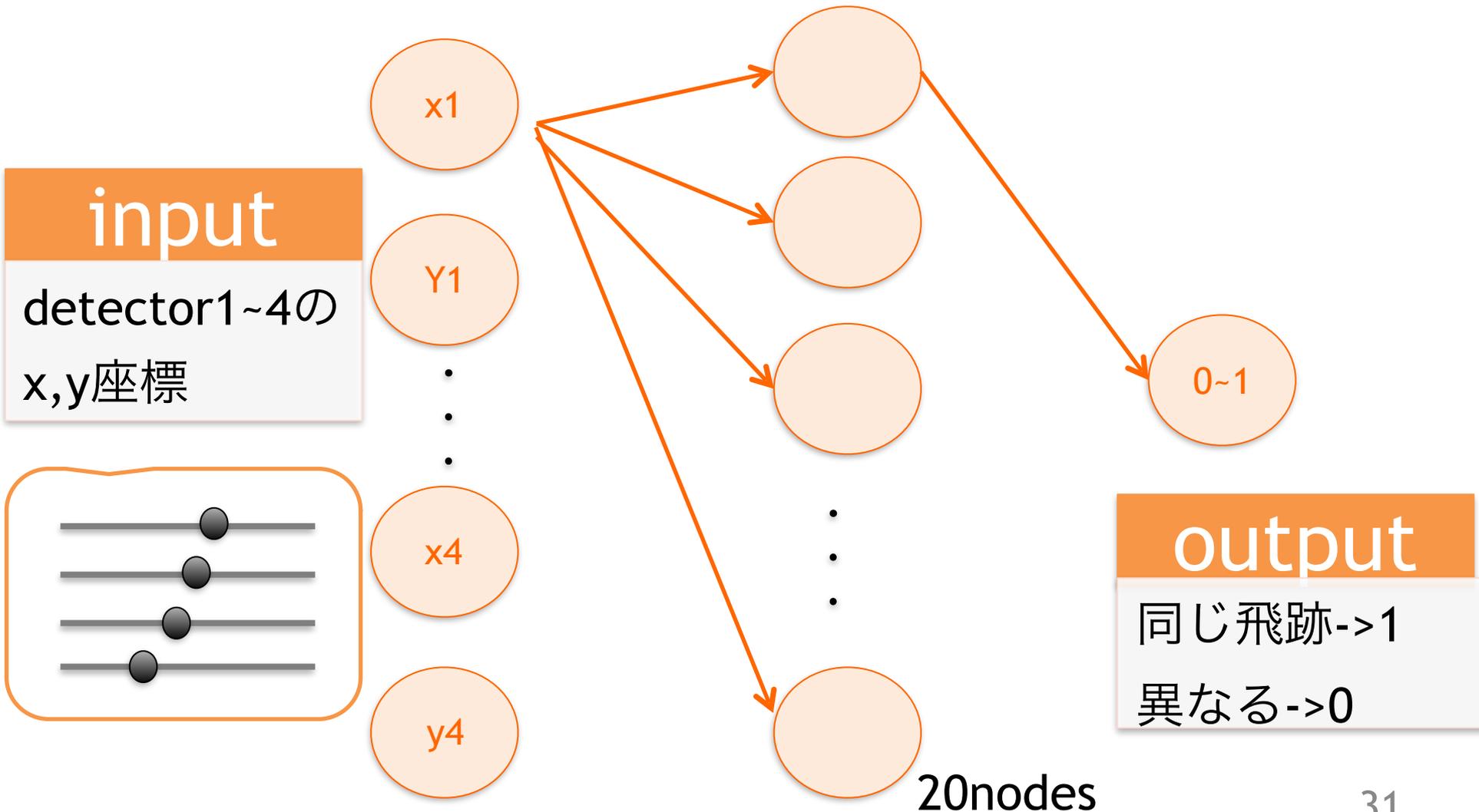


Neural Network

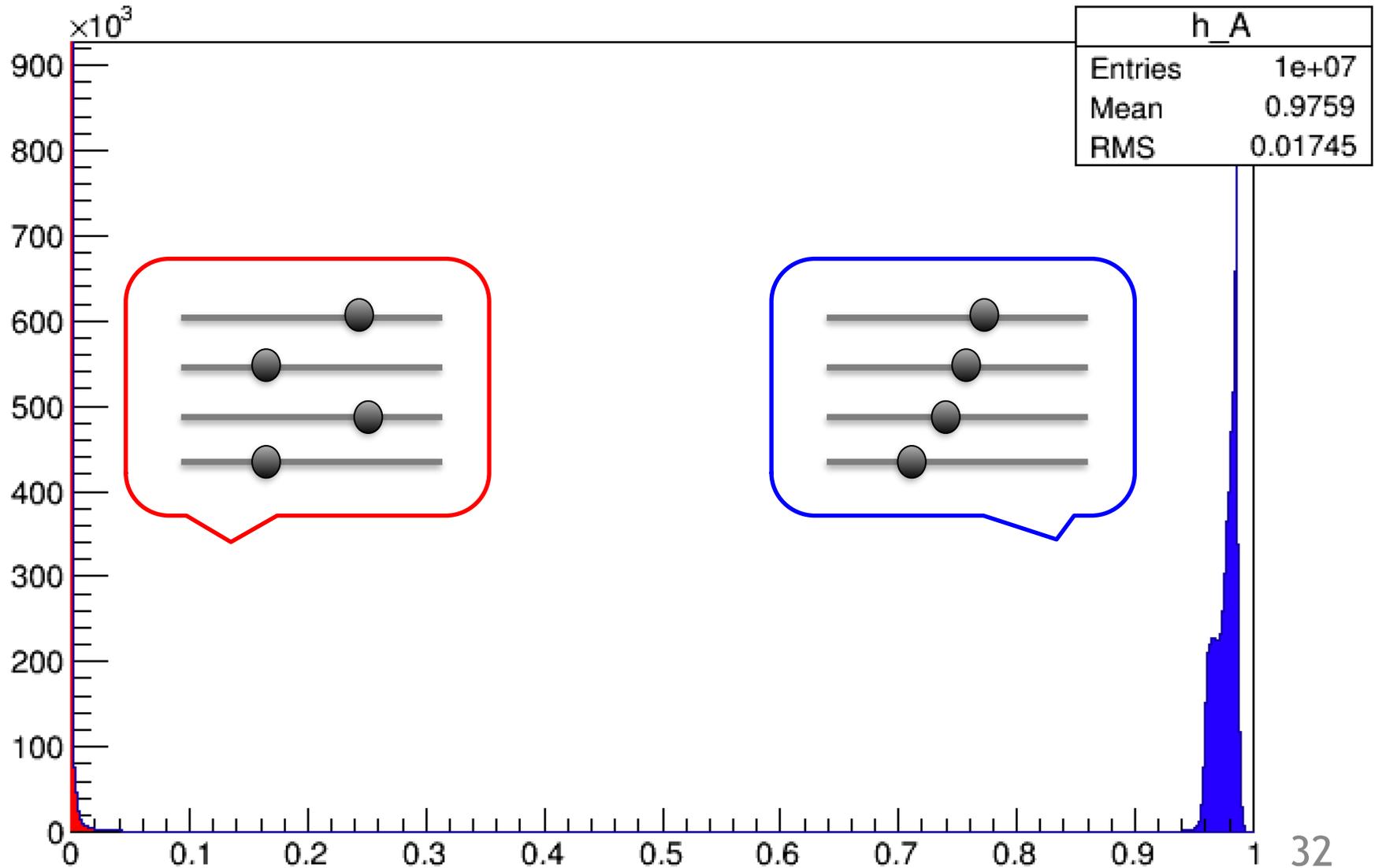


訓練により適切な出力が得られるように重みを計算する

4 Hits Net



Training Result

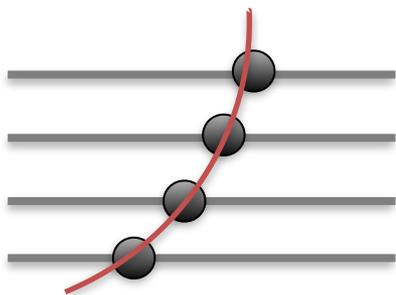


Hit on Track Net

input

detector1~3の
3点を通る飛跡
のparameter

detector4の
x,y座標



$1/R$

φ

x_4

y_4

⋮

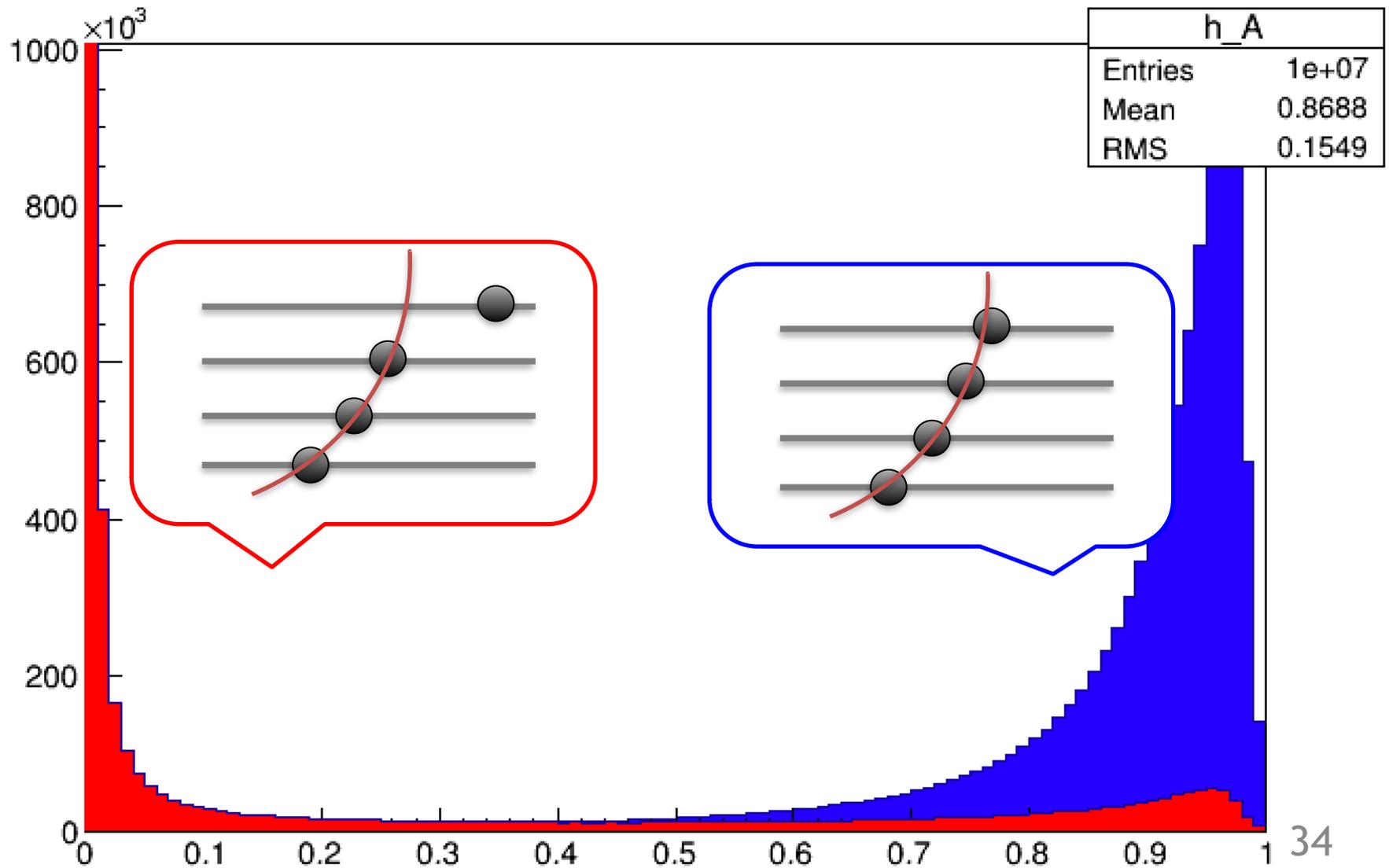
30nodes

0~1

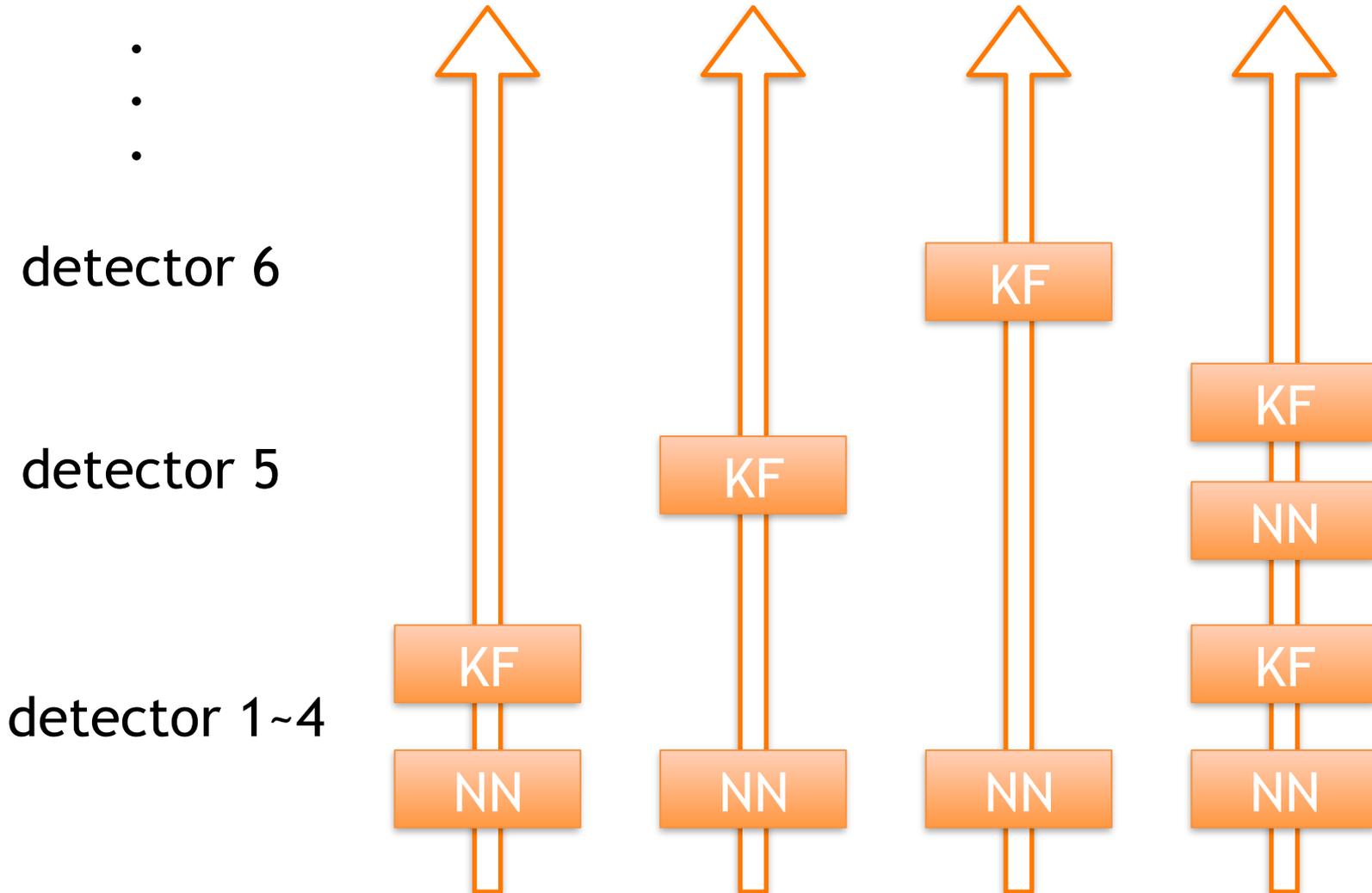
output

同じ飛跡->1
異なる->0

Training Result

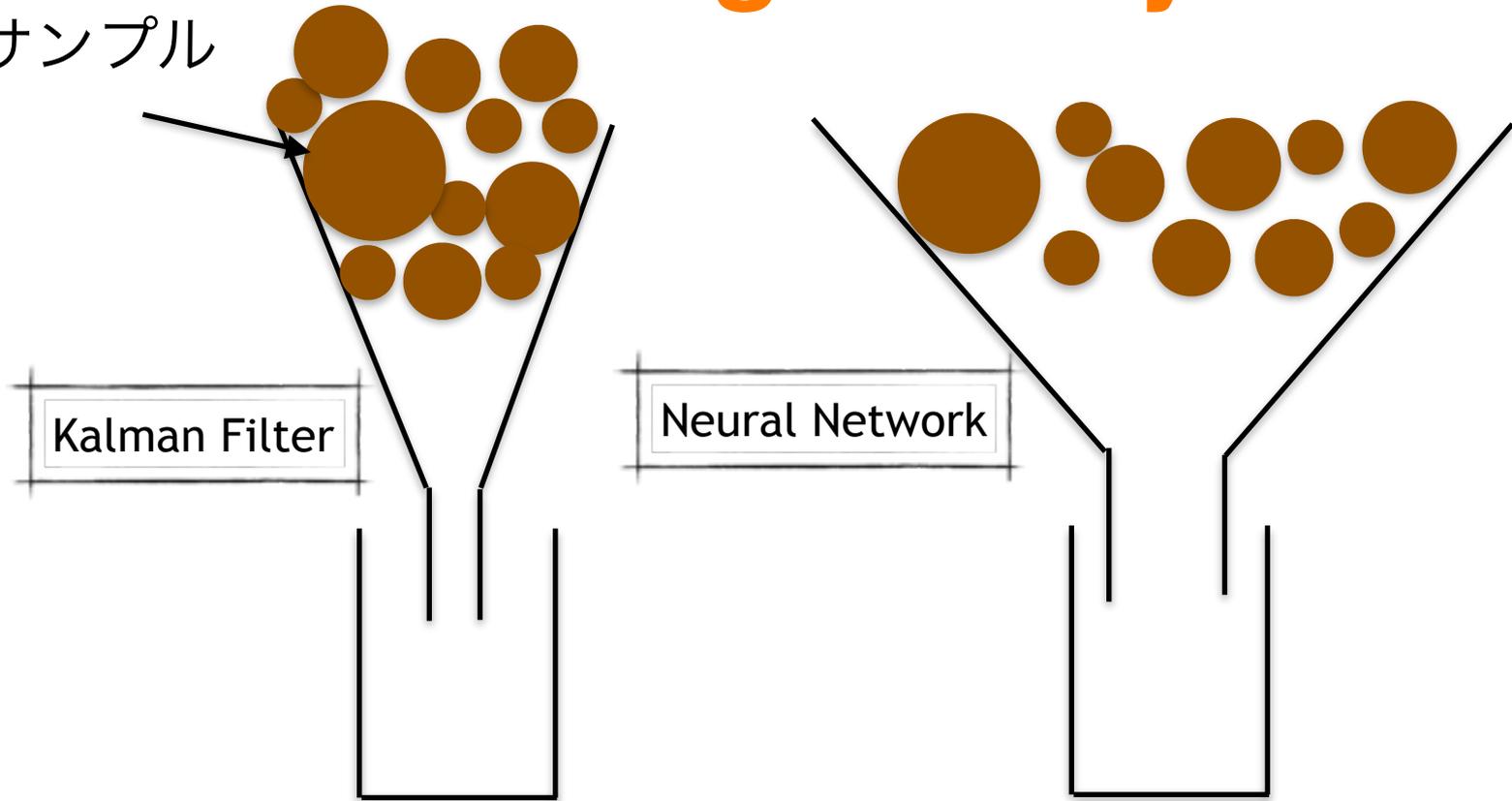


How to Use the Network?



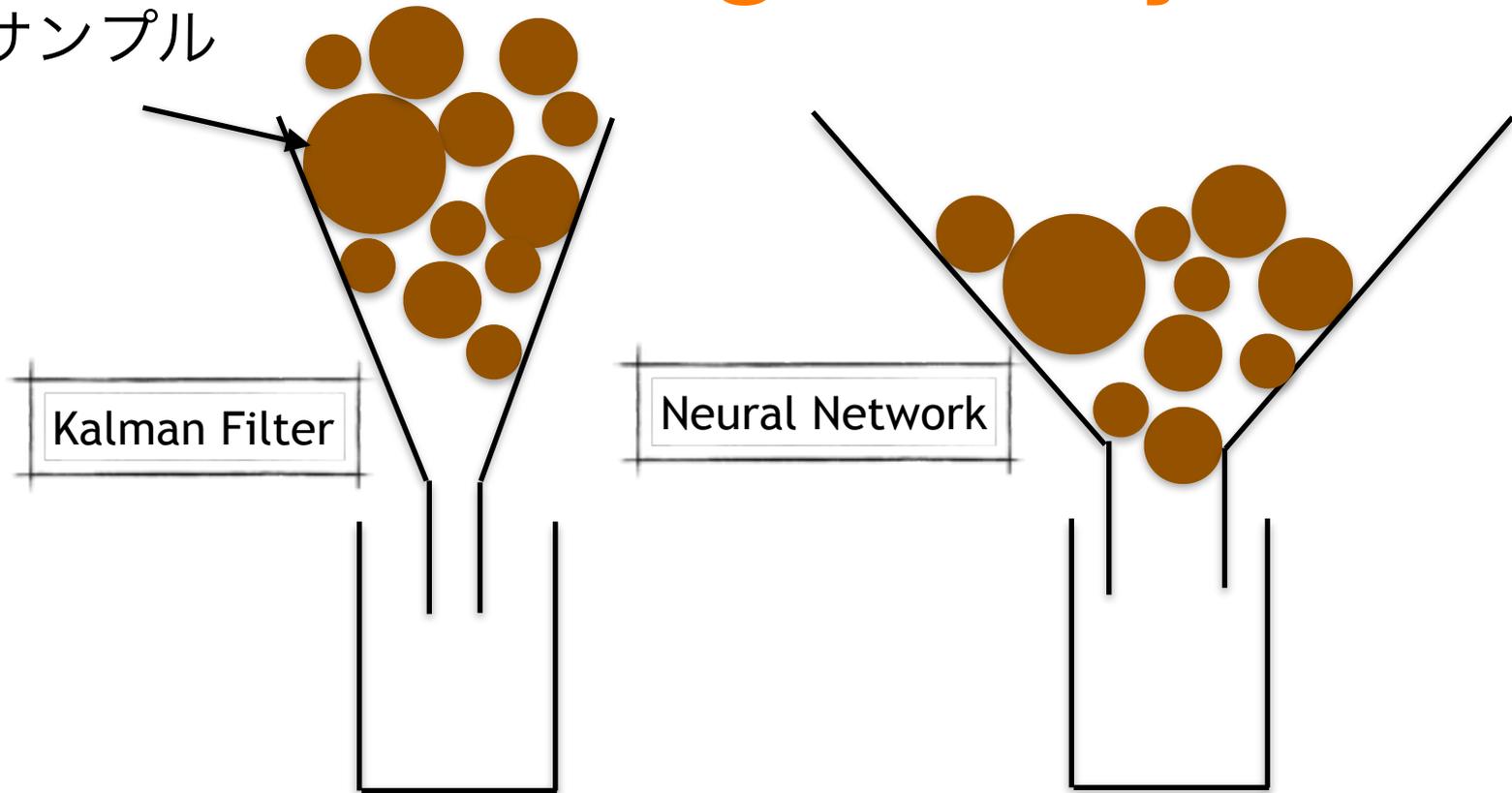
Filtering Quality

計算サンプル



Filtering Quality

計算サンプル

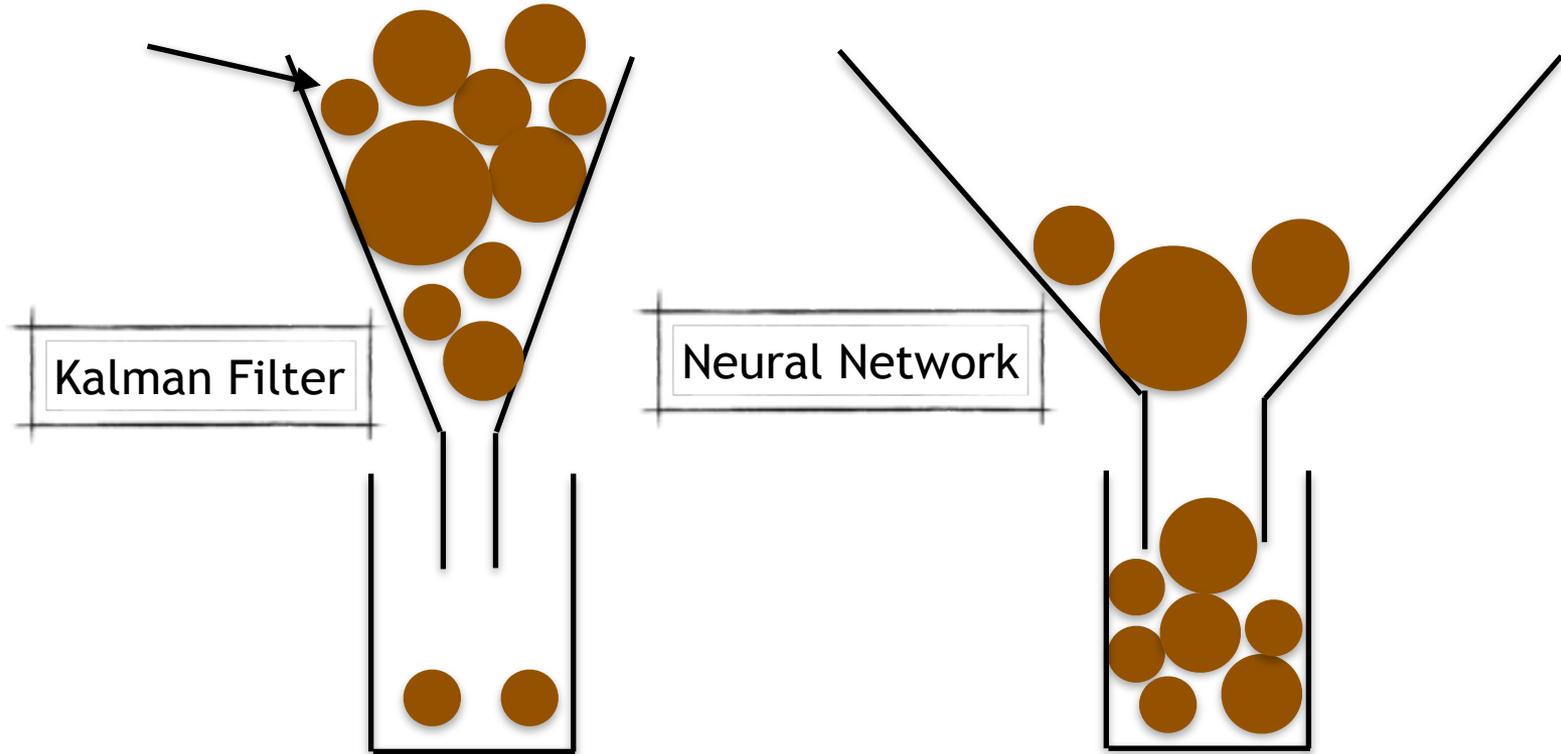


速さ

Kalman Filter < Neural Network

Filtering Quality

計算サンプル



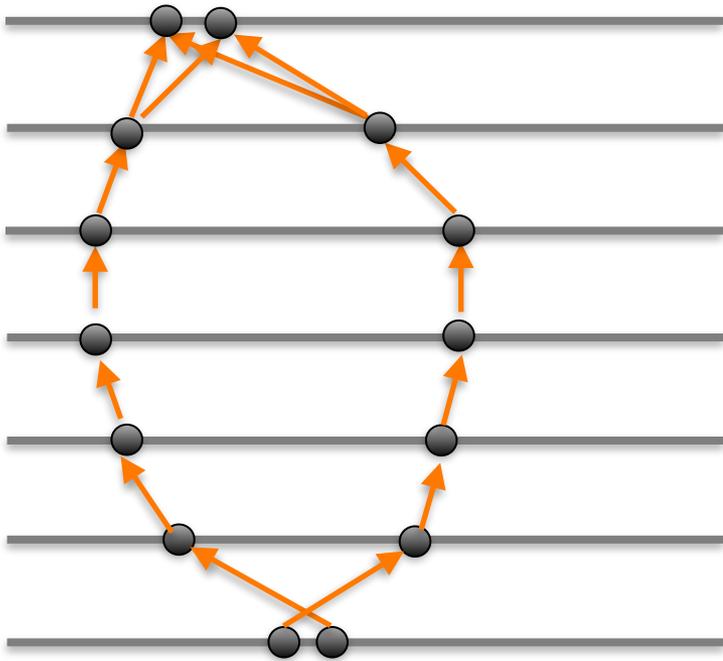
速さ

Kalman Filter < Neural Network

精度

Kalman Filter > Neural Network

Kalman Filter

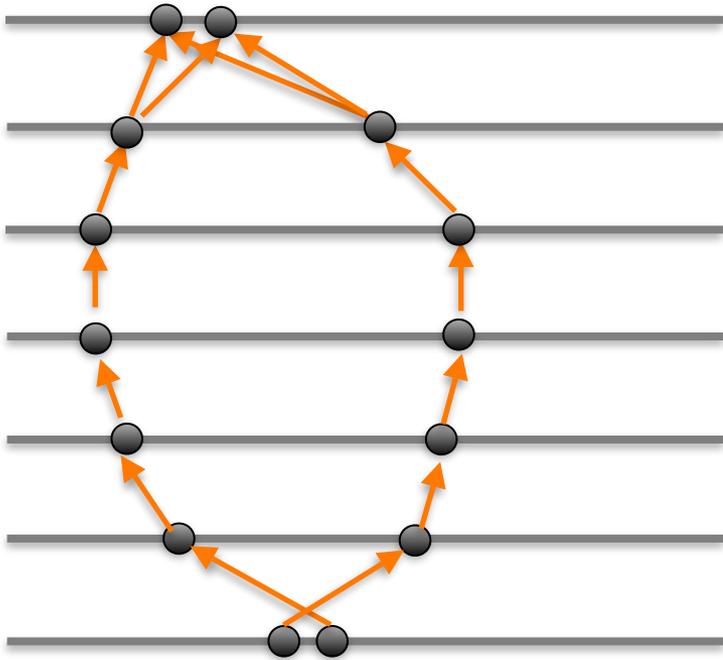


$$2^4 + 2 \times 2 + 2 \times 2 + 2 \times 2 = 28$$



元々の#track

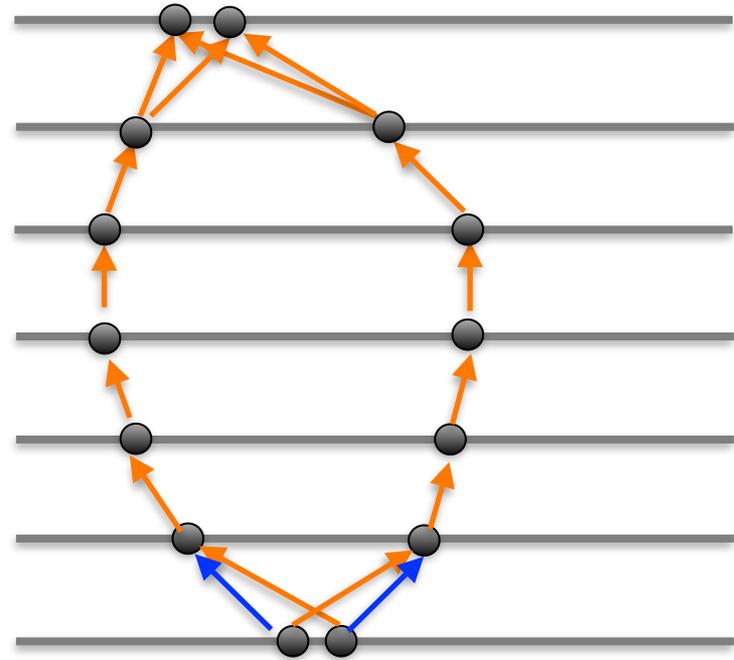
Kalman Filter



$$2^4 + 2 \times 2 + 2 \times 2 + 2 \times 2 = 28$$

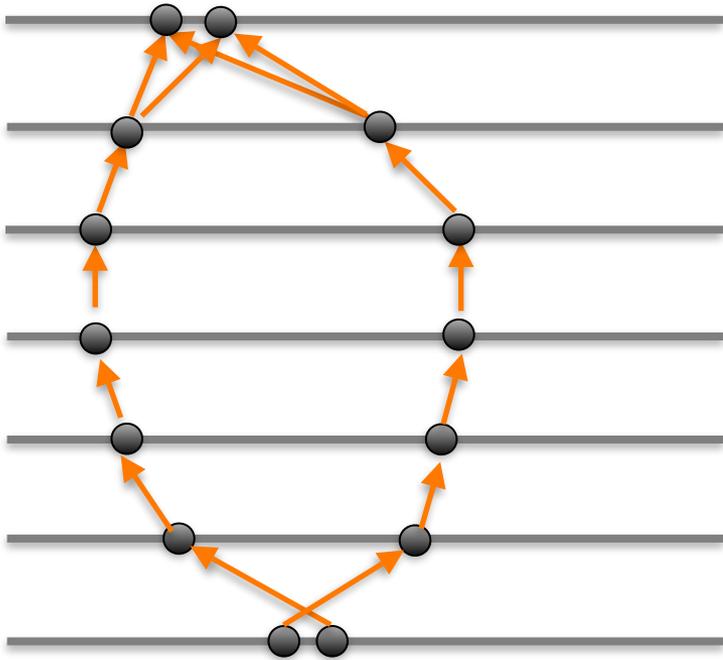


元々の#track

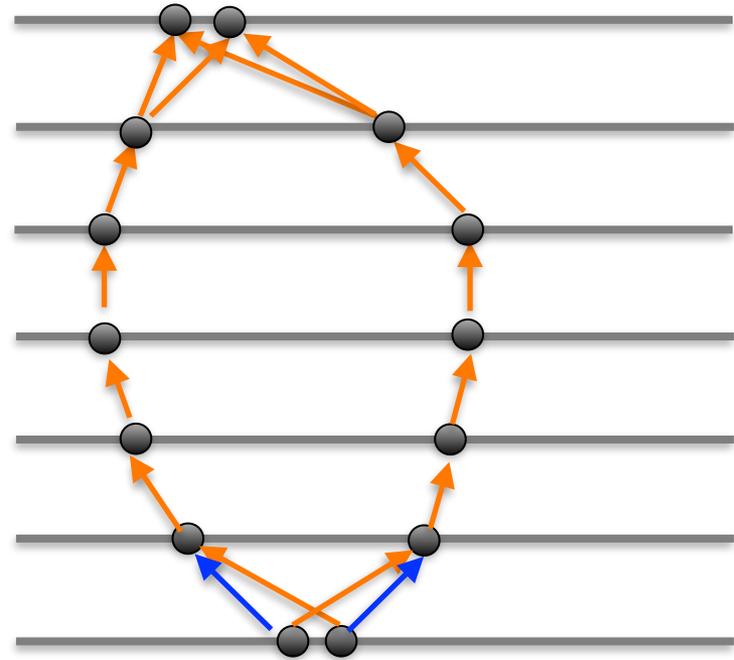


$$2^4 + 4 \times 2 + 4 \times 2 + 4 \times 2 = 40$$

Kalman Filter



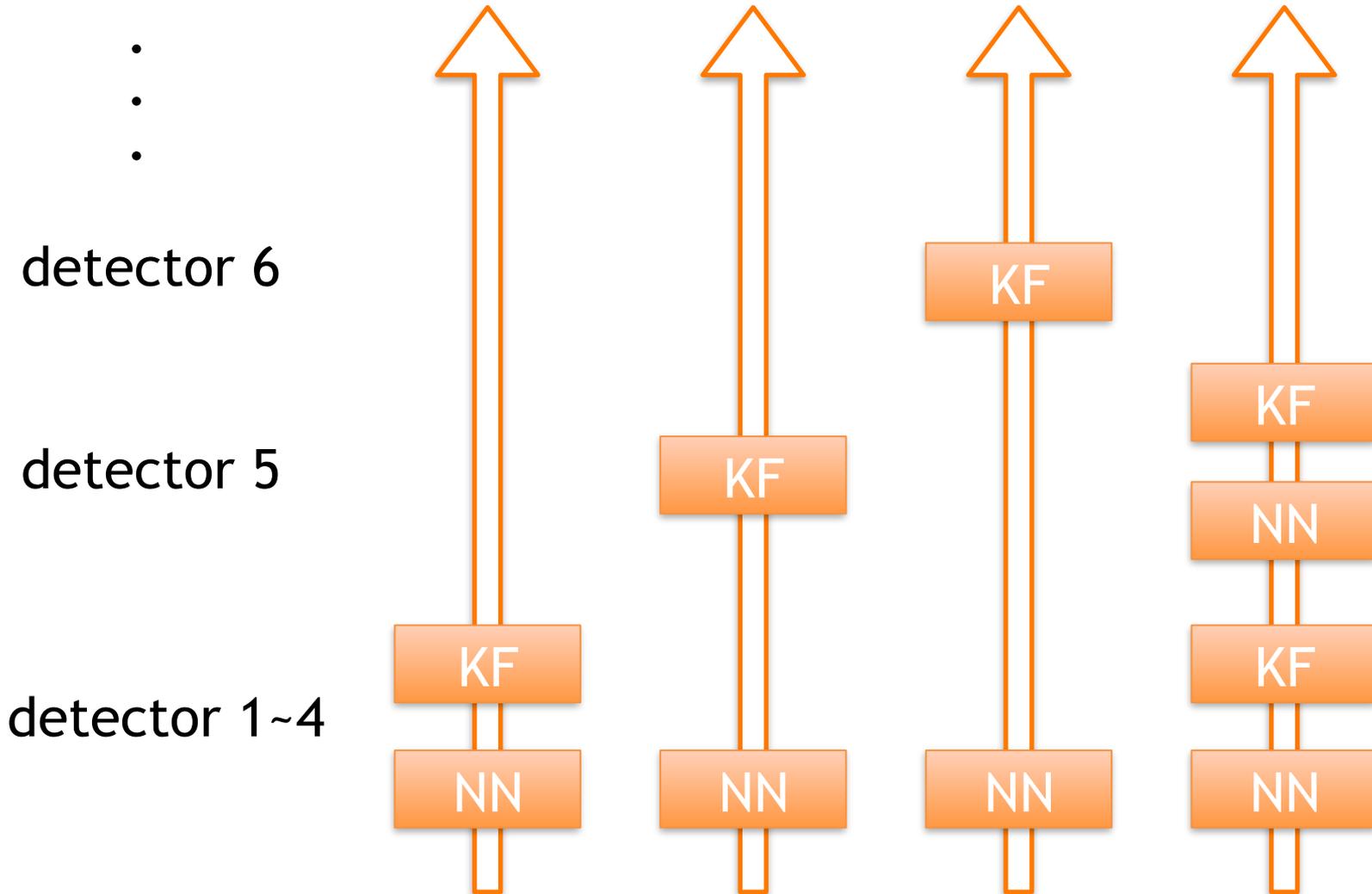
$$2^4 + 2 \times 2 + 2 \times 2 + 2 \times 2 = 28$$



$$2^4 + 4 \times 2 + 4 \times 2 + 4 \times 2 = 40$$

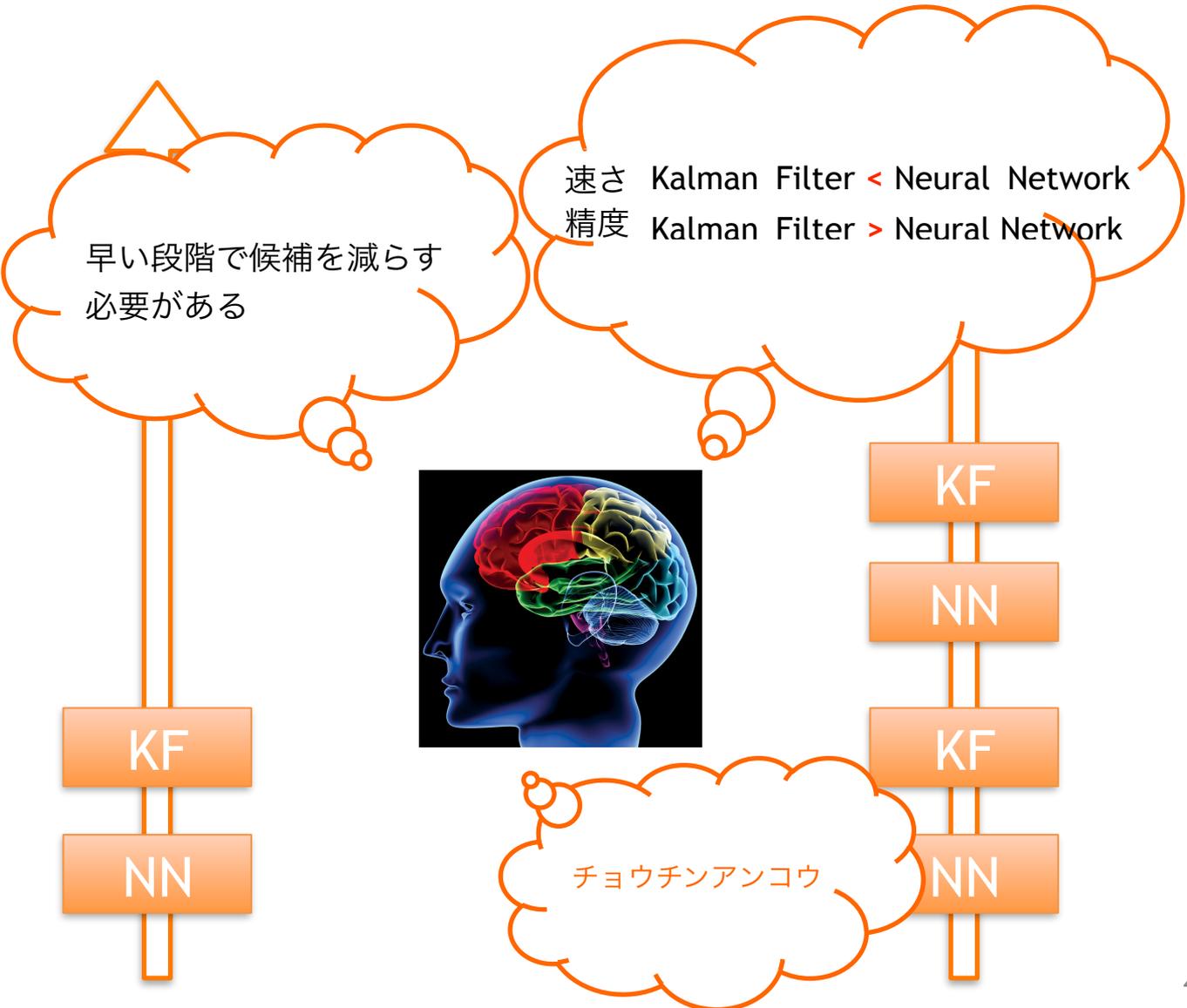
早い段階で候補を減らす必要がある

How to Use the Network?

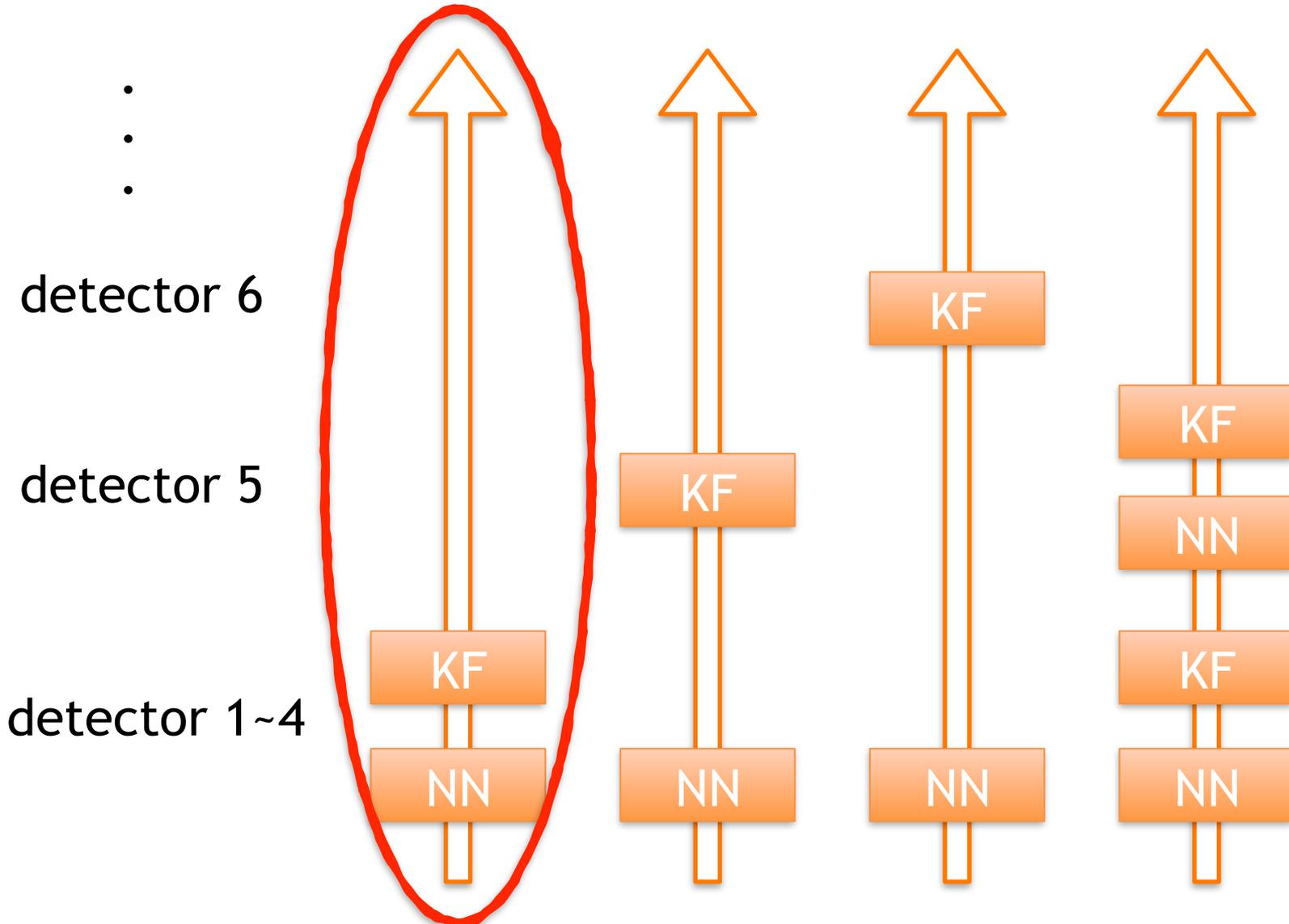


How to Use the Network?

- -
 -
- detector 6
- detector 5
- detector 1~4



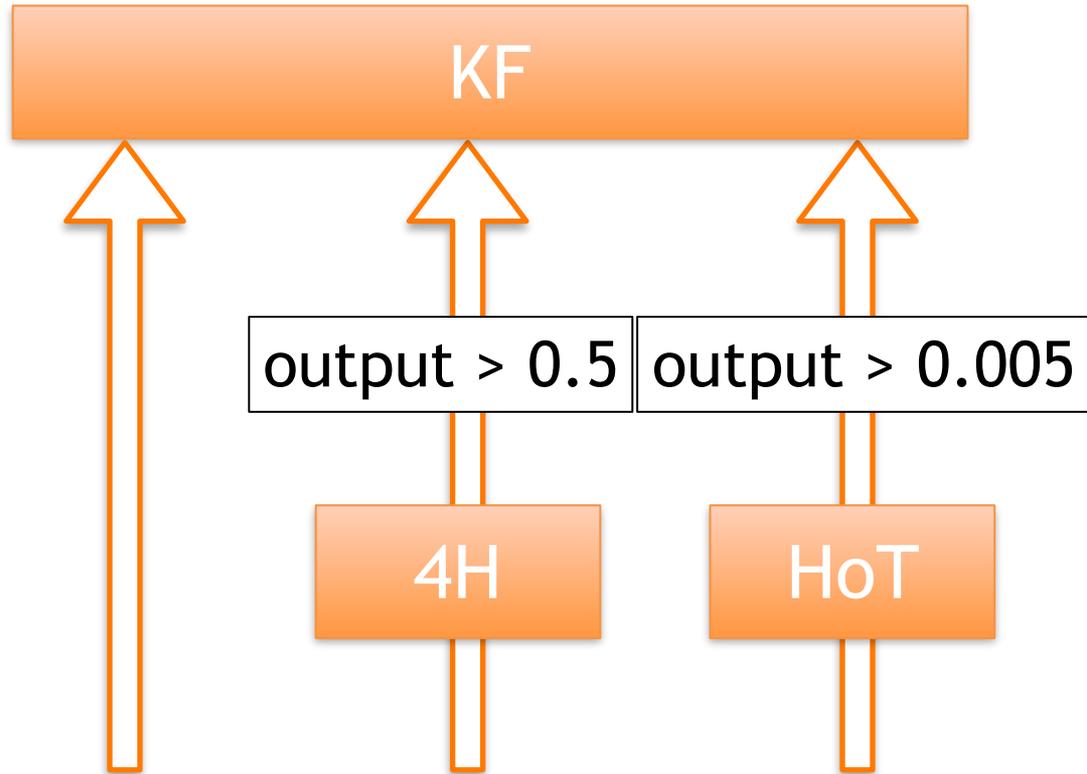
How to Use the Network?



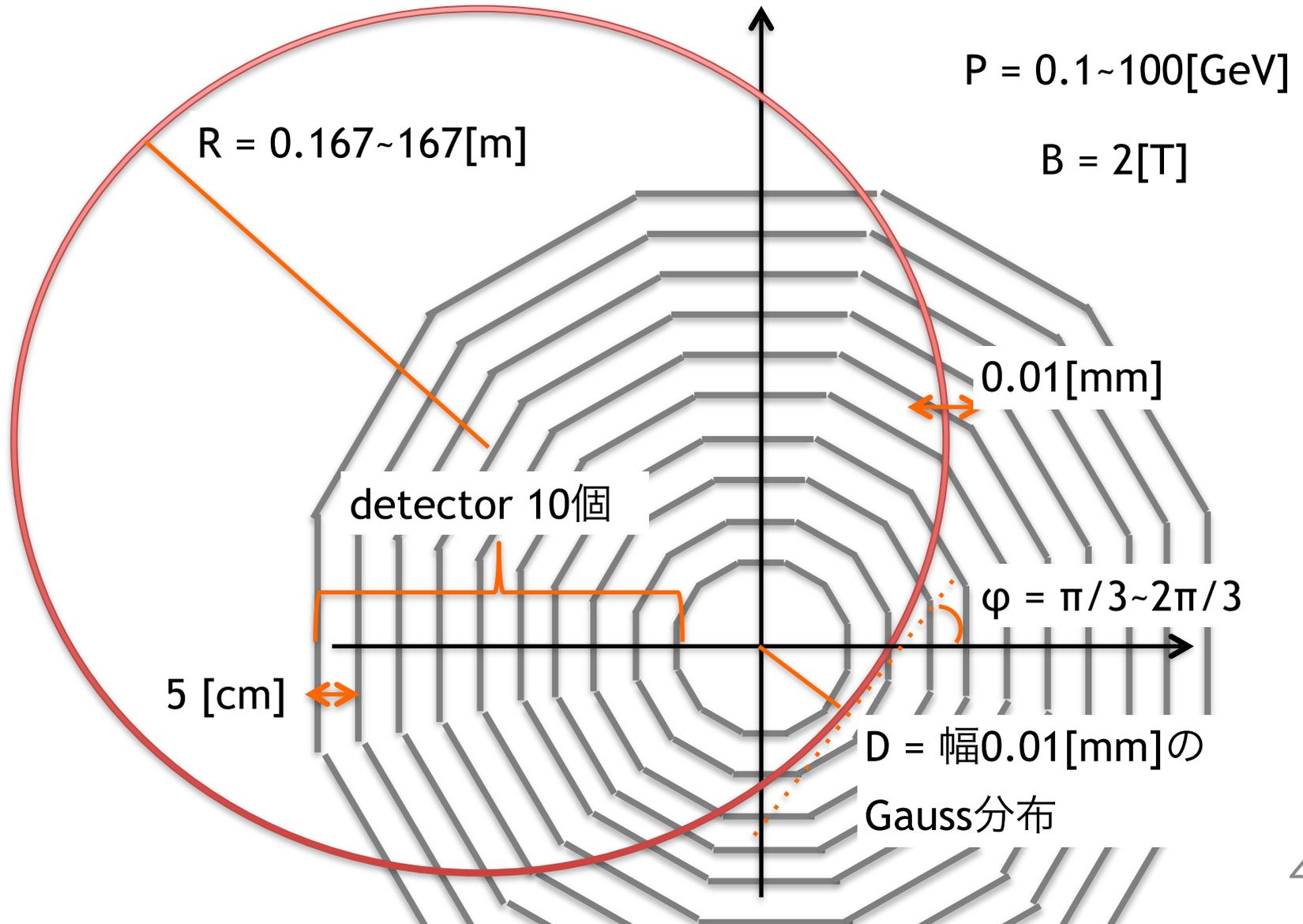
Calculation Methods

飛跡を計算

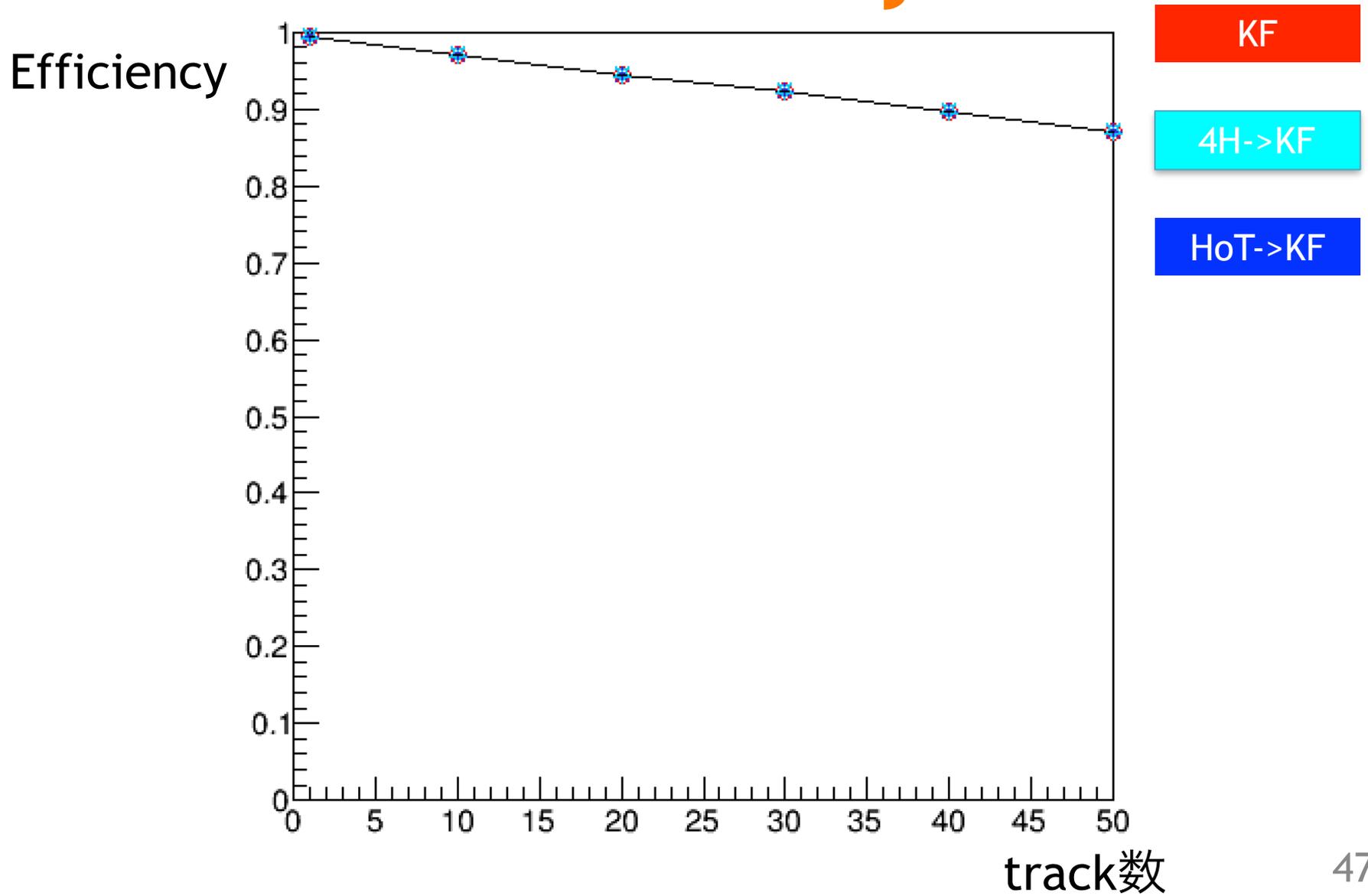
detector1~4のhitsの
組み合わせが同一飛
跡に乗るか判定



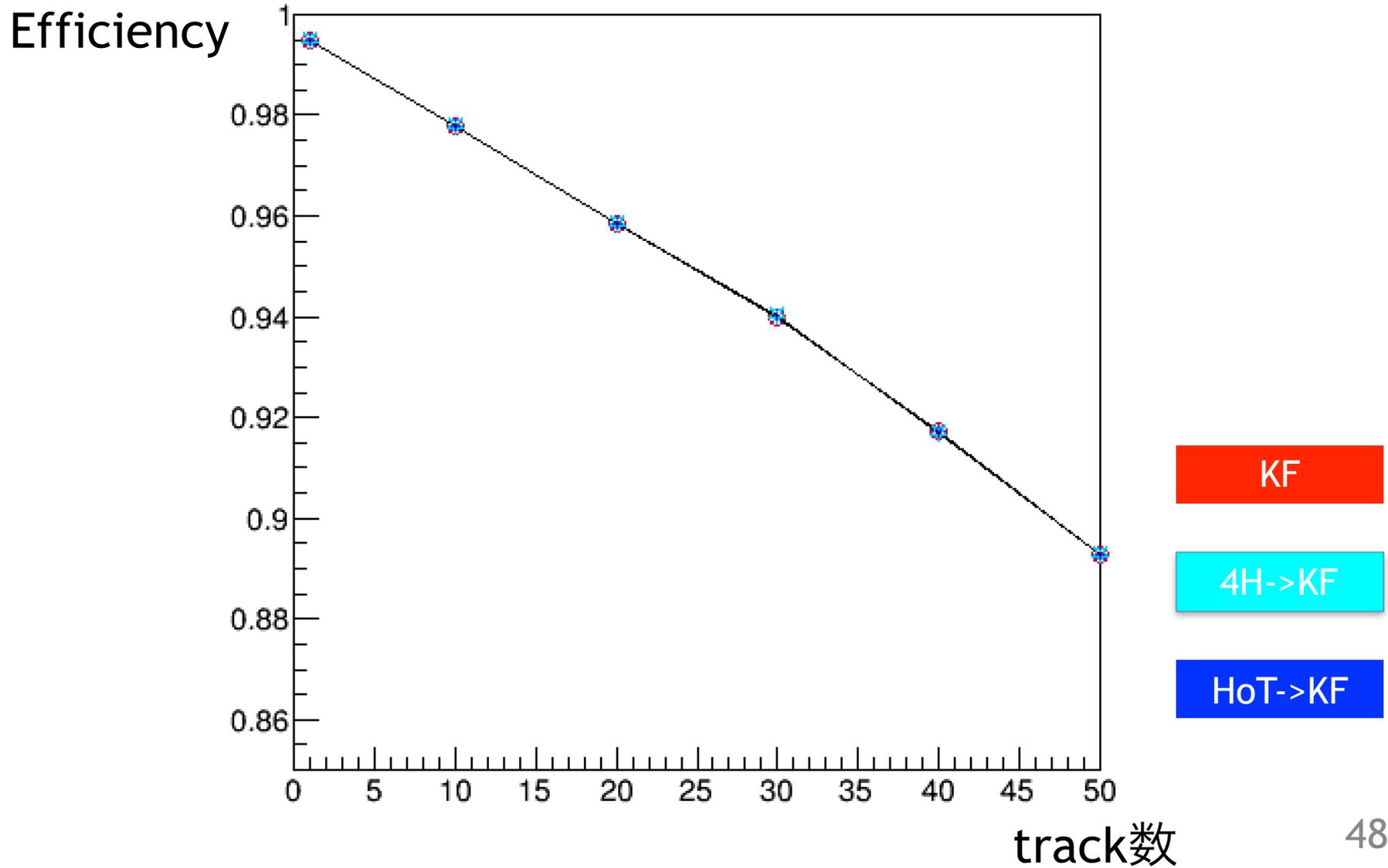
Setting for Reconstruction



Efficiency

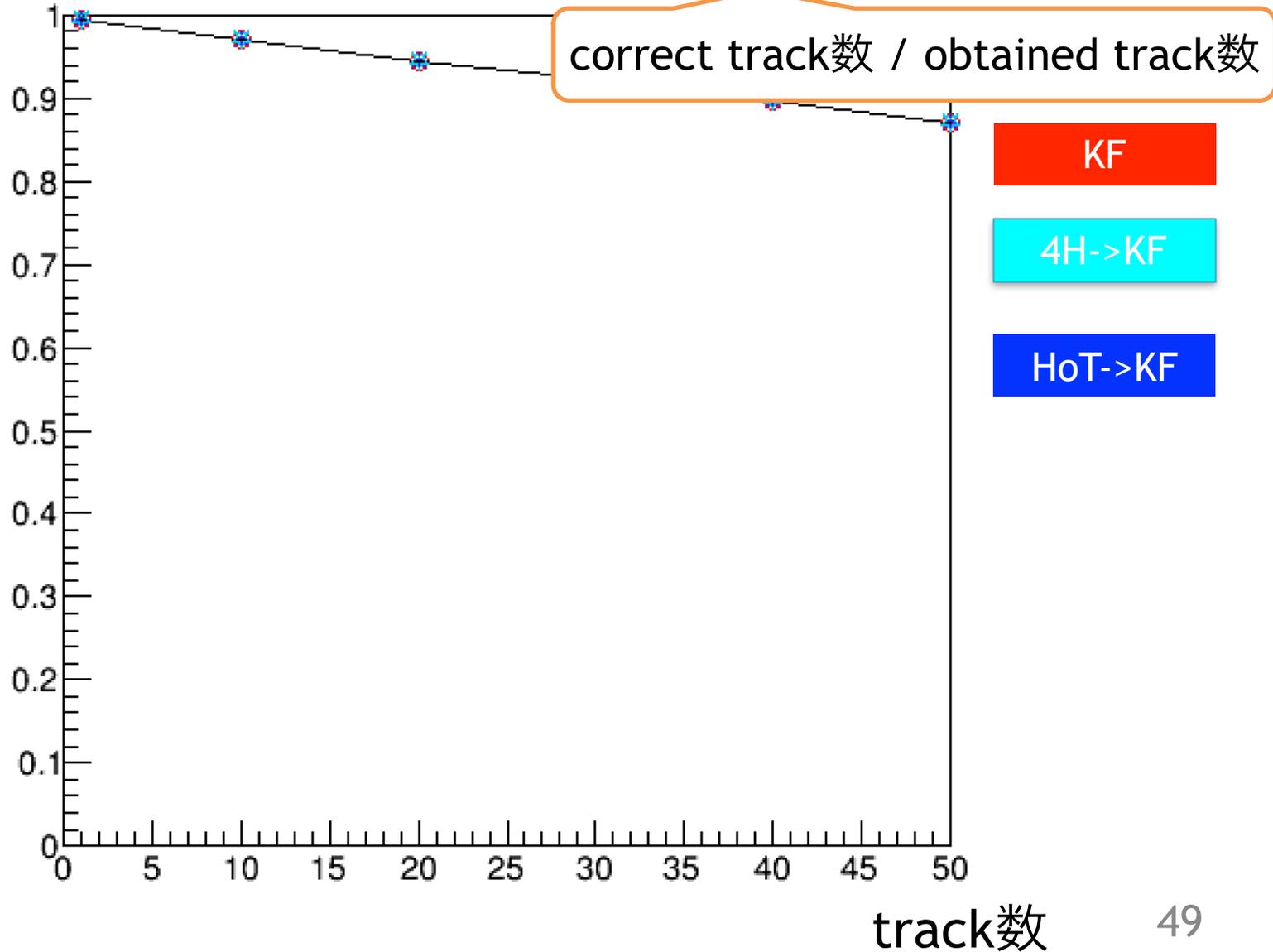


Efficiency



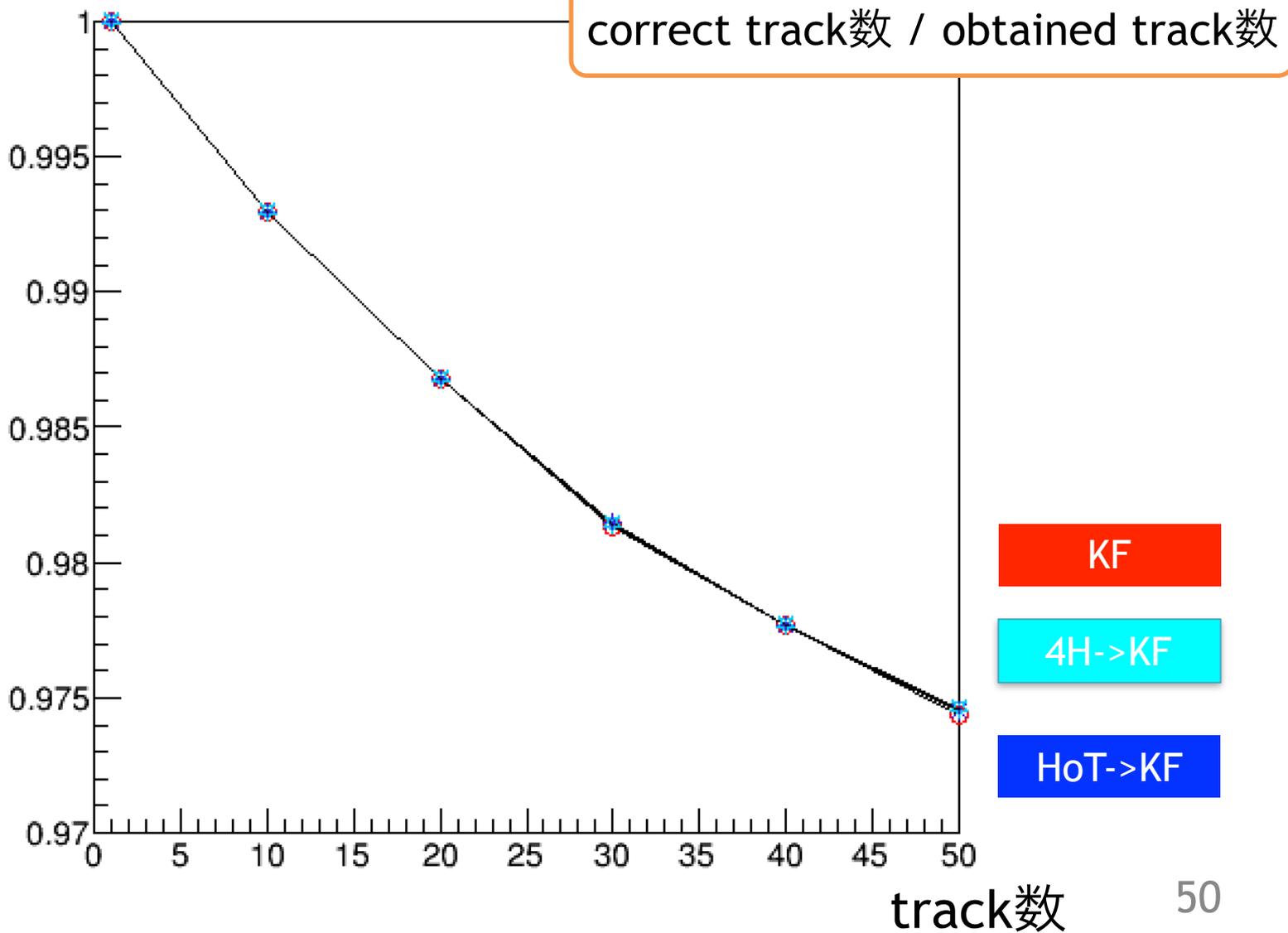
Accuracy Rate

Accuracy Rate



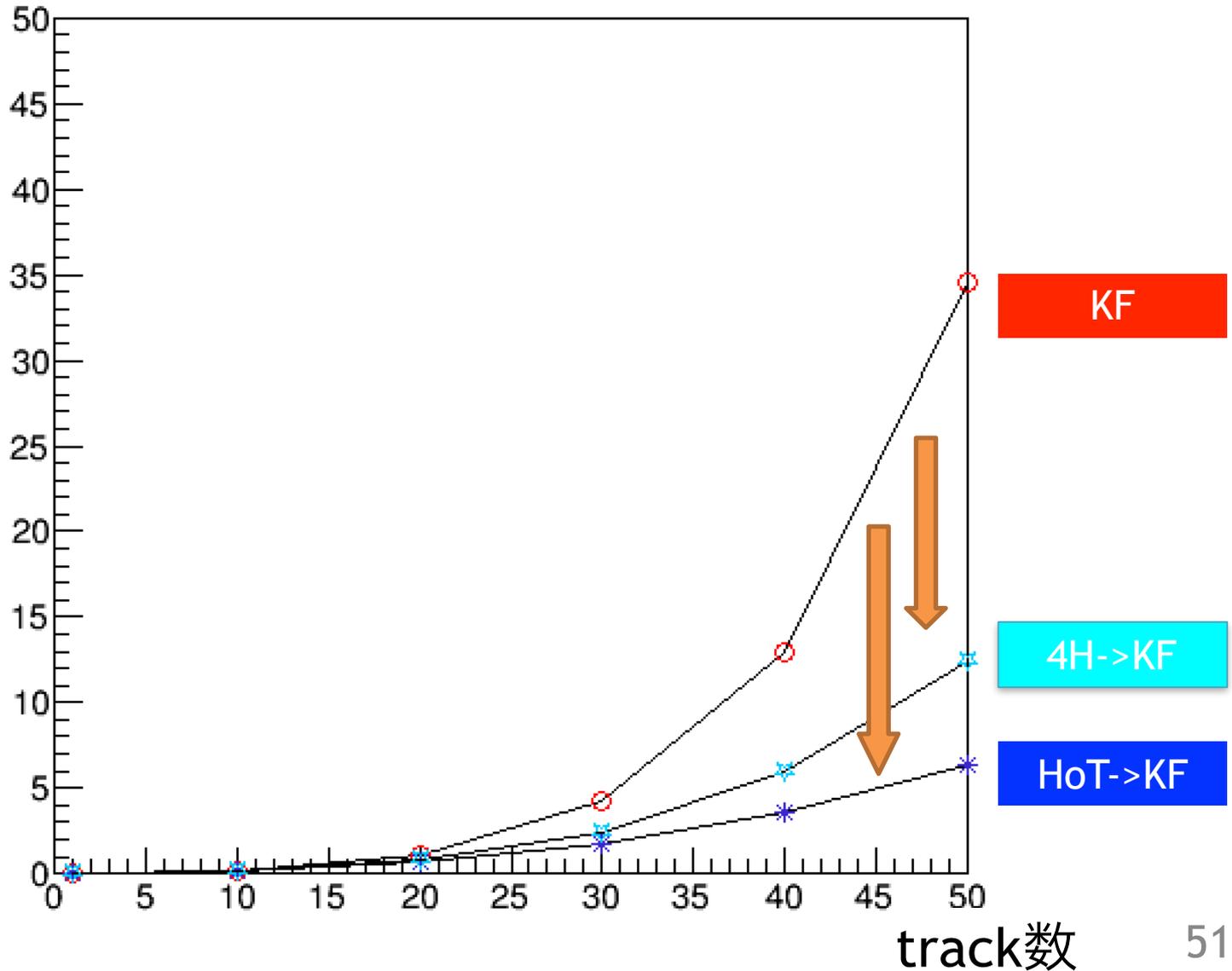
Accuracy Rate

Accuracy Rate



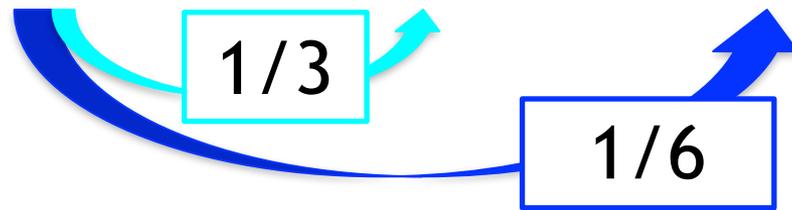
Calculation Time

Calculation Time [s]

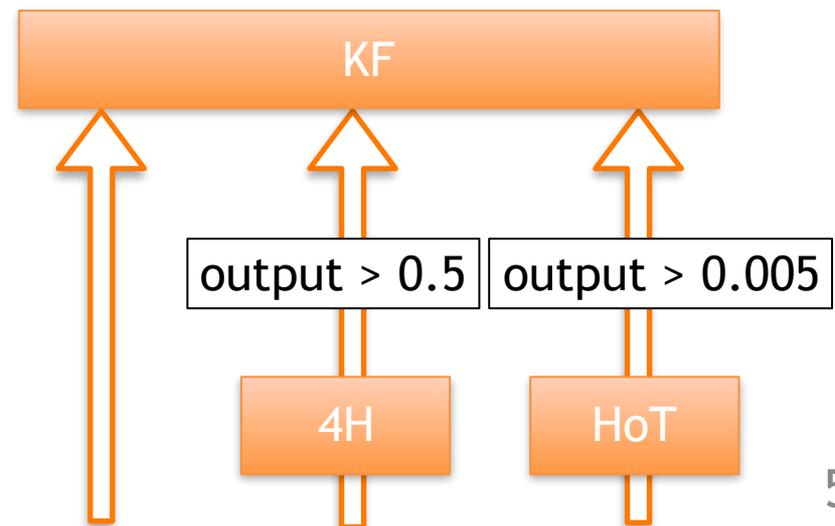


Calculation Time for 50 tracks

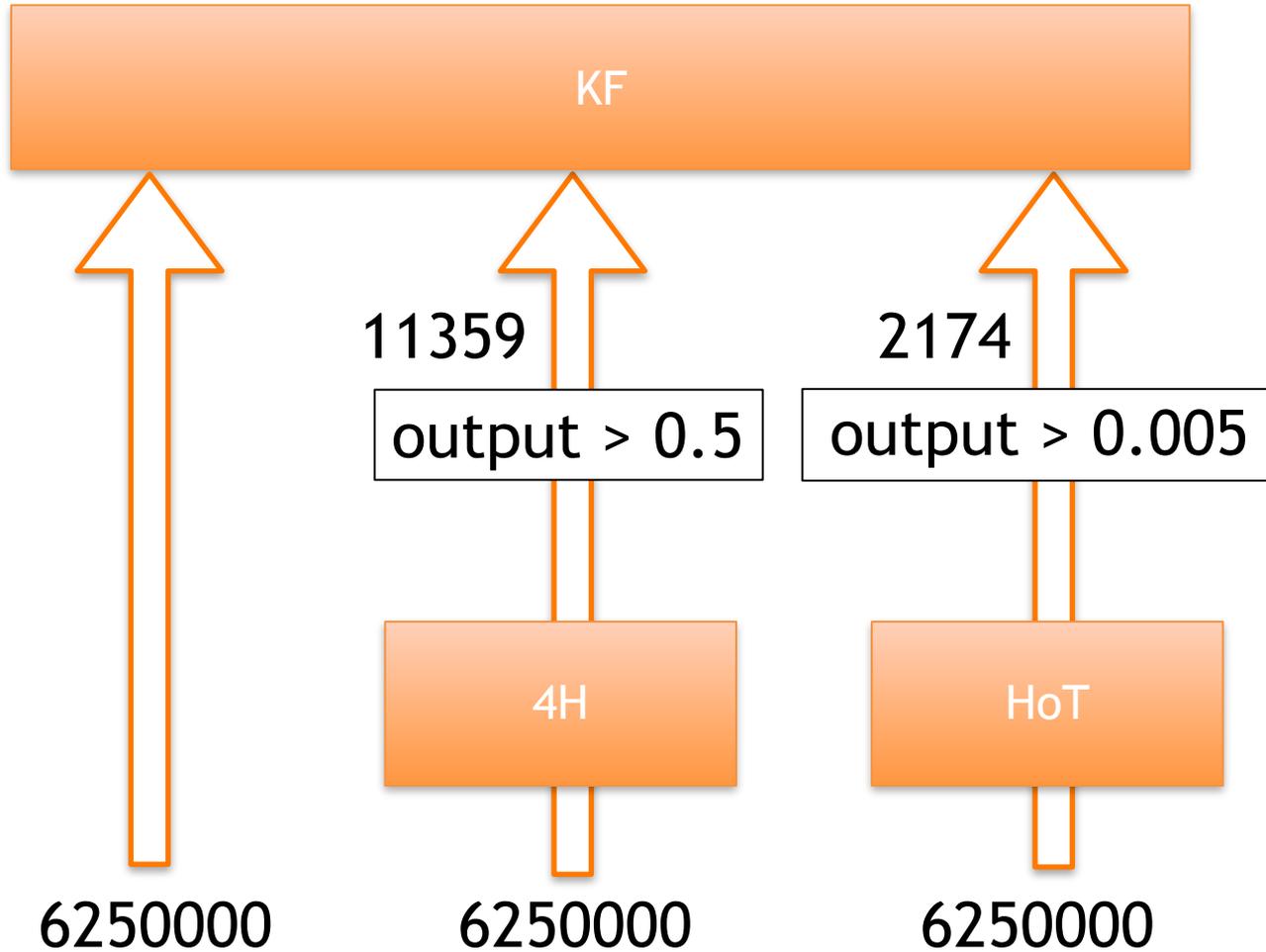
KF	4H -> KF	HoT -> KF
34.55s	12.40s	6.35s



計算する組み合わせの数を
大幅にカットできる



Inputs for 50 tracks



$$50^4 =$$

6250000

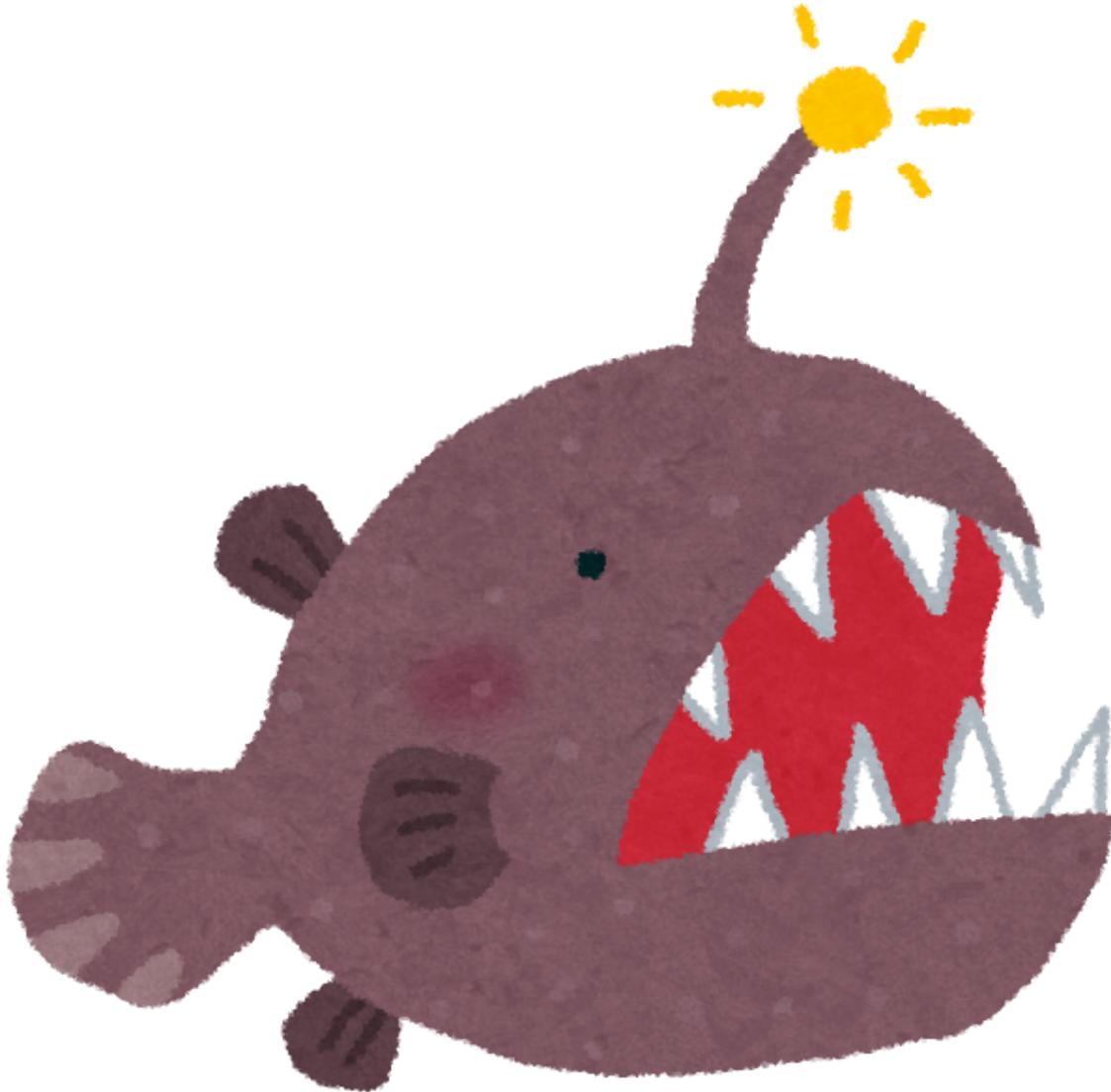
6250000

6250000

Conclusion

✓ Neural Networkを用いることにより、飛跡再構成を同等の効率で短時間に行うことができた

➤ 実際の実験条件に近づける

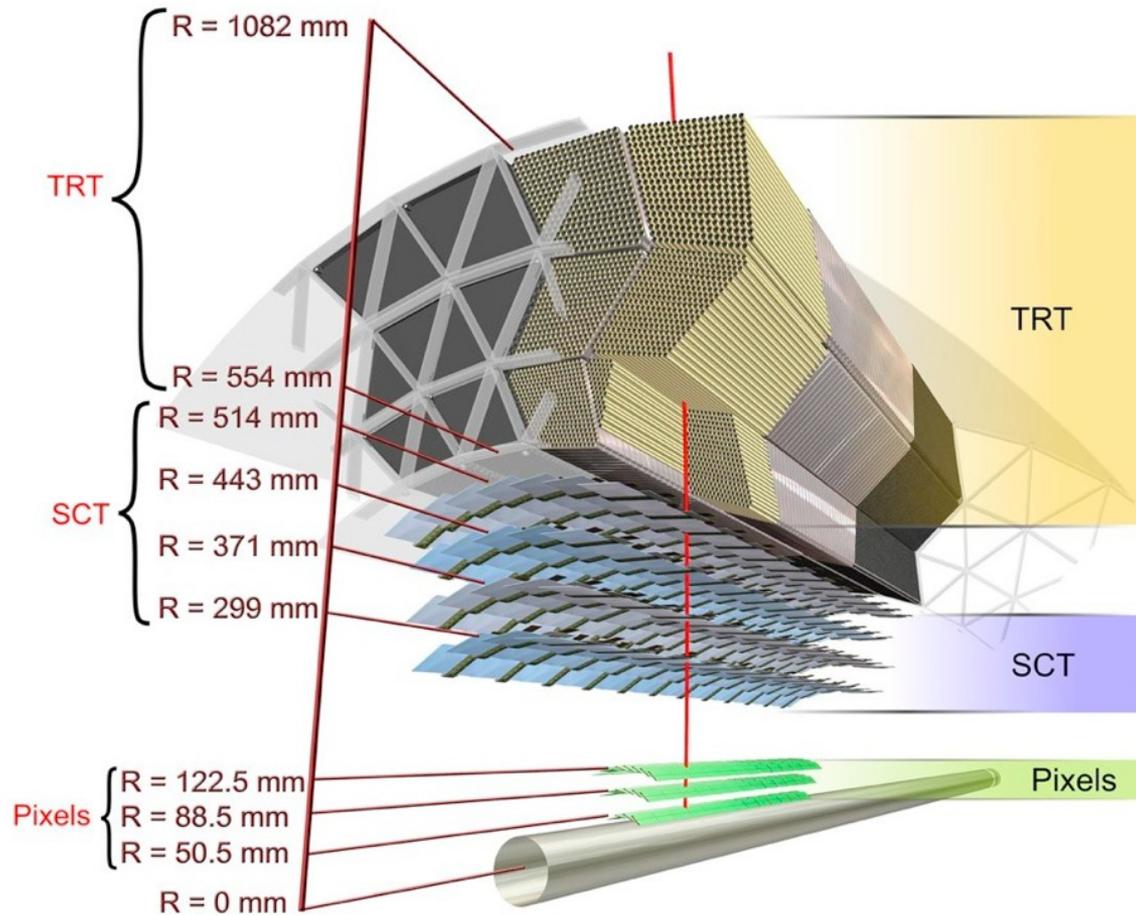


Backup

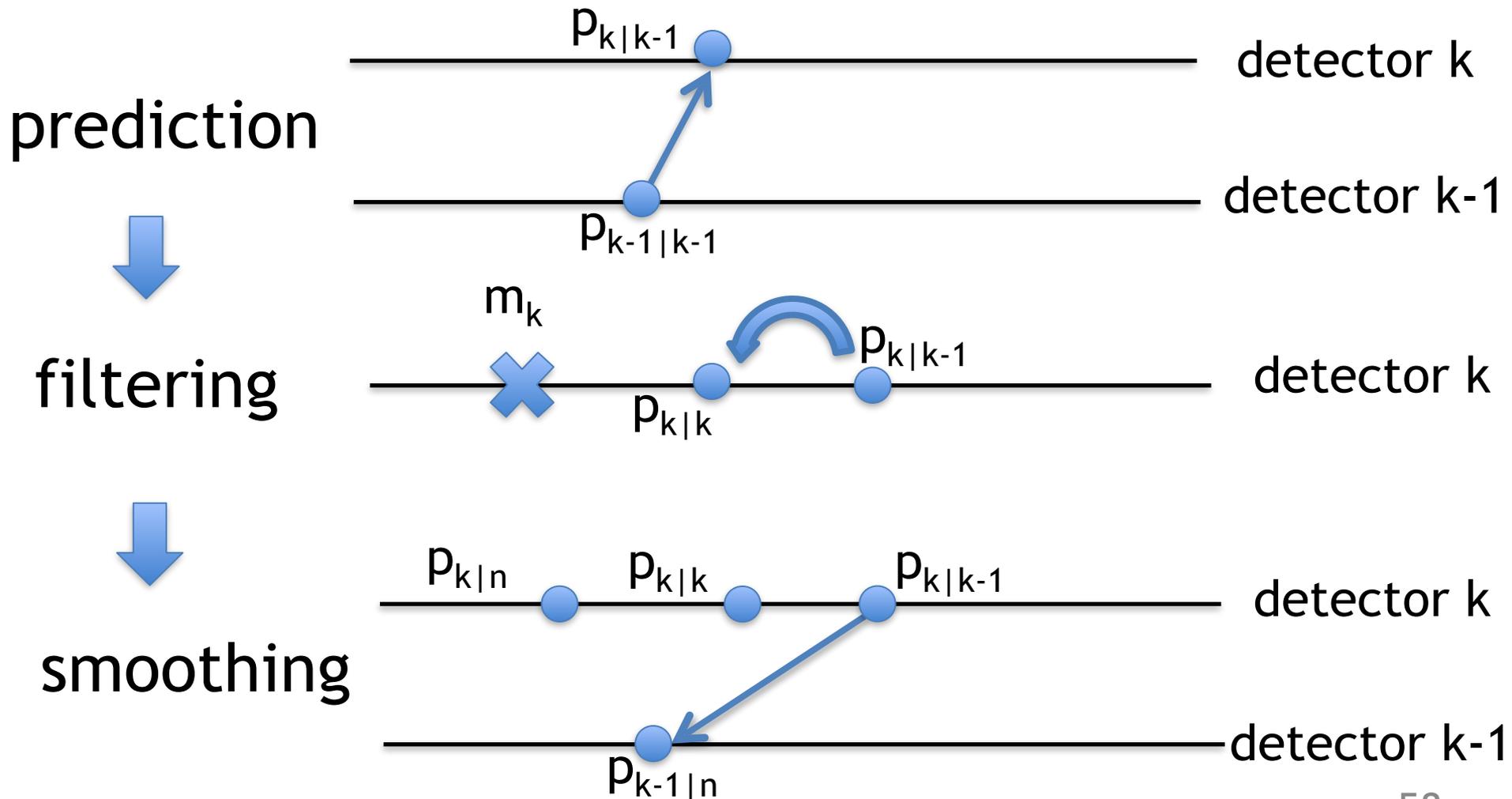
参考文献

- 足立修一 ・ 丸田一郎 (2012) 『カルマンフィルタの基礎』 東京電機大学出版局
- R.Fruhwirth (1987) “Application of Kalman Filtering to Track and Vertex Fitting”, Nuclear Instruments and Methods in Physics Research A262 p.444-450
- 斎藤康毅 (2016) 『ゼロから作るDeep Learning』 オライリー・ジャパン

ATLAS の内部飛跡検出器



Kalman Filter



Kalman Filter

n次元状態ベクトル $x(k)$ 、観測データ $y(k)$ が

$$x(k+1) = Ax(k) + Bv(k)$$

$$y(k) = C^T x(k) + w(k)$$

という状態空間モデルに従うとする。

ただし、 A は $n \times n$ 行列、 B は $n \times r$ 行列、 C は $p \times n$ 行列

v は平均値0、共分散行列 Q の r 次元システム雑音ベクトル。 w は平均値0、共分散行列 R の p 次元観測雑音ベクトル。

更新アルゴリズム

❖ 予想ステップ

$$\text{事前状態推定値} : \mathbf{x}_{k|k-1} = \mathbf{A} \mathbf{x}_{k-1|k-1}$$

$$\text{事前誤差共分散行列} : \mathbf{P}_{k|k-1} = \mathbf{A} \mathbf{P}_{k-1|k-1} \mathbf{A}^T + \mathbf{B} \mathbf{Q} \mathbf{B}^T$$

❖ フィルタリングステップ

$$\text{カルマンゲイン行列} : \mathbf{G}_k = \mathbf{P}_{k-1|k-1} \mathbf{C}^T (\mathbf{C} \mathbf{P}_{k-1|k-1} \mathbf{C}^T + \mathbf{R})^{-1}$$

$$\text{状態推定値} : \mathbf{x}_{k|k} = \mathbf{x}_{k|k-1} + \mathbf{G}_k (\mathbf{y}(k) - \mathbf{C} \mathbf{x}_{k|k-1})$$

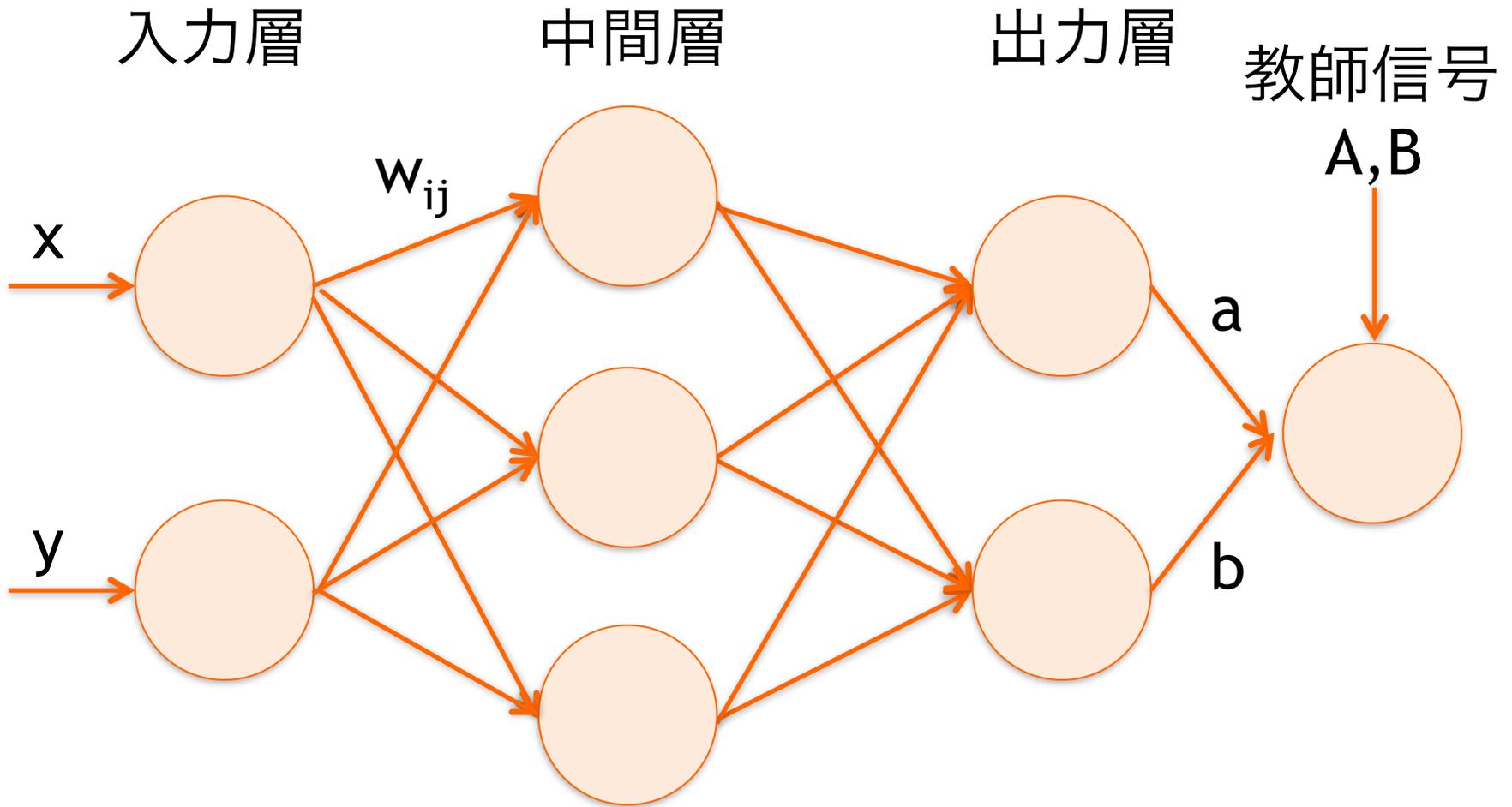
$$\text{事後誤差共分散行列} : \mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{G}_k \mathbf{C}) \mathbf{P}_{k|k-1}$$

❖ 平滑ステップ

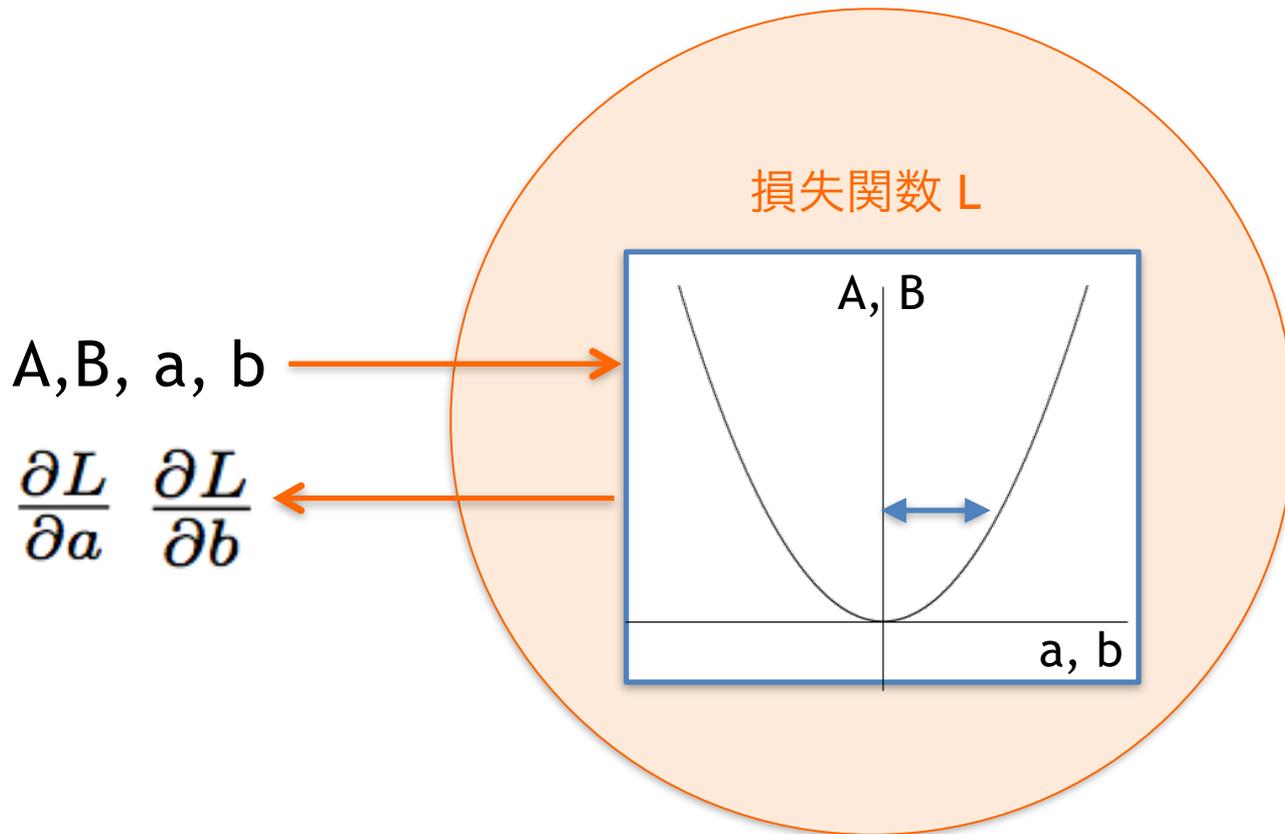
$$\mathbf{x}_{k|n} = \mathbf{x}_{k|k} + \mathbf{P}_{k|k} \mathbf{A} \mathbf{P}_{k+1|k}^{-1} (\mathbf{x}_{k+1|n} - \mathbf{x}_{k+1|k})$$

$$\mathbf{P}_{k|n} = \mathbf{P}_{k|k} + \mathbf{P}_{k|k} \mathbf{A} \mathbf{P}_{k+1|k}^{-1} (\mathbf{P}_{k+1|n} - \mathbf{P}_{k+1|k}) (\mathbf{P}_{k|k} \mathbf{A} \mathbf{P}_{k+1|k}^{-1})^T$$

Training Phase

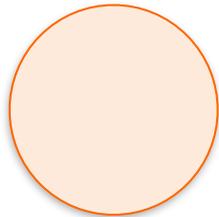
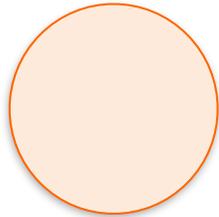


Training Phase

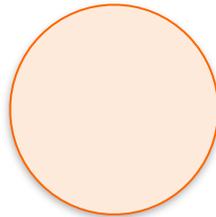
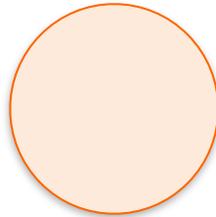
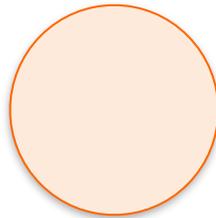


Training Phase

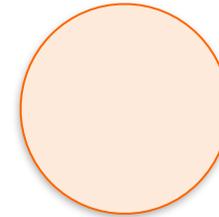
入力層



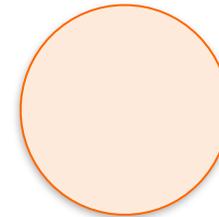
中間層



出力層



← $\frac{\partial L}{\partial a}$



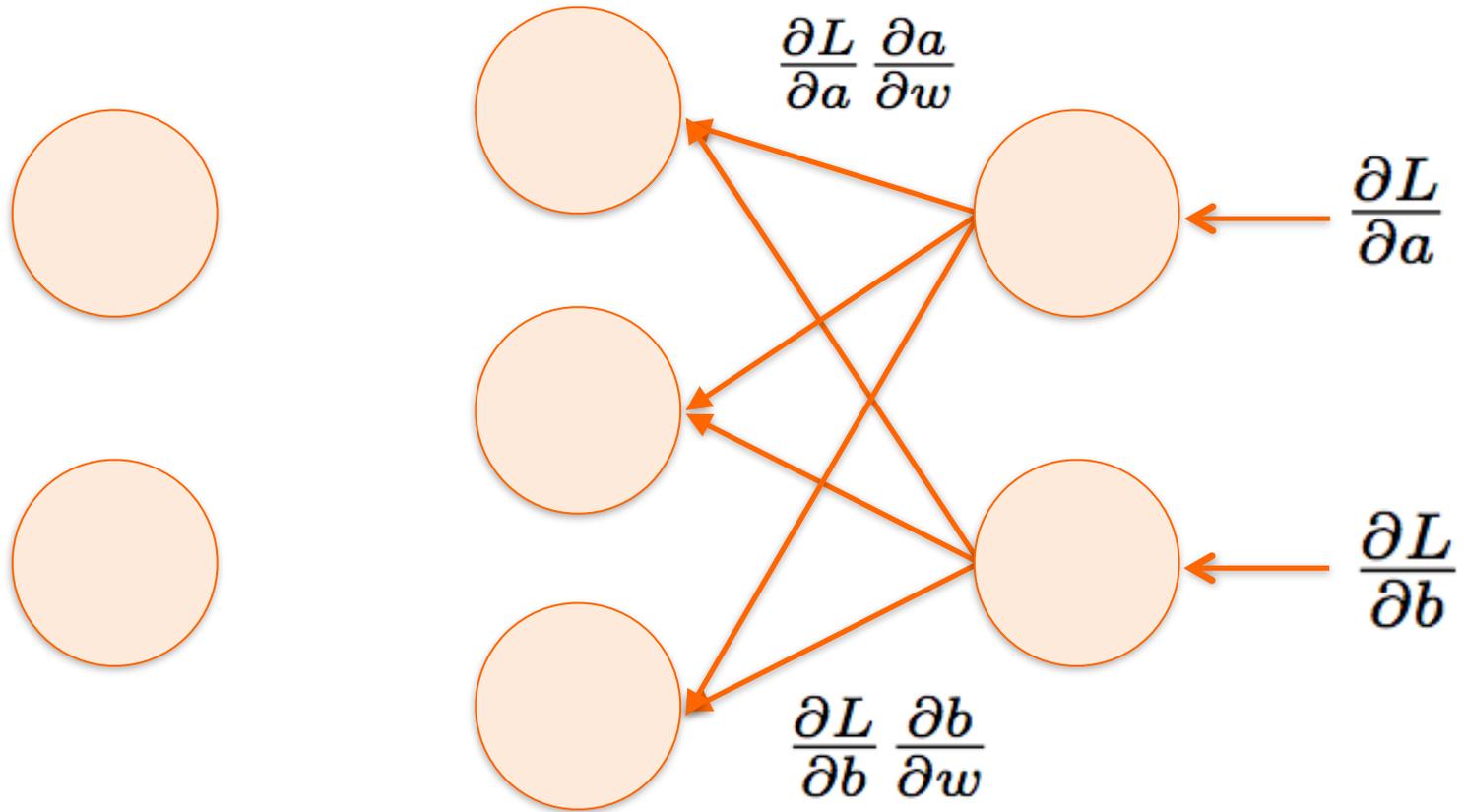
← $\frac{\partial L}{\partial b}$

Training Phase

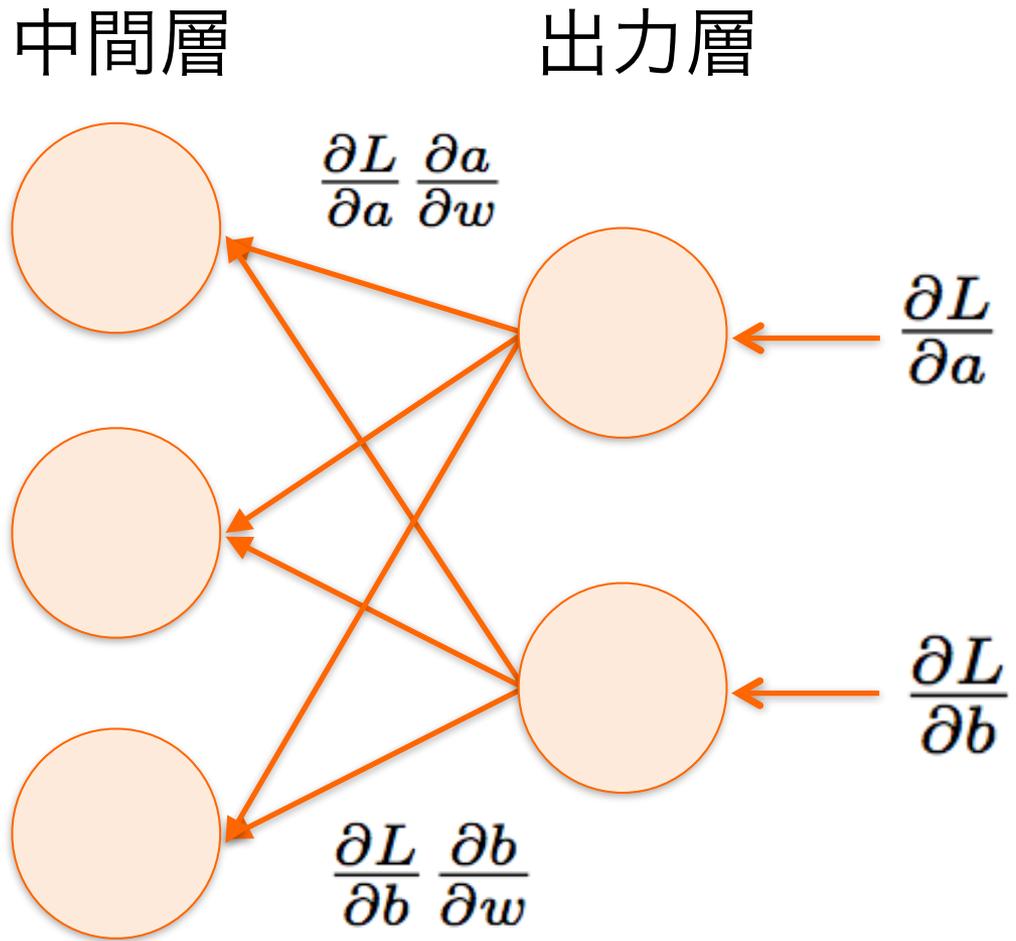
入力層

中間層

出力層



Training Phase



$$w = w - \eta \frac{\partial L}{\partial w}$$

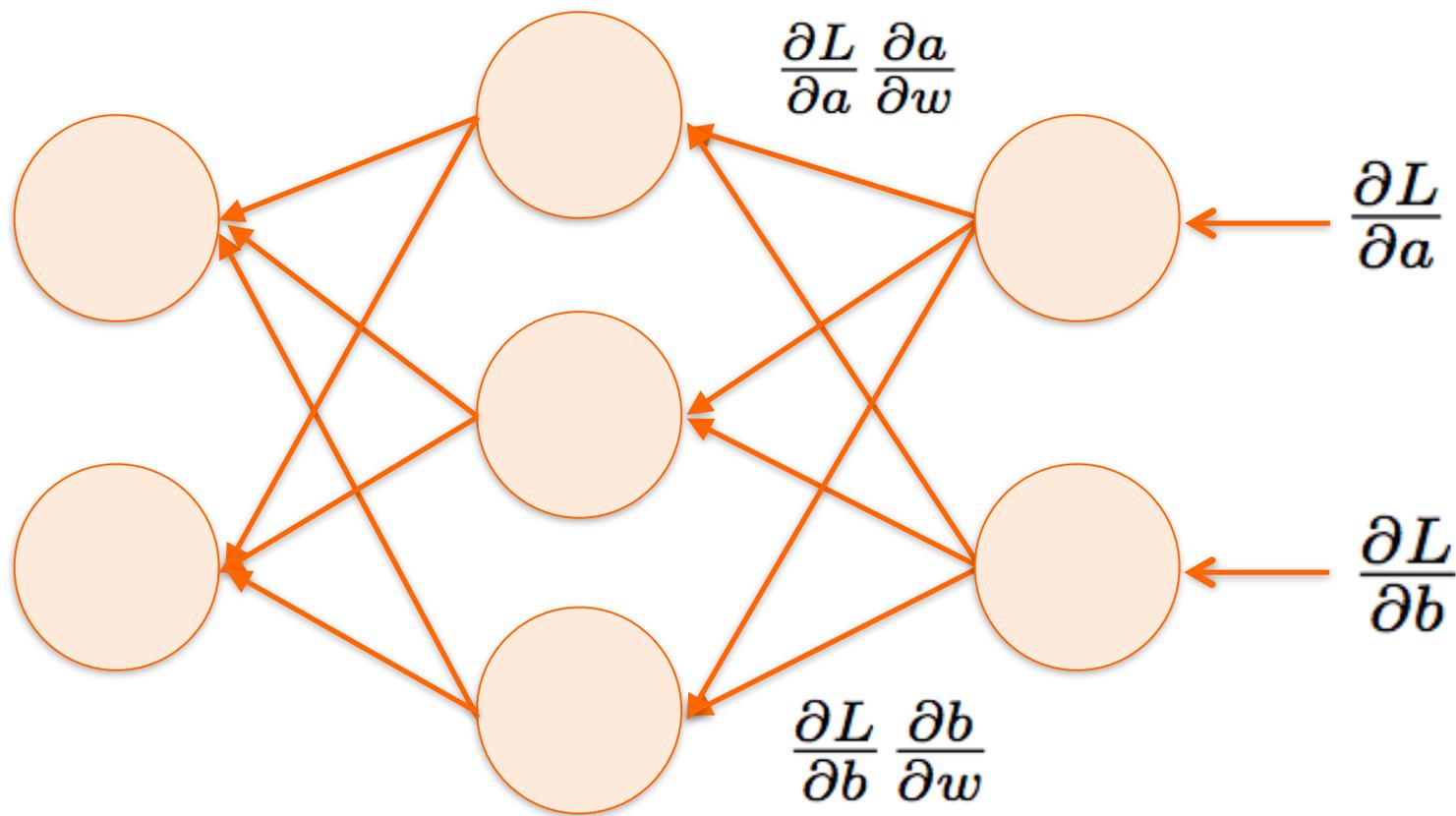
η : 学習係数

Training Phase

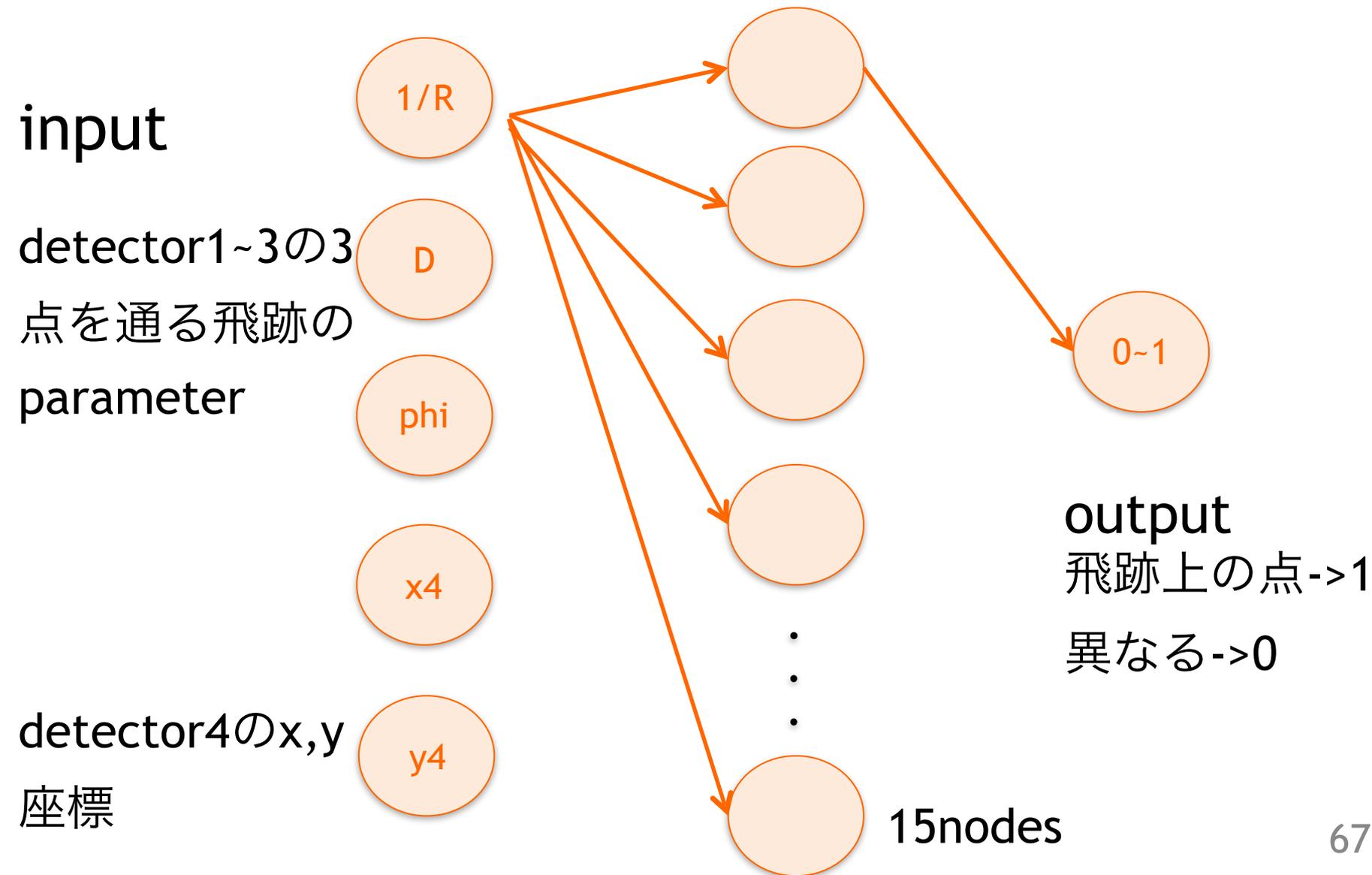
入力層

中間層

出力層

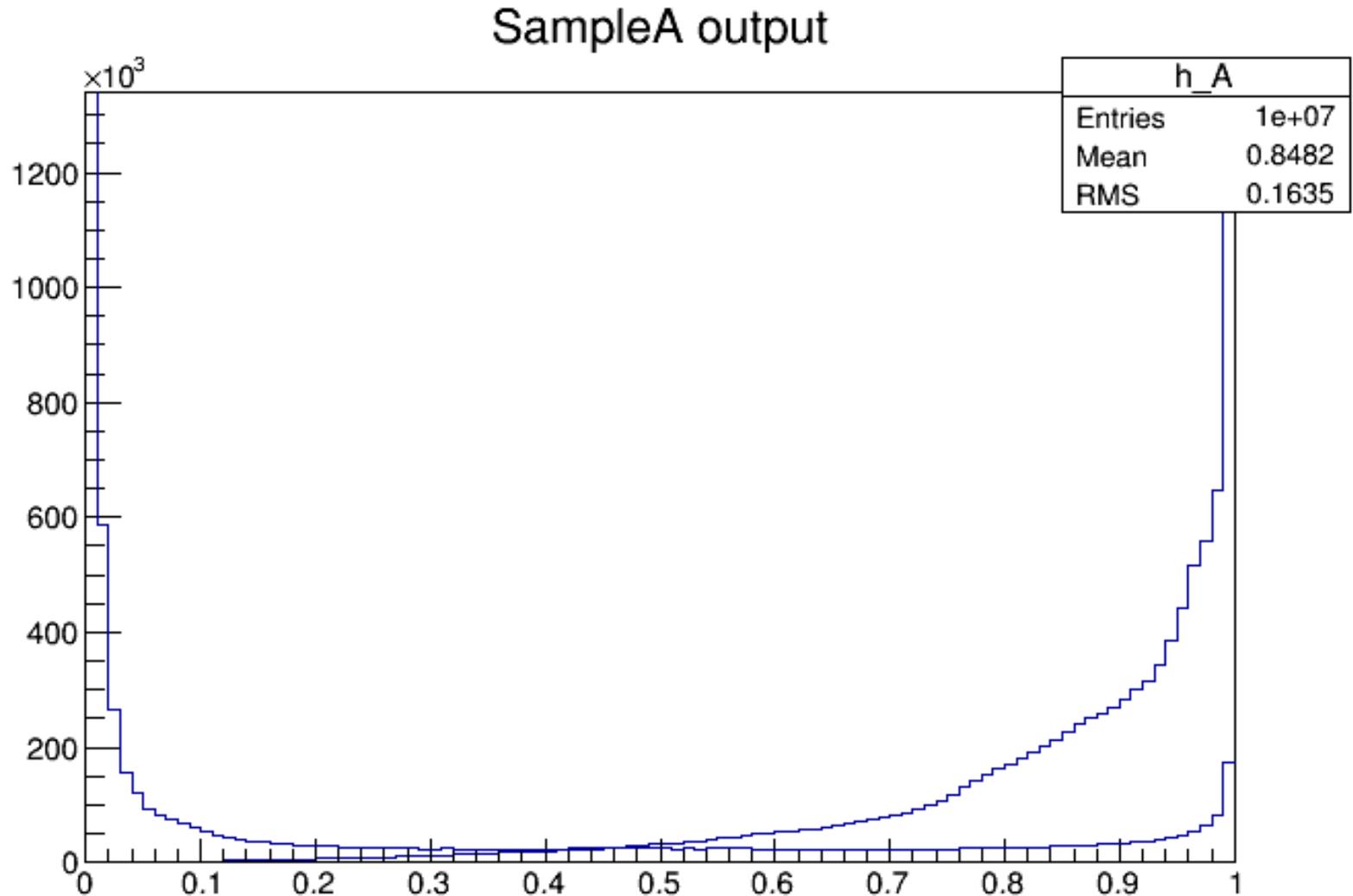


Hit on Track Net with D



Training Result

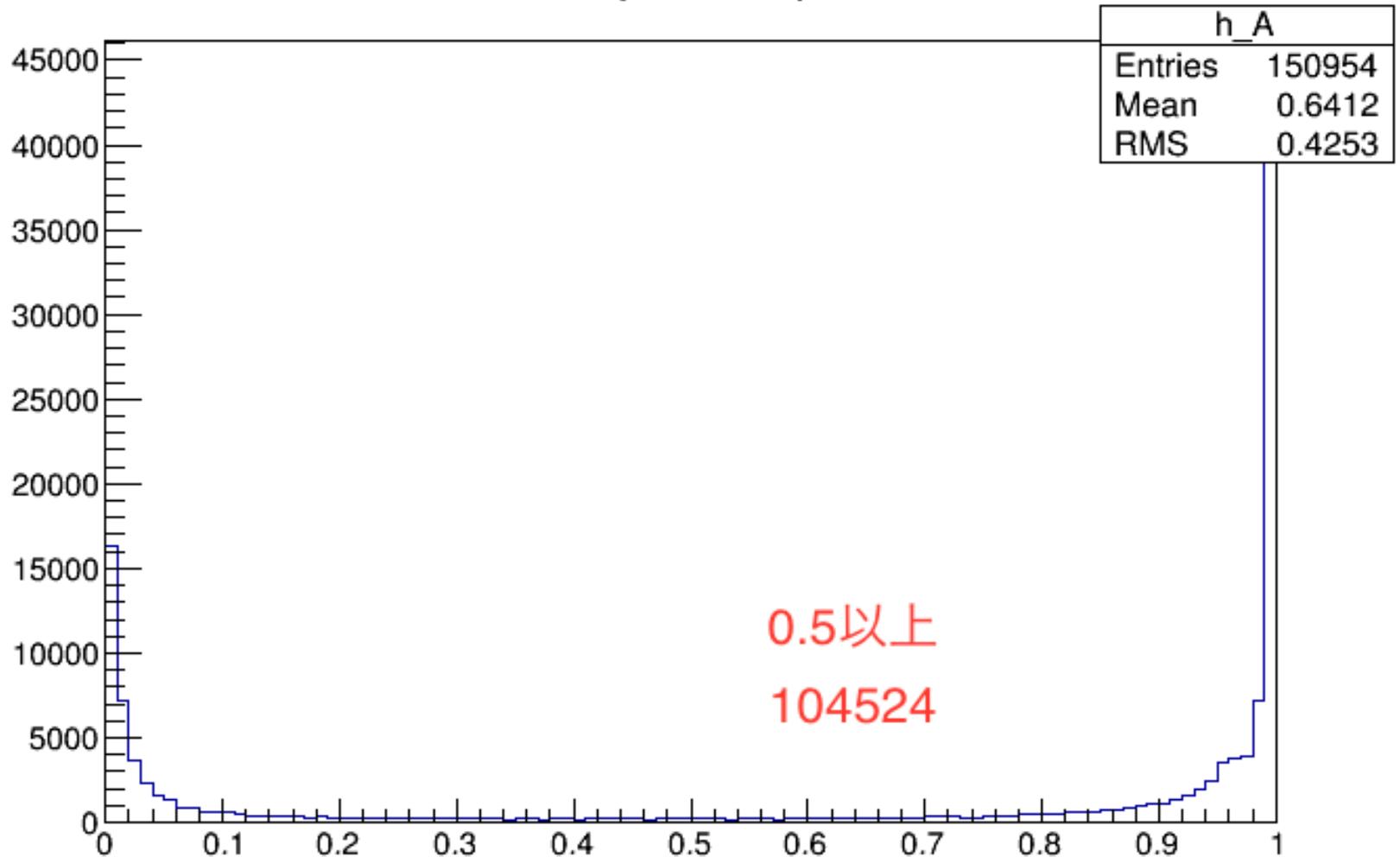
input parameters = true parameters



NeuralNet Output

input parameters = calculated

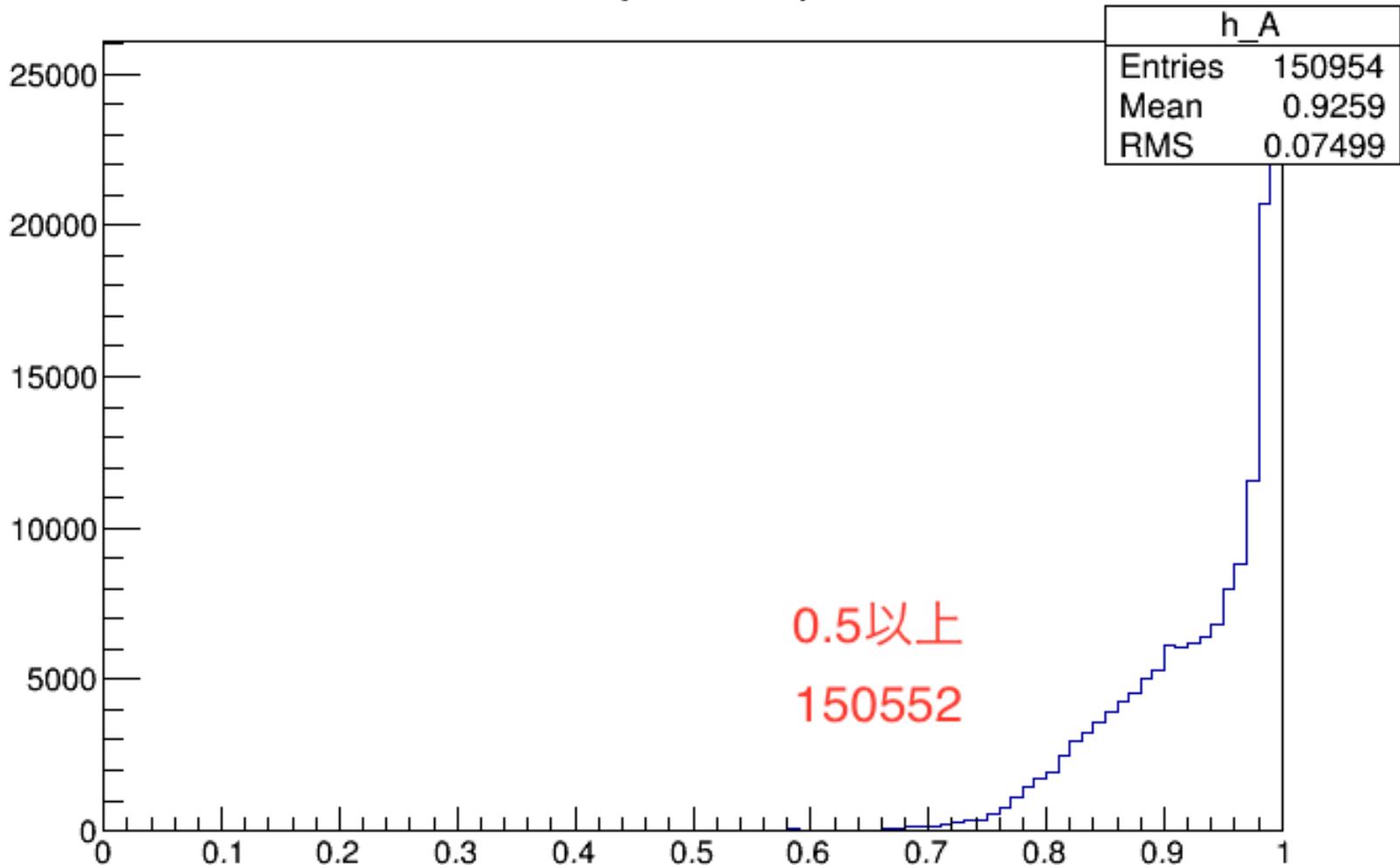
SampleA output



NeuralNet Output

input parameters = true parameters

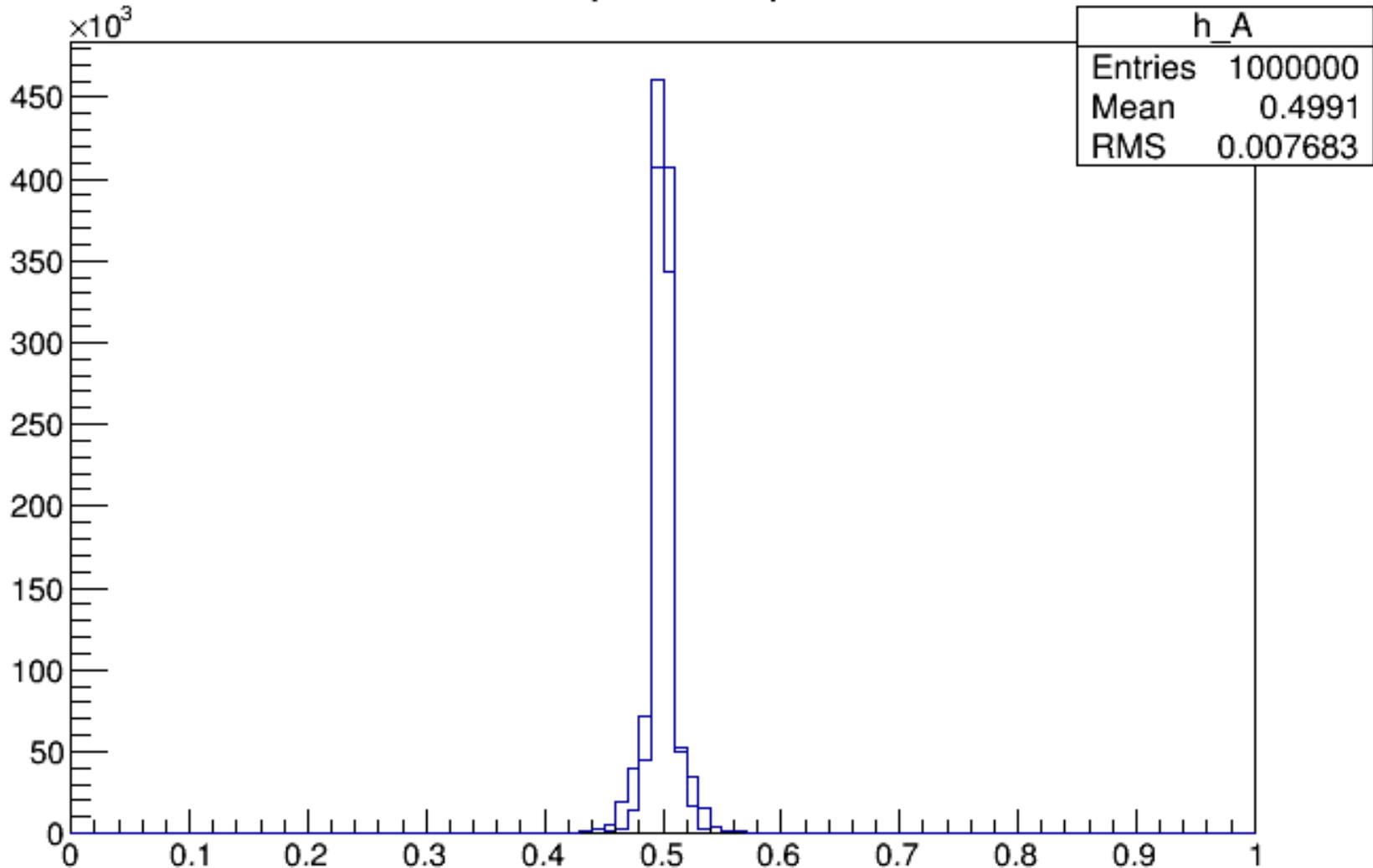
SampleA output



Training Result

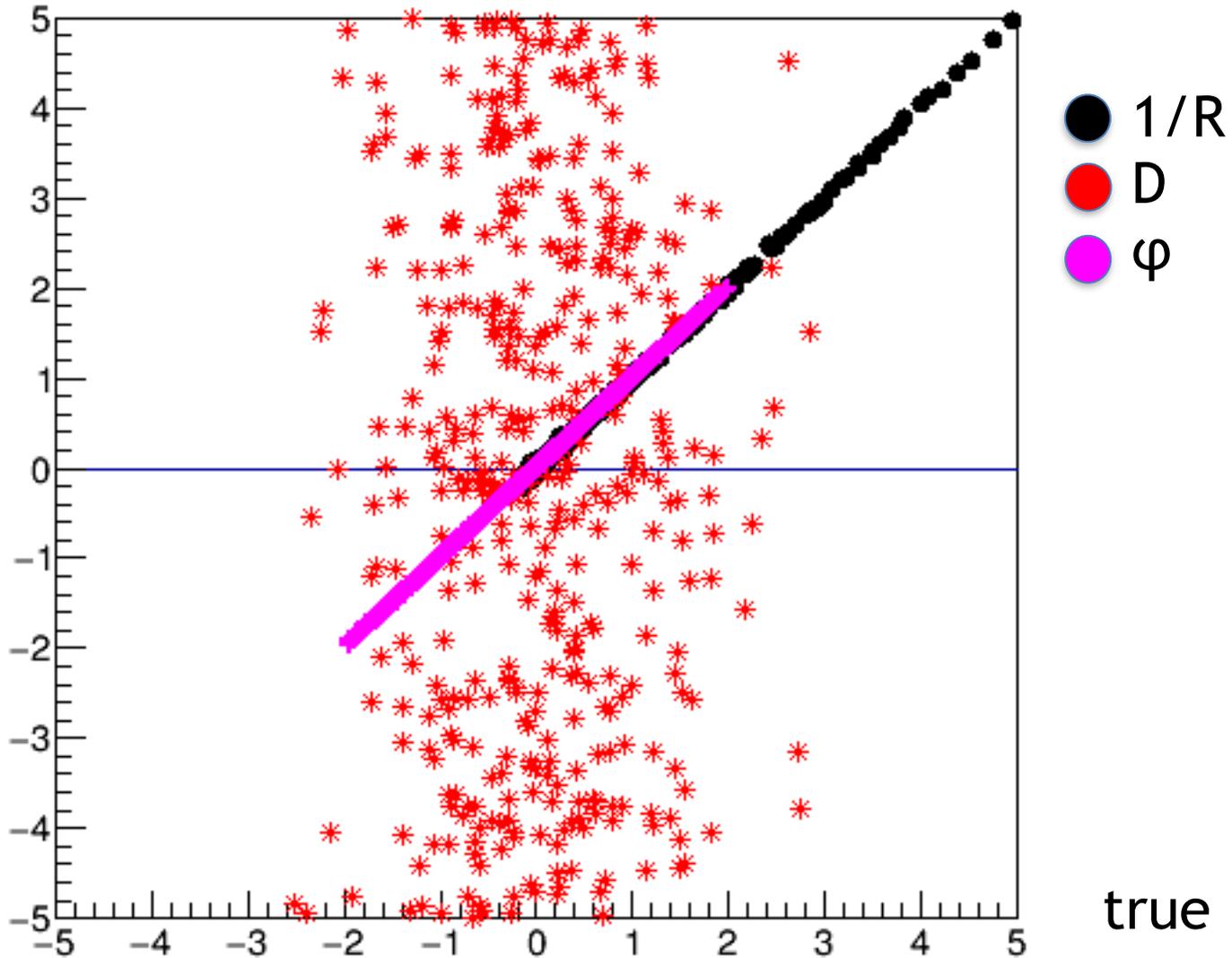
input parameters = calculated

SampleA output

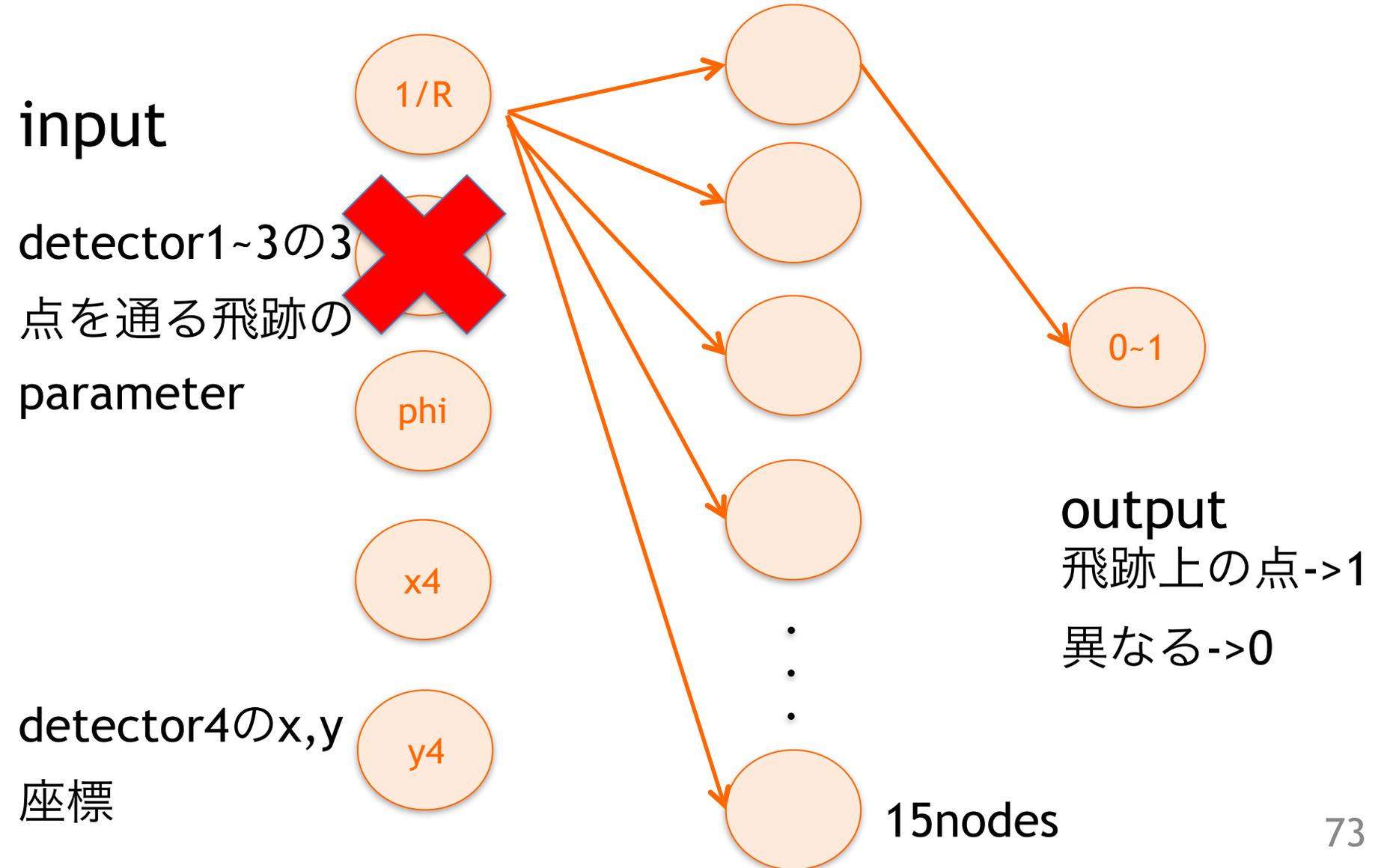


true-cal parameters

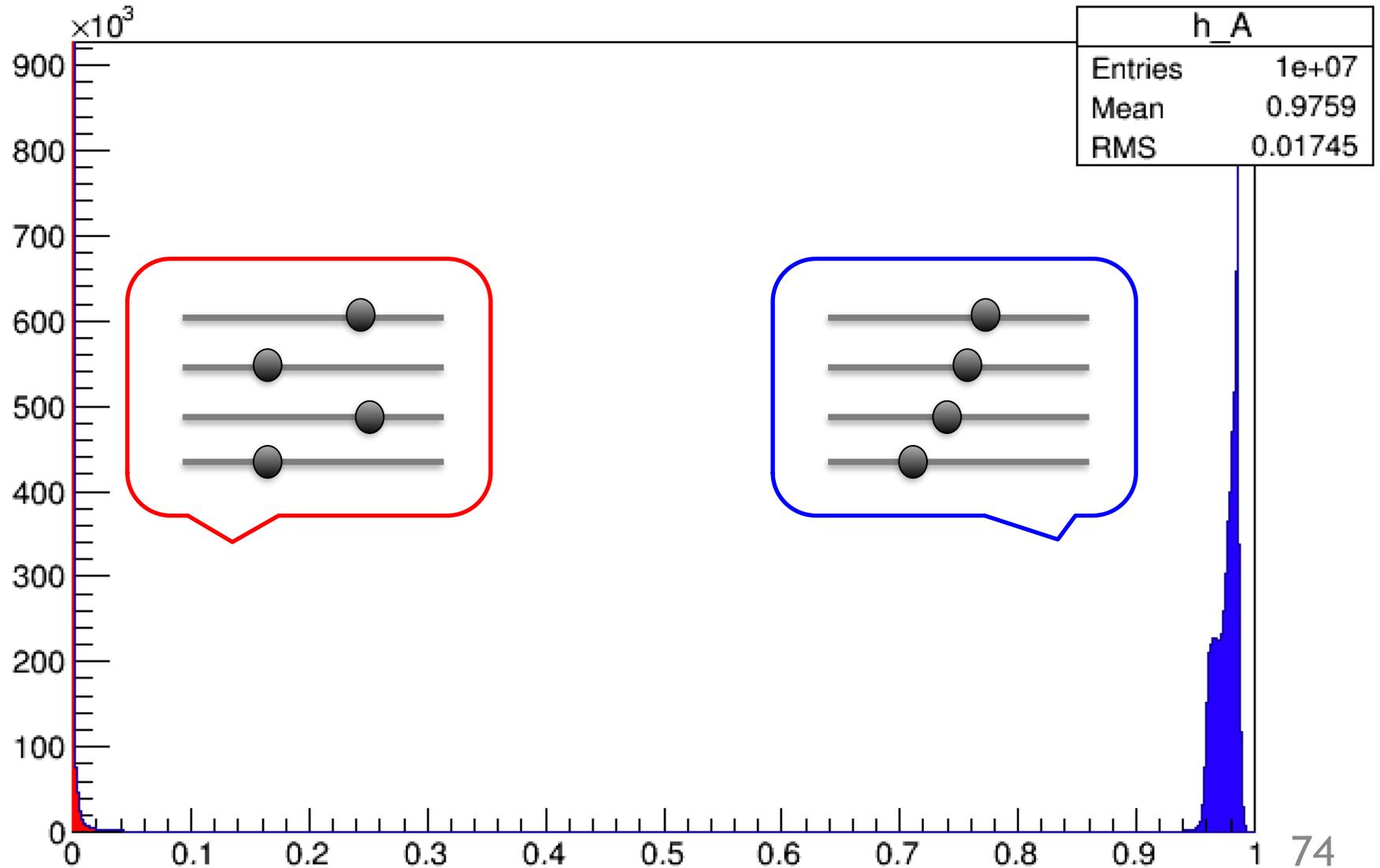
calculated



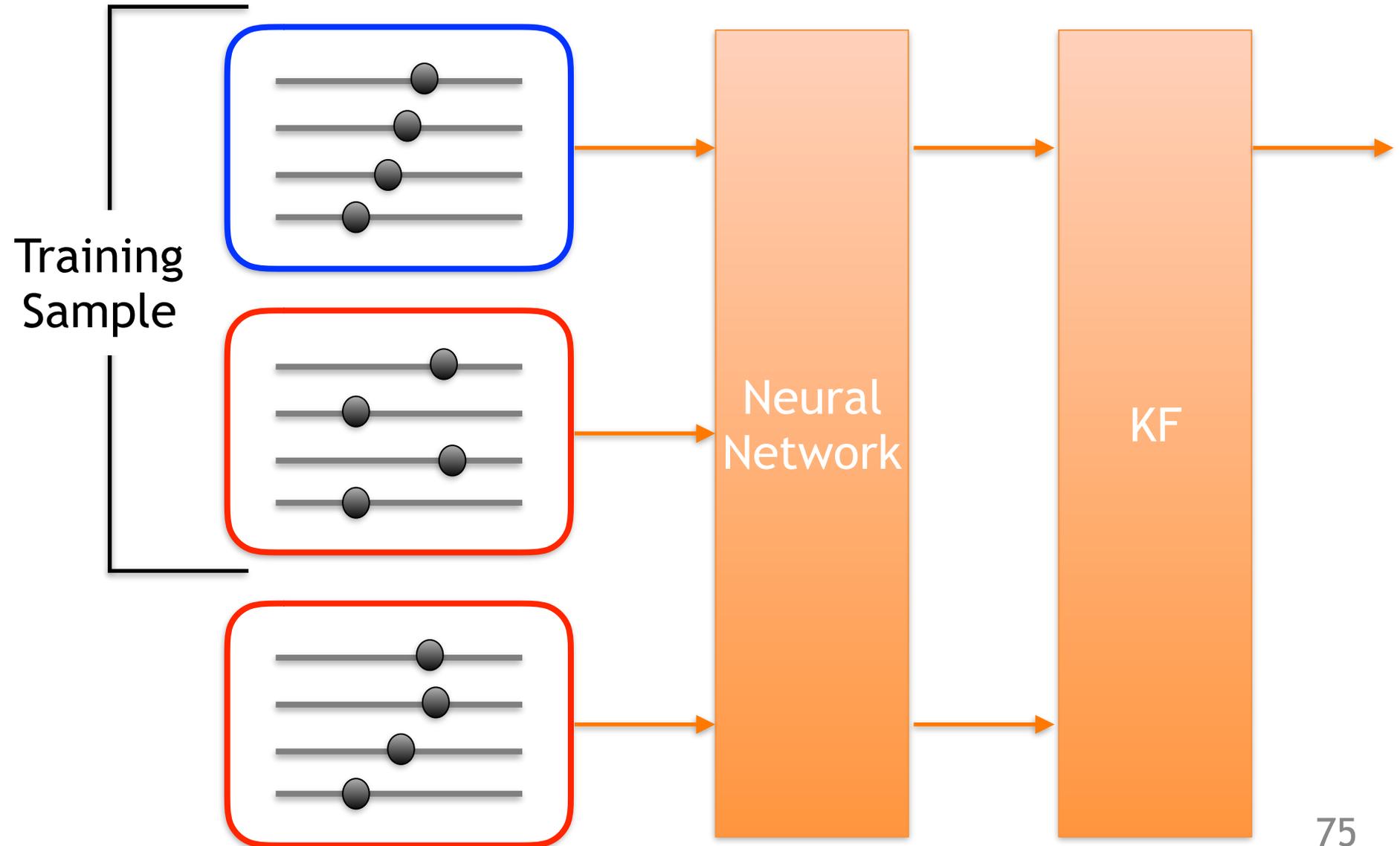
Hit on Track Net without D



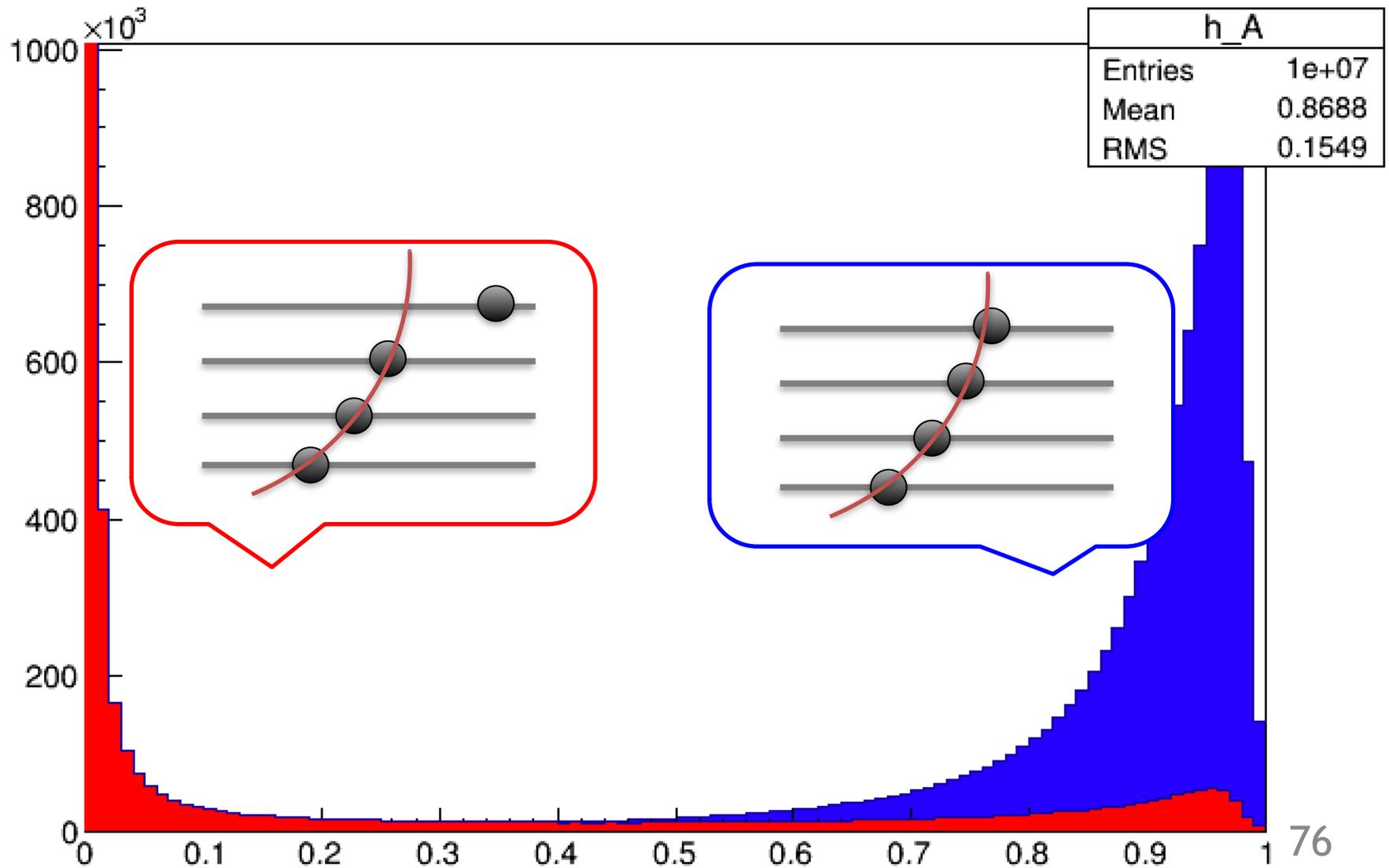
Training Result



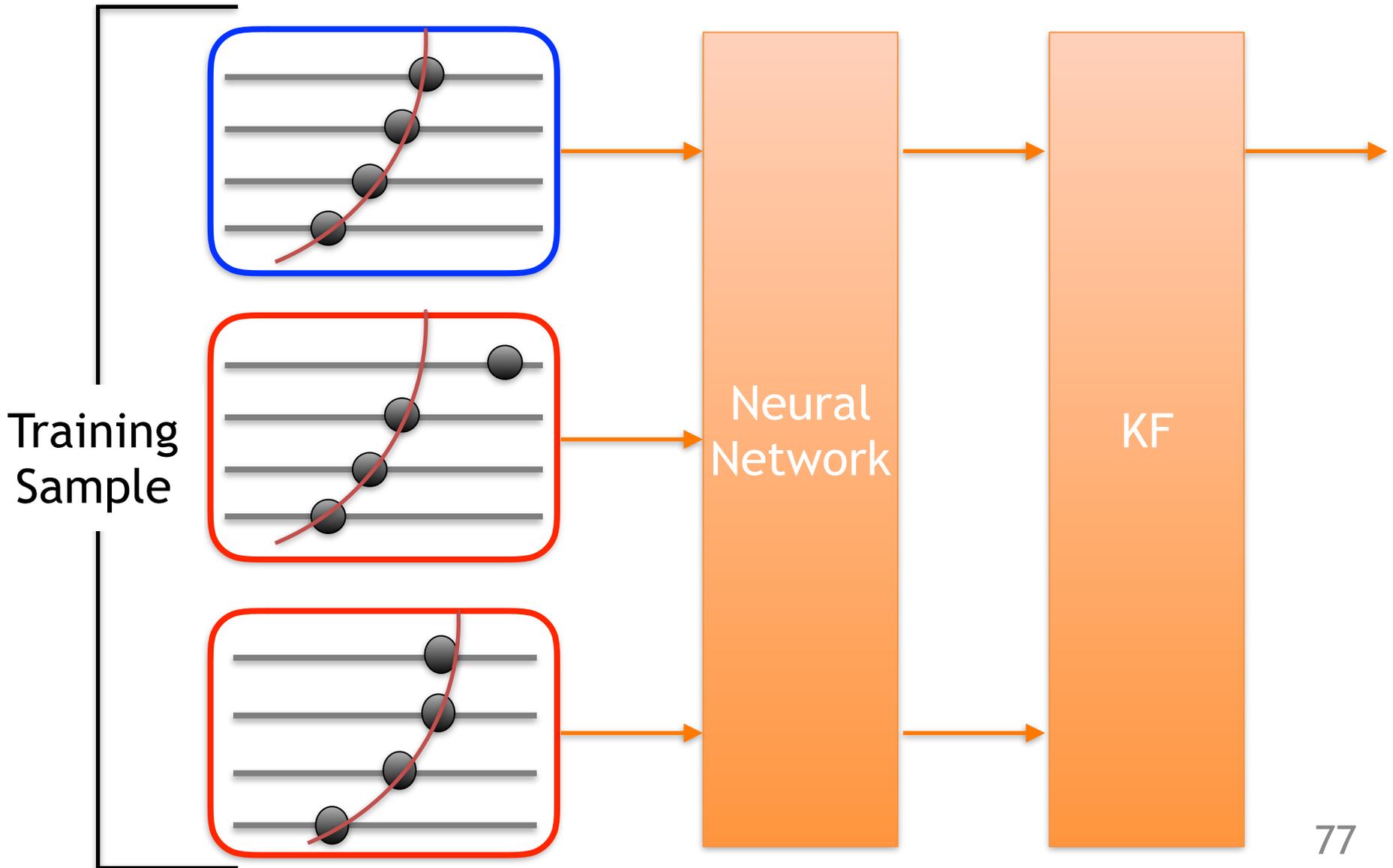
Use of 4 Hits Net



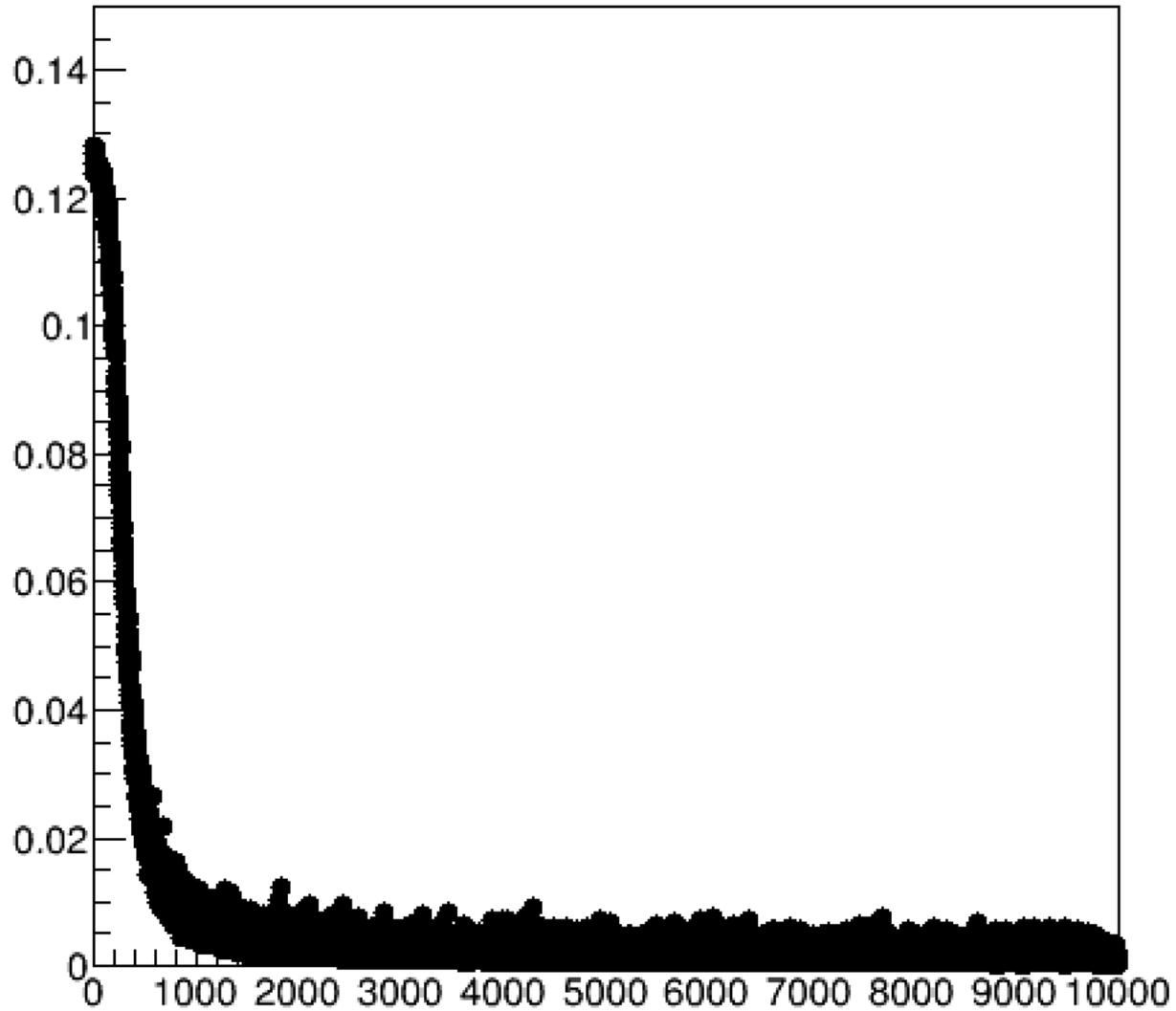
Training Result



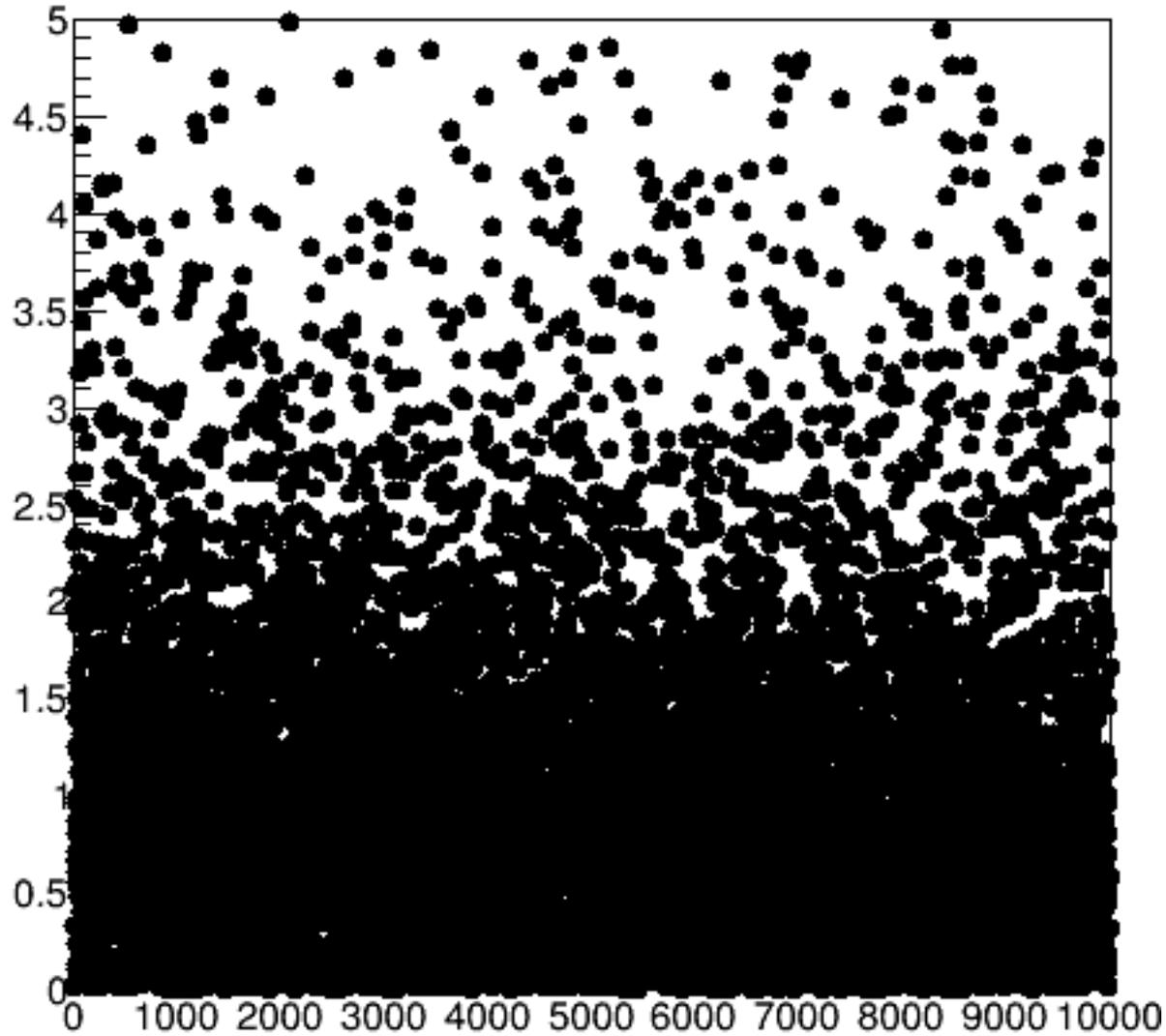
Use of Hit on Track Net



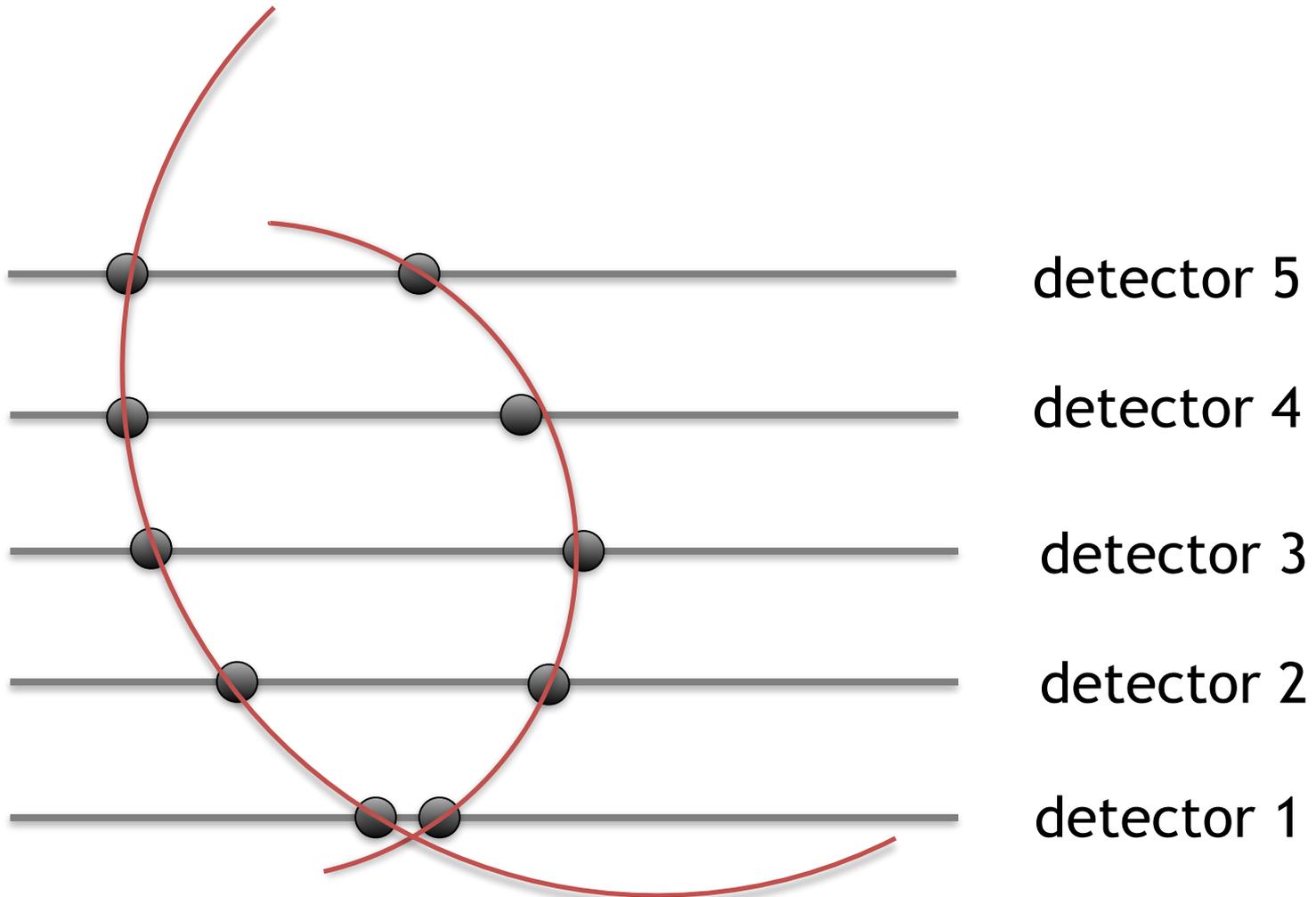
Loss Function



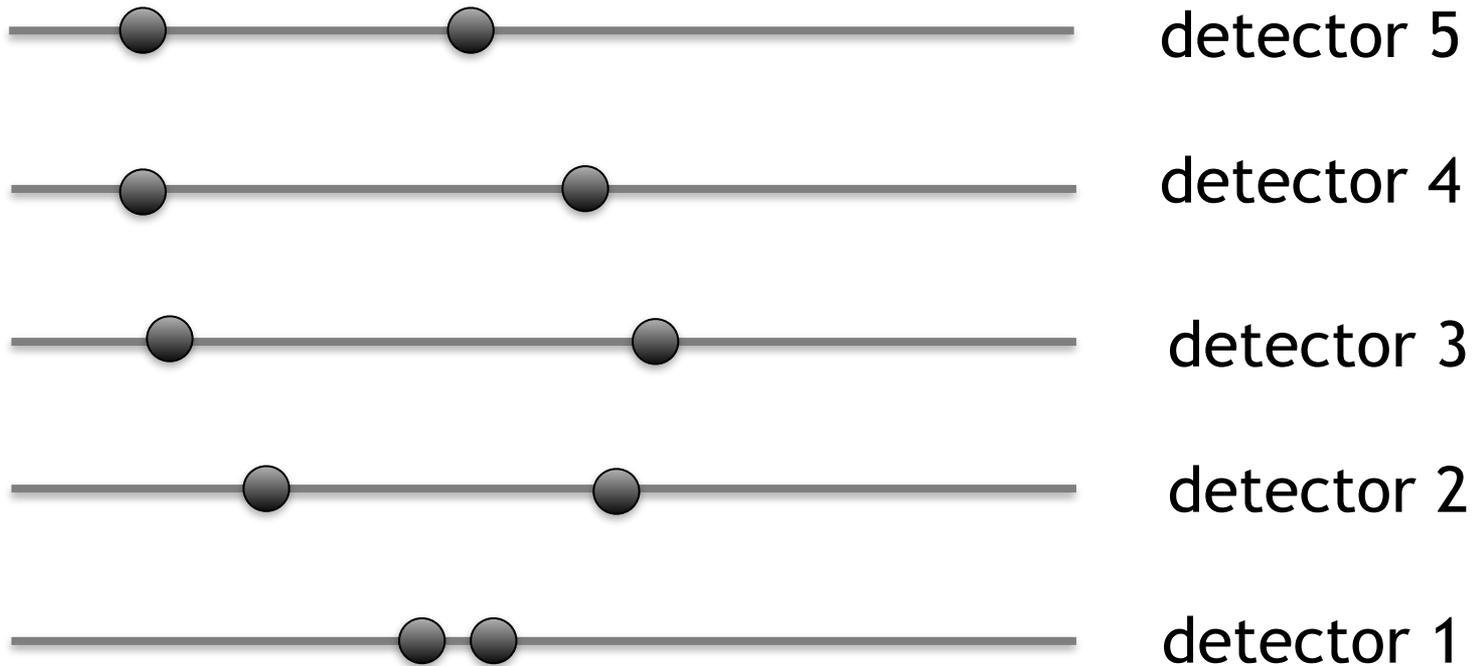
Loss Function



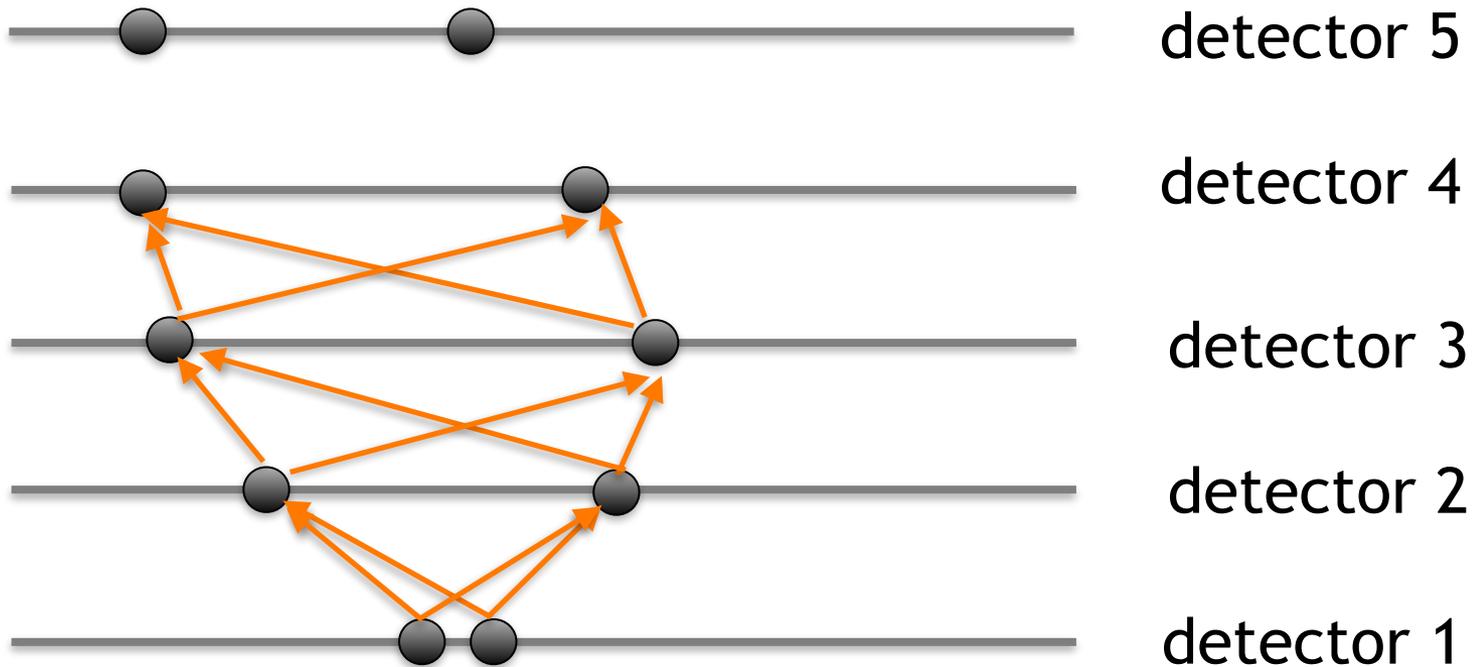
Calculation Methods



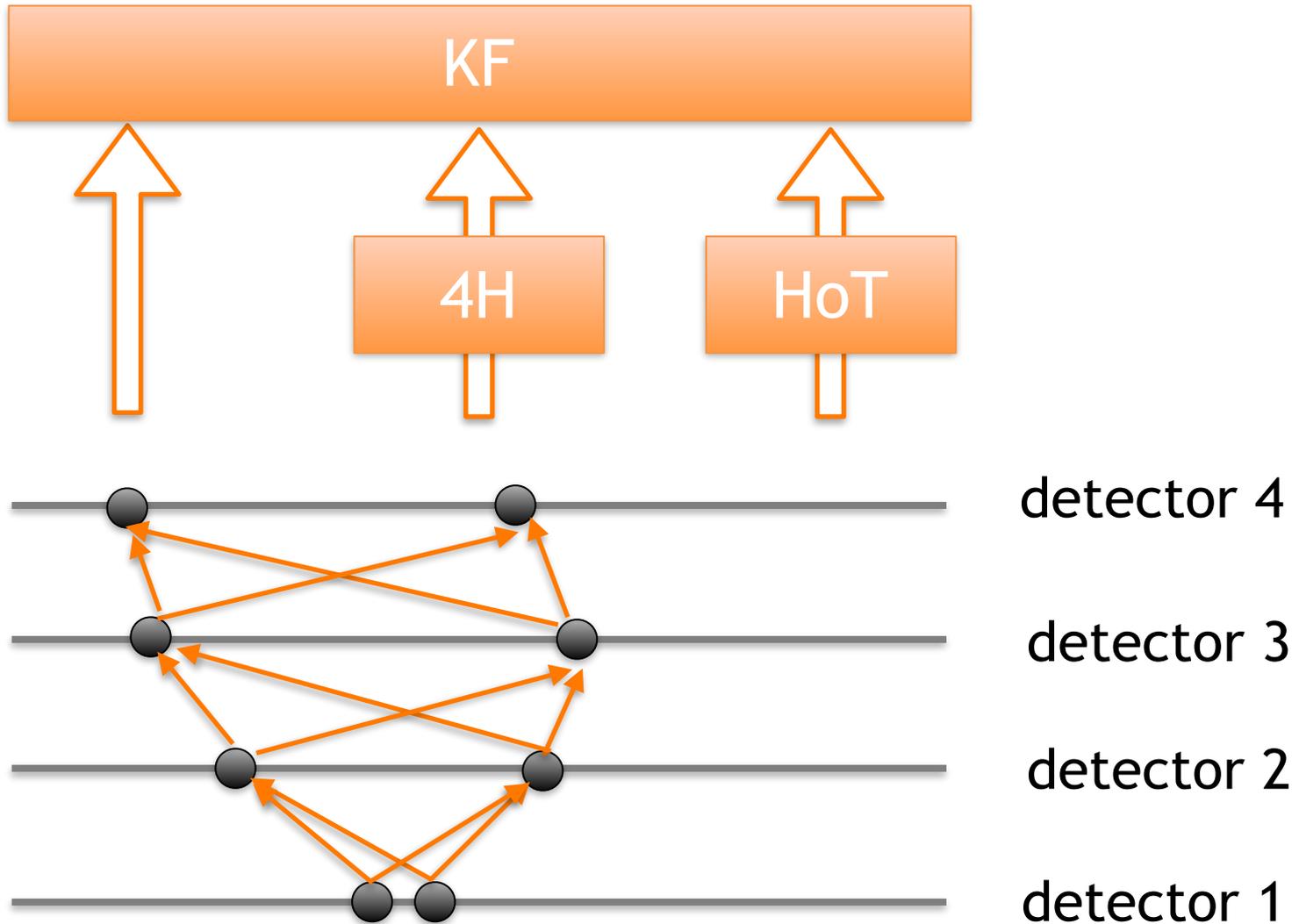
Calculation Methods



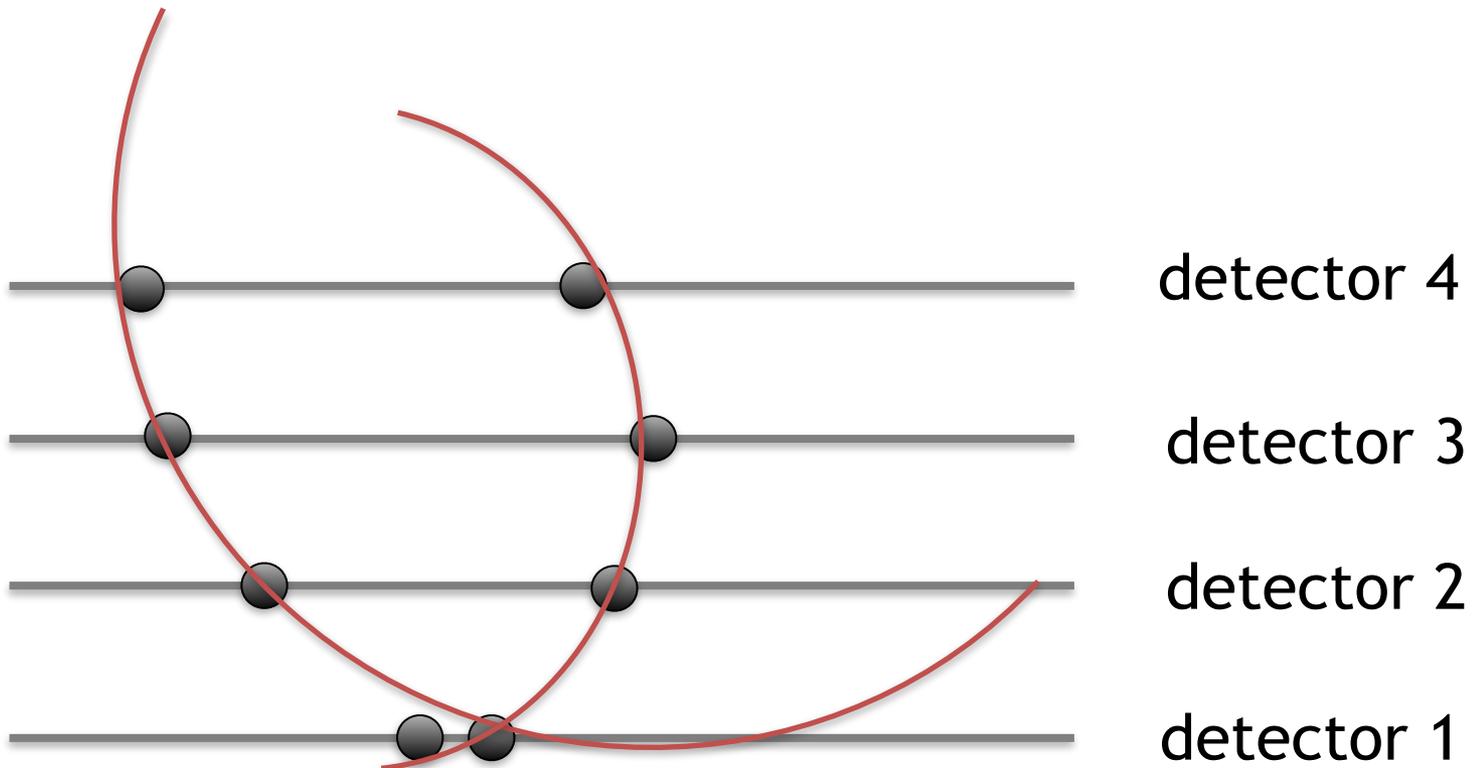
Calculation Methods



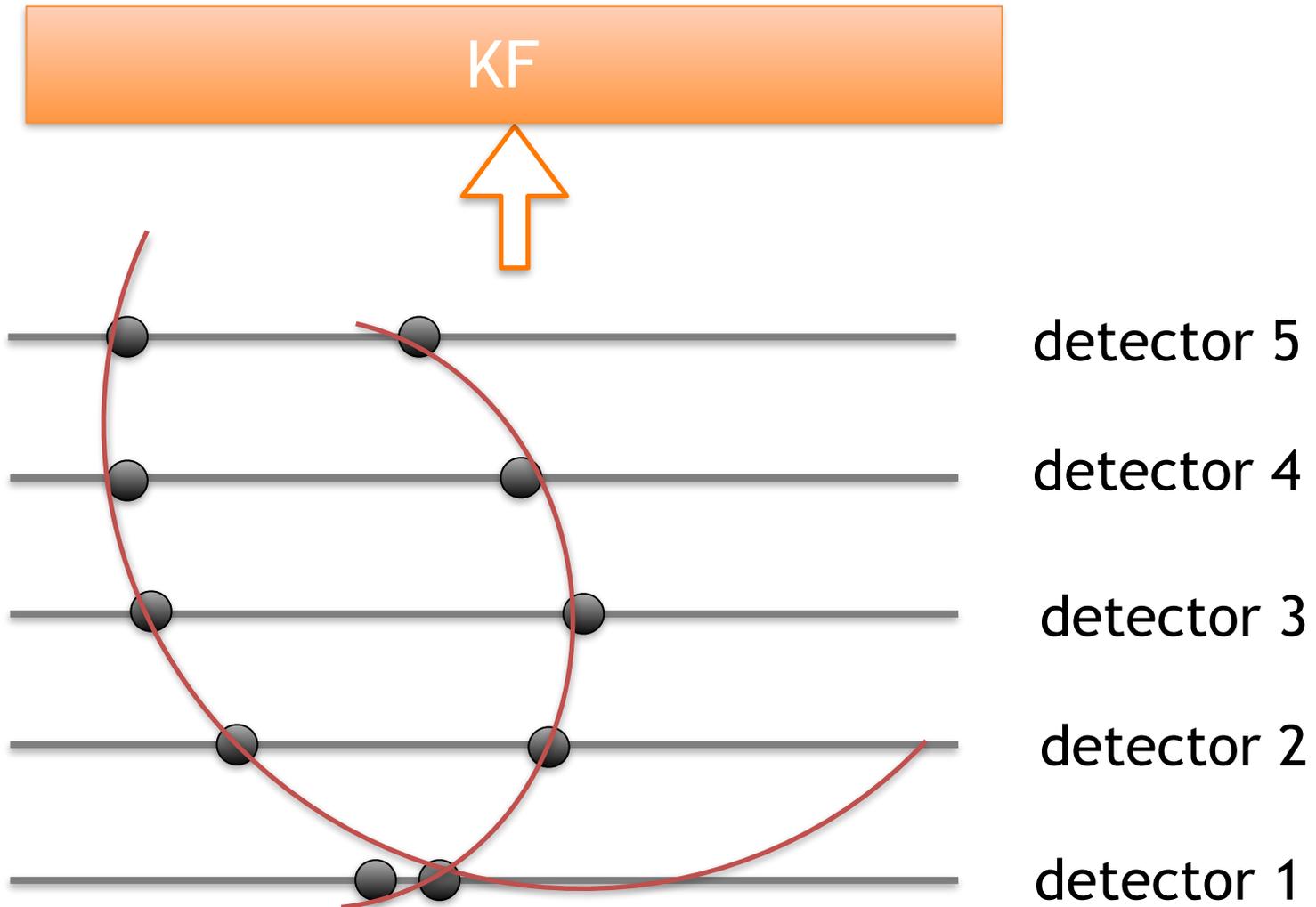
Calculation Methods



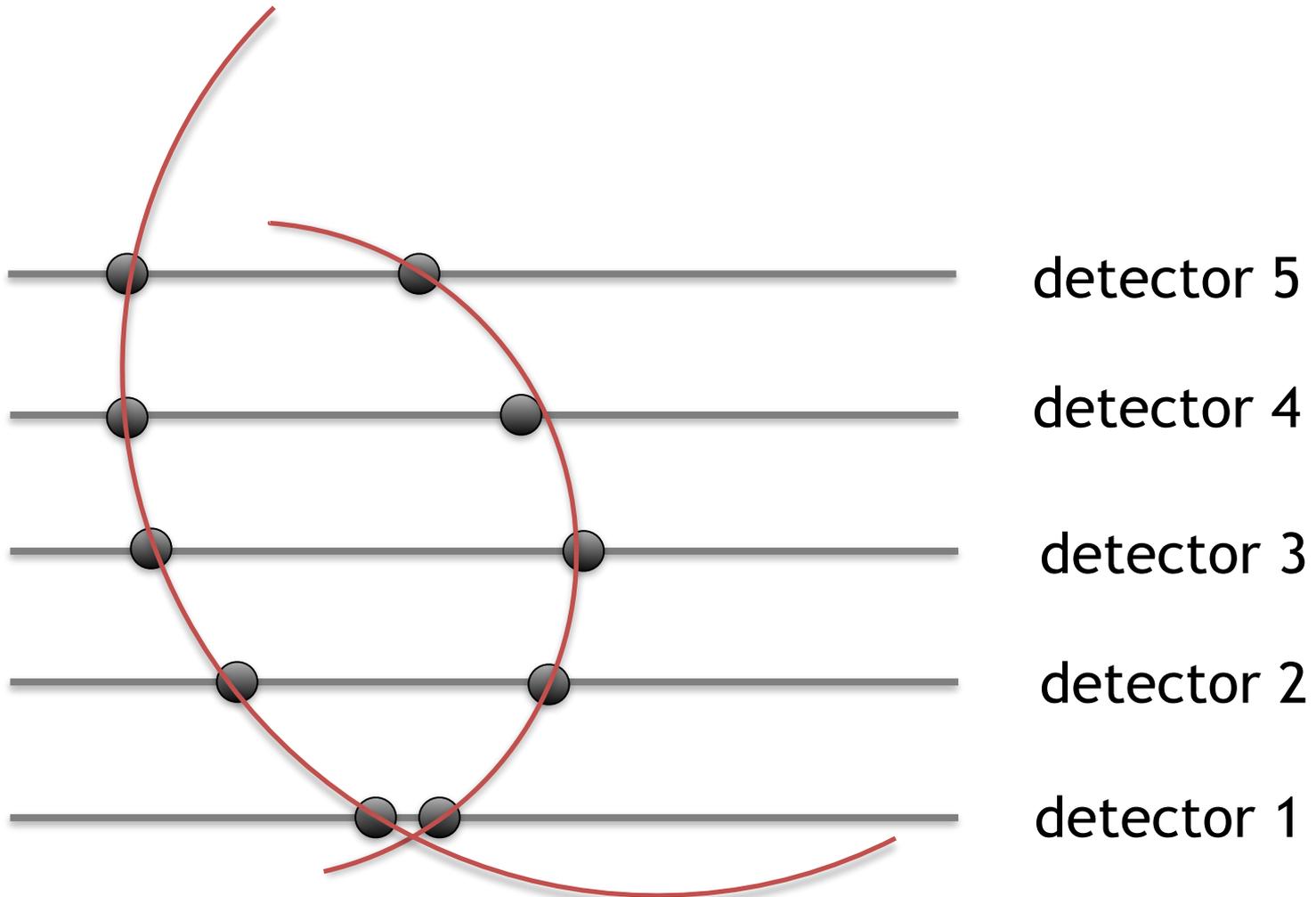
Calculation Methods



Calculation Methods



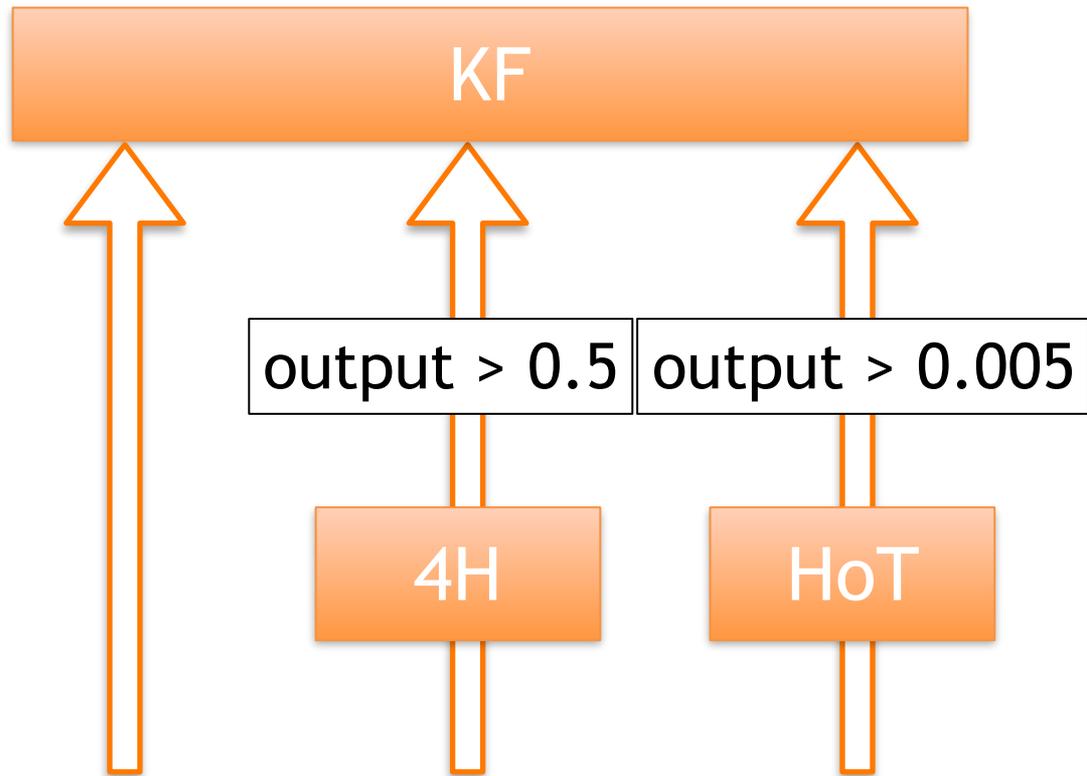
Calculation Methods



Calculation Methods

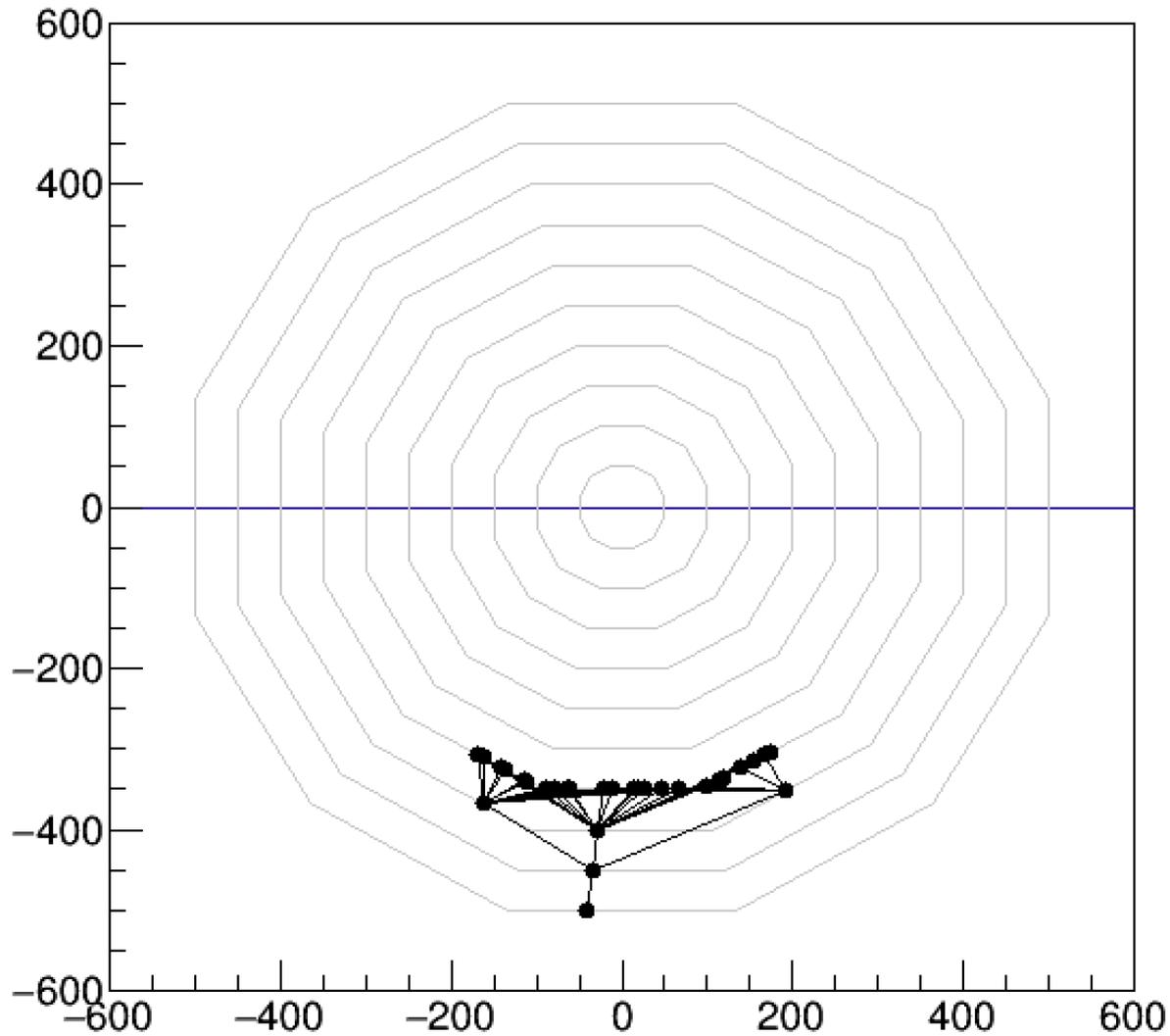
飛跡を計算

detector1~4のhitsの
組み合わせが同一飛
跡に乗るか判定



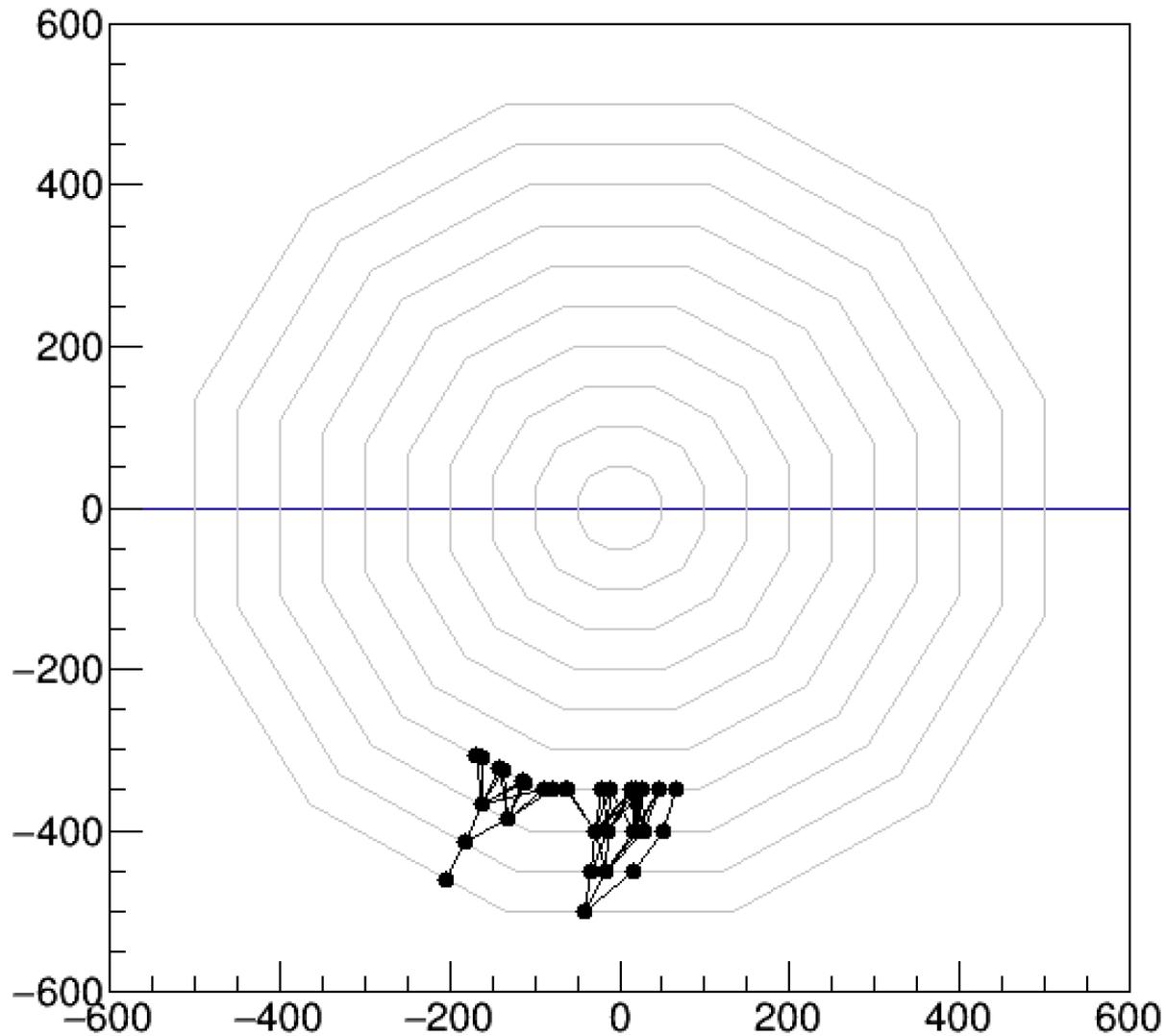
KF Inputs for 50 tracks

#50



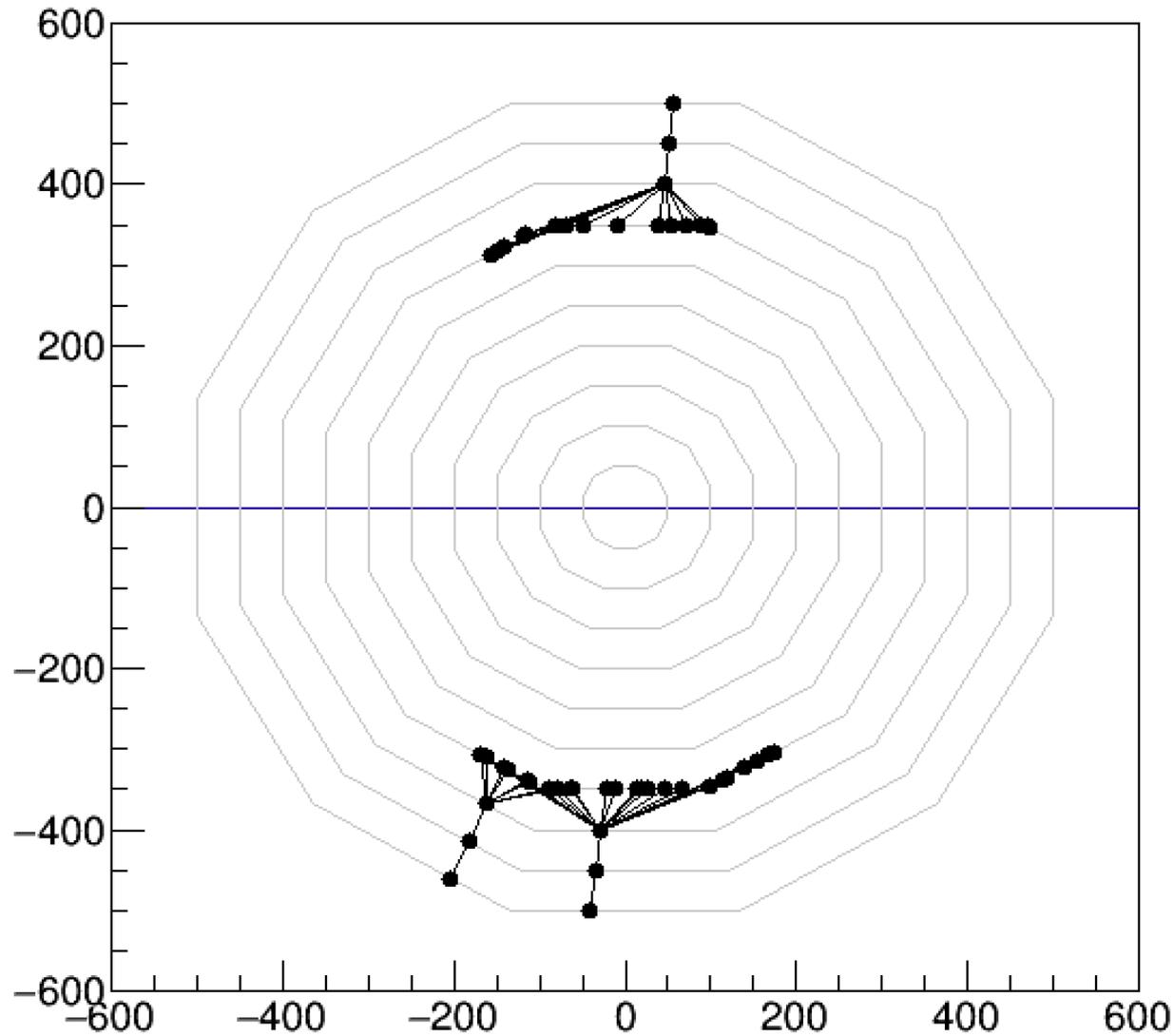
KF Inputs for 50 tracks

#50
with 4H



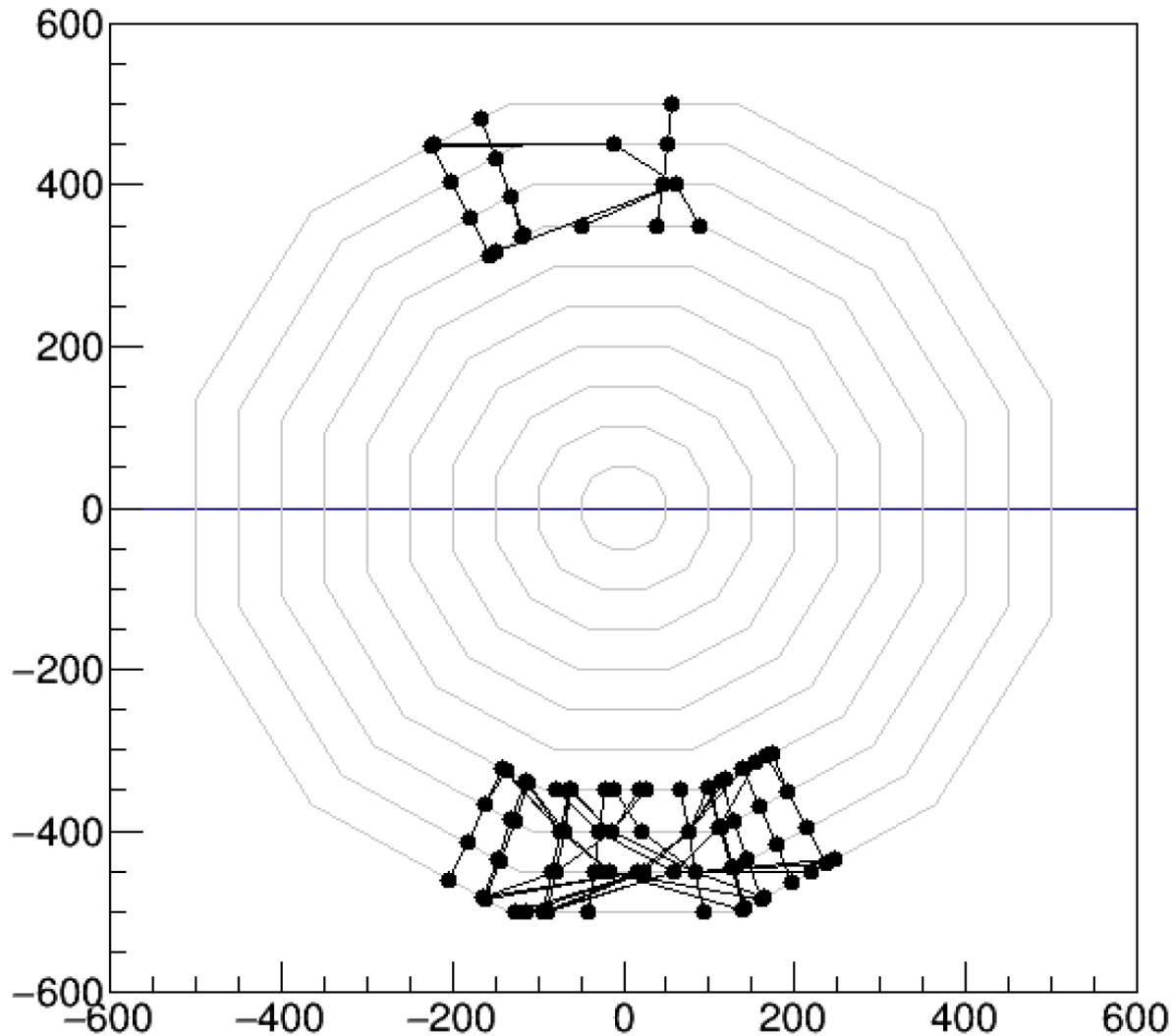
KF Inputs for 50 tracks

#50
with HoT



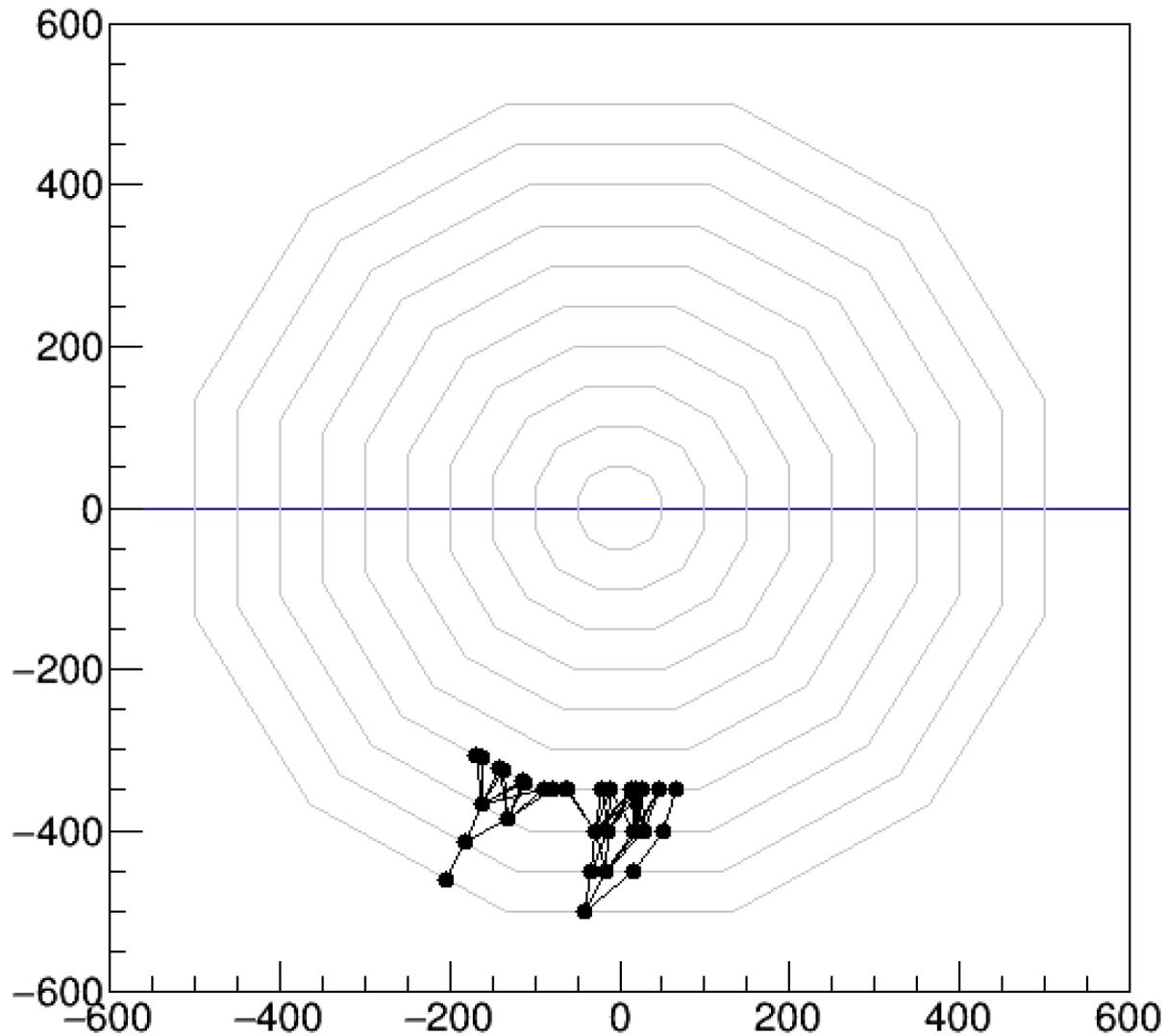
KF Outputs for 50 tracks

#50

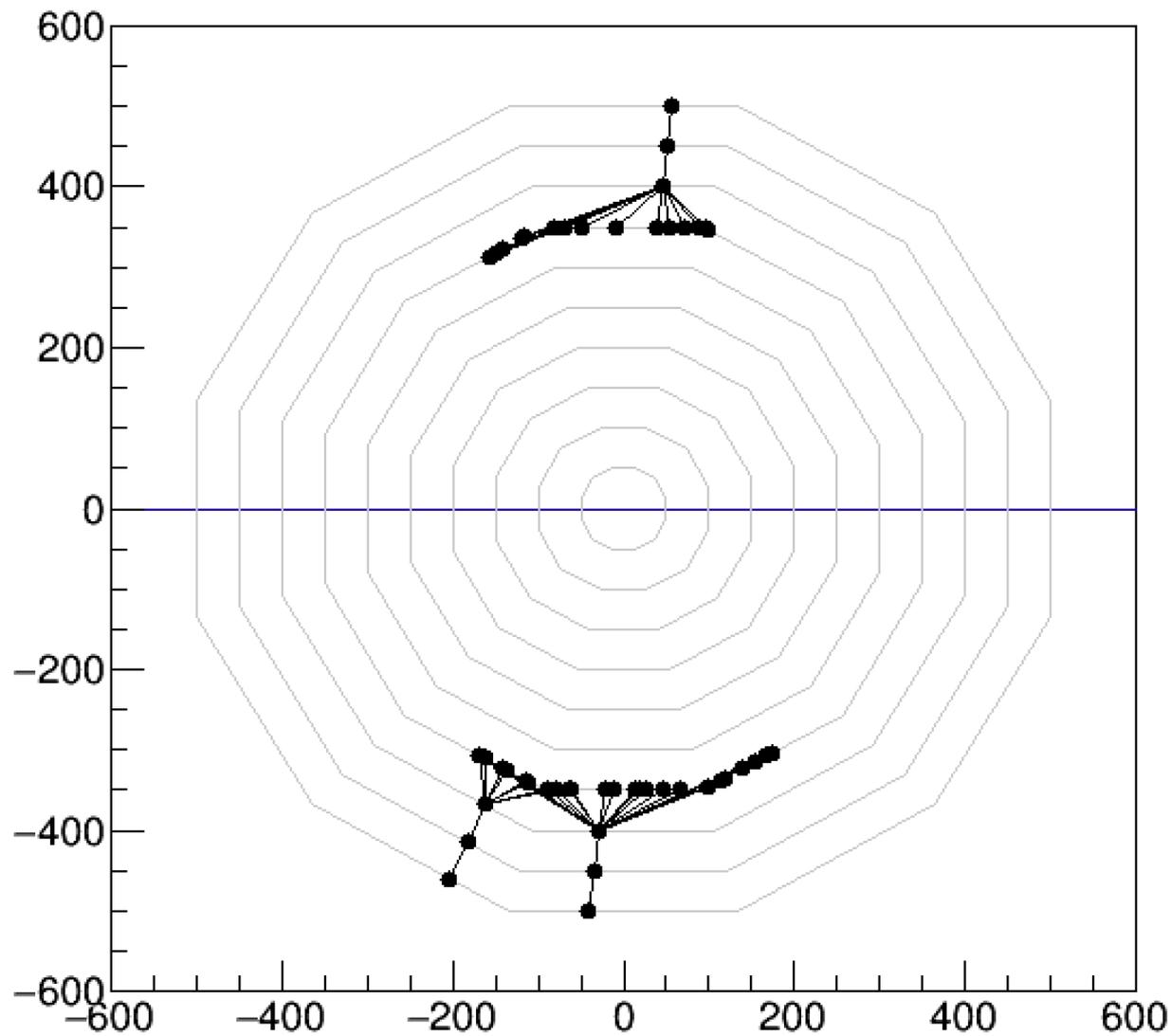


4H Outputs for 50 tracks

#50



HoT Outputs for 50 tracks



流れ

- 3点でparameter初期化
 - $|D| > 10$ をカット
 - 点を加えてparameter更新 ← $v > 10\sigma$ をカット
 - $v > 10\sigma$ をカット
 - parameter収束判定
 - $|D| > 10$ をカット
 - $S/n < 10 \rightarrow$ OK
 - 加えたdataの $v > 10\sigma \rightarrow$ 加えずに次のdetectorへ
-