Propagation of Ultrahigh Energy Neutrinos in the Black-Body Neutrino Field

Shigeru Yoshida

Department of Physics, Tokyo Institute of Technology, Tokyo 152, Japan

(To be published in Astroparticle Physics)

ABSTRACT

We discuss the propagation of ultrahigh energy (UHE) neutrinos with energies ranging up to the grand unification (GUT) scale. In this energy range, the Universe becomes opaque for UHE neutrinos due to interactions with black-body neutrinos (ν_{bb}) and the UHE neutrinos propagate cascading if the sources produce neutrinos at high redshift epochs ($z \gg 10$). The absorption due to the energy loss and the cascade development in the black-body neutrino field may lead to a pile-up and a cut-off in the the energy spectrum of UHE neutrinos emitted at an early epoch. However, these modifications will be smeared if many sources distributed universally from early epochs to the present contribute to the bulk of the UHE neutrinos. In this case, the main effect of the cosmic neutrino background on the UHE neutrino spectrum is to change the spectral slope and knowledge of the primary energy distribution of UHE neutrinos and the source evolution in the Universe is indispensable for reliable inferences of the presence of the cosmic neutrino background from observations of the UHE neutrino spectrum.

1. INTRODUCTION

The idea of production of UHE particles in the present Universe due to the annihilation or collapse of topological defects (TDs) such as monopoles, cosmic strings, etc., has been discussed recently [1-3]. The maximum energy of the particles produced from TDs can reach the typical GUT energy scale of $\sim 10^{15}\ GeV$. The produced particles are in the form of supermassive gauge bosons and Higgs bosons which are usually referred to as X-particles. The X-particles can decay into quarks, gluons, leptons, etc., which materialize into UHE nucleons, photons, and neutrinos with energies up to the GUT scale. This suggests that it is possible that UHE neutrinos with energies of $\sim 10^{15}\ GeV$ exist in the Universe.

If UHE neutrinos of $\sim 10^{15}\ GeV$ exist, their interactions with the black-body neutrinos cannot be neglected because the collision energy can reach the mass of the Z_0, $\sim 90\ GeV$, and the cross section rapidly increases due to the resonance effect. The secondary products of muons, tauons, and pions emit neutrinos through their decay processes and further contribute to the neutrino "cascade". Therefore, the existence of black-body neutrinos would modify the energy distribution of UHE neutrinos during their propagation. This suggests that black-body neutrinos can be "searched" for through the spectral shape of UHE neutrinos. Bhattacharjee, Hill, and Schramm derived UHE neutrinos spectra from TDs taking into account the absorption of neutrinos due to interactions with black-body neutrinos [2]. However, their calculation took into account only the absorption due to black-body neutrinos and neglected the effect of the neutrino cascade. In this paper, we study the propagation of UHE neutrinos with energies up to the GUT energy scale taking into account the neutrino cascade produced by interactions with black-body neutrinos and examine the resulting modifications in the energy spectrum of UHE neutrinos. We also discuss that possibility that observations of these modifications could provide the first observational evidence of the existence of the black-body neutrinos.

2. UHE NEUTRINO CASCADES IN THE UNIVERSE

The most important channel of the interaction of UHE neutrinos in the ν_{bb} field (present temperature $1.9\ K$) is the coupling of neutrinos and anti-neutrinos through Z_0 in the s-channel, which has resonant behavior. Schematically,

$\nu_e\bar{\nu}_e \rightarrow e^+e^-, \quad \mu^+\mu^-, \quad \tau^+\tau^-, \quad \nu\bar{\nu}, \quad q\bar{q}$
$\nu_\mu\bar{\nu}_\mu \rightarrow ....$
$\nu_\tau\bar{\nu}_\tau \rightarrow ....$

The cross section is

$$\sigma(s) = \frac{G_w^2}{3\pi}\frac{M_Z^4}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2}s(C_V^2+C_A^2), \tag{1}$$

where s is the Lorentz invariant parameter, G_w is the weak-interaction coupling constant, M_Z is the mass of the Z_0, Γ_Z is the decay width of the Z_0, and C_V and C_A are the vector and pseudovector coupling constant respectively, whose values depend on the produced fermions.

The t-channel $W^\pm$ exchange also produces leptons. This interaction is most important channel for coupling of neutrinos with different flavors. In Fig.1 are shown cross sections of $\nu_\mu\bar{\nu}_\mu \rightarrow e^+e^-$ (Z_0 exchange in the s-channel) and $\nu_e\bar{\nu}_\mu \rightarrow e^-\mu^+$ (W exchange in the t-channel) as a function of the total center-of-mass energy $\sqrt{s}$. It is seen that the $W-t$ process becomes important at energies beyond and below the Z_0-peak.

Muons, tauons produced by these processes emit neutrinos through their decay process. Quarks produced by the process of the s-channel Z_0 exchange fragment and produce jets of hadrons. Most of the hadrons in the jets are pions which emit UHE neutrinos. We use the following hadronic fragmentation formula (Hill, Schramm, and Walker 1987 [3]):

$$\frac{dN_h}{dx} \simeq 0.08\exp[2.6\sqrt{\ln(1/x)}](1-x)^2[x\sqrt{\ln(1/x)}]^{-1}. \tag{2}$$

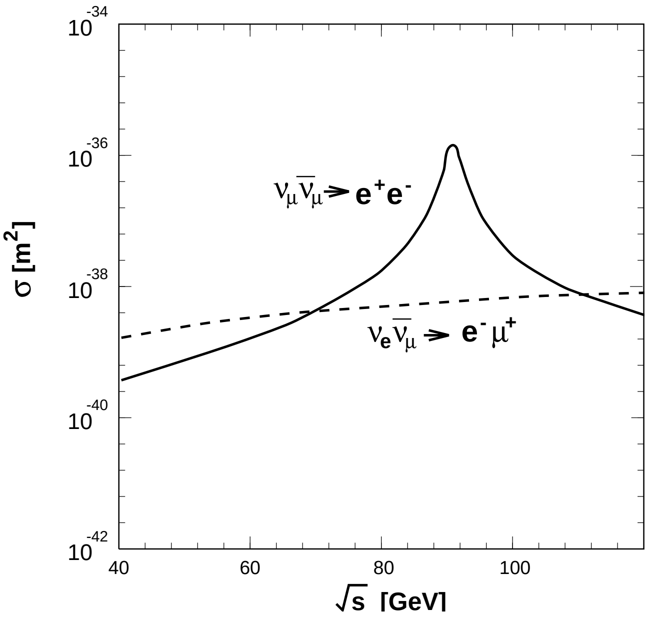

Figure 1: Cross sections for $\nu_\mu \bar{\nu}_\mu \rightarrow e^+e^-$ (Z_0 exchange in the s-channel) and $\nu_e \bar{\nu}_\mu \rightarrow e^-\mu^+$ (W exchange in the t-channel) as a function of the total center-of-mass collision energy.

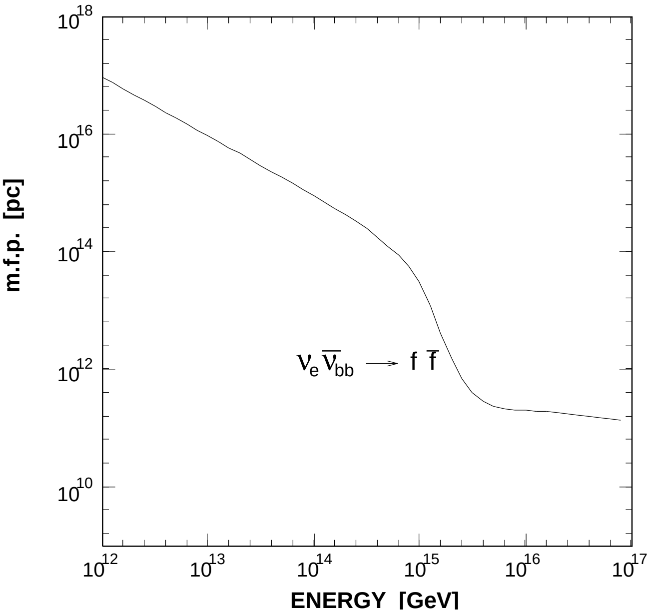

Figure 2: The mean free path of UHE electron neutrinos for collision with ν_{bb} in the present Universe. We assume that all neutrinos are massless Dirac neutrinos.

Here, $x = E/E_{jet}$ is the energy of hadron in the jet and N_h is the number of hadrons with a fraction, x, of the energy in the jet, E_{jet}. We assume that 97% of the hadrons in the jet are pions (see refs.[1,2]). The UHE neutrinos propagate "cascading" via these processes if the mean free path is much shorter than the propagation length. The mean free path is given by

$$\lambda = c\left(\frac{1}{4\pi}\int ds\sigma(s)\int d\Omega(1+\cos\theta)\frac{dn_{\nu_{bb}}}{dE_{\nu_{bb}}}(s,\theta)\frac{dE_{\nu_{bb}}}{ds}\right)^{-1}, \tag{3}$$

where θ is the collision angle between the UHE neutrinos and ν_{bb}, and $n_{\nu_{bb}}$ is the number density of the black-body neutrinos. s can be expressed in the Cosmic Ray Laboratory Frame as

$$s = 2E_\nu E_{\nu_{bb}}(1+\cos\theta), \tag{4}$$

where E_ν is the energy of the UHE neutrino. The mean free path of UHE neutrinos for $\nu\bar{\nu_{b}}_{b} \rightarrow f\bar{f}$ is shown in Fig.2. The effect of the Z_0 resonance decreases the mean free path above 10^{15} GeV to $\sim 10^{11}$ pc. This path is much longer than the radius of the present Universe, however, the evolution of the ν_{bb} field shortens the mean free path. For example, at the epoch when the redshift $z \simeq 100$, the mean free path of UHE neutrinos becomes $\sim 100kpc$ at $10^{14}GeV$. This suggests that the UHE neutrinos emitted from sources at high redshift epochs form cascades of neutrinos.

In the following calculation for the neutrino cascades, we take into account almost all relevant contributions to interactions of UHE neutrinos in the black-body neutrino field shown as follows:

$\nu_i\bar{\nu}_i \rightarrow f\bar{f}$	(s-channel Z_0 exchange)
$\nu_i\bar{\nu}_j \rightarrow \nu_i\bar{\nu}_j$ $(i \neq j)$	(t-channel Z_0 exchange)
$\nu_i\bar{\nu}_j \rightarrow l_i\bar{l}_j$ $(i \neq j)$	(t-channel $W^\pm$ exchange)
$\nu_i\bar{\nu}_i \rightarrow l_i\bar{l}_i$	(s-channel Z_0 and t-channel $W^\pm$ exchange and their interference)
$\nu_i\bar{\nu}_i \rightarrow \nu_i\bar{\nu}_i$	(s-channel and t-channel Z_0 exchange and their interference)
$\nu_i\nu_j \rightarrow \nu_i\nu_j$ $(i \neq j)$	(t-channel Z_0 exchange)
$\nu_i\nu_i \rightarrow \nu_i\nu_i$	(u-channel and t-channel Z_0 exchange and their interference)

We calculated the cross sections of these processes analytically using approximation that the masses of the produced fermions can be neglected. One can find their explicit expressions in Ref.[17] for example. The annihilation into Z_0Z_0, W^+W^- and Z_0H were neglected for simplicity because of their relatively low cross sections. All neutrinos were assumed to be massless Dirac particles.

To calculate the propagation of UHE neutrinos in the ν_{bb} field taking into account cascading of the neutrinos we use the following transport equation:

$$\begin{aligned}\frac{dN_i}{dL}(k_i,z) = -\frac{N_i(k_i,z)}{\lambda_i(k_i,z)} + \sum_{j=\nu_e,\nu_\mu,\nu_\tau}\int_{k_i}^{k_i^{max}} dk_j^{'}N_j(k_j^{'},z)\frac{dN_{j\rightarrow i}(k_j^{'},k_i,z)}{dk_i dL} \\ +\frac{d}{dk_i}[H_0(1+z)^{\frac{3}{2}}k_iN_i(k_i,z)], \qquad i = \nu_e,\nu_\mu,\nu_\tau\end{aligned} \tag{5}$$

where k_i is the energy of UHE neutrino of type i, N_i is the number of UHE neutrinos with energy k_i at the epoch of redshift z, λ_i is the mean free path as a function of k_i and z, $N_{j\rightarrow i}$ is the number of UHE neutrinos of type i produced by the interactions of type j neutrinos with energies of k_j, H_0 is the Hubble constant, and L is the propagation length. We assume that our Universe is an Einstein-de Sitter type and dL is related to the redshift z as

$$dL = -\frac{1}{H_0}\frac{dz}{(1+z)^{\frac{5}{2}}}. \tag{6}$$

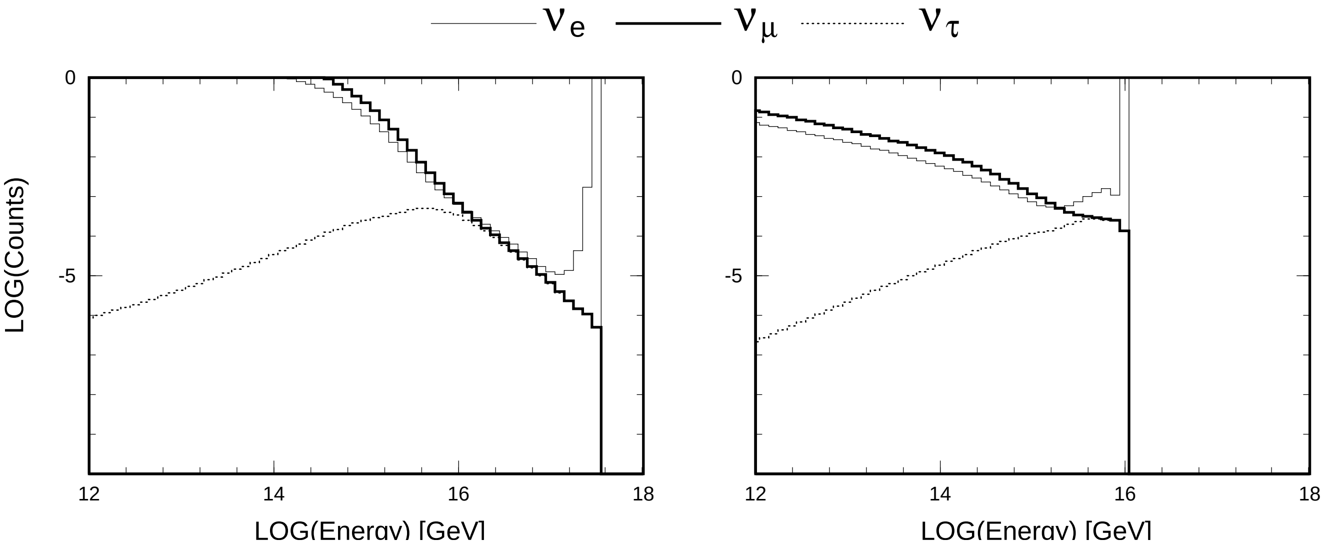

Figure 3: The energy distribution of UHE neutrinos in the cascade after propagation through $1Gpc$ in the present Universe. The primary input spectra are electron neutrinos with energies of 3×10^{17} GeV and 10^{16} GeV respectively.

The last term on the right hand side of Eq.(5) represents the adiabatic loss due to the redshift. The interaction term $N_{j \to i}$ is calculated by the differential cross sections and the energy distribution of the decay products of π's, μ's etc.. In terms of the Lorentz invariant parameter s, the energy distribution of the secondary neutrinos from the collision is obtained by

$$\frac{dN_{j \to i}}{dLdk_i} = \frac{1}{4\pi} \int \frac{dn}{dk_i}(E_{rec}, k_i) dE_{rec} \int \frac{d\sigma}{dE_{rec}}(s) ds \int (1 + \cos\theta) \frac{dn_{\nu_{bb}}}{dE_{\nu_{bb}}}(s, \theta) \frac{dE_{\nu_{bb}}}{ds} d\Omega, \tag{7}$$

where E_{rec} is energy of the recoil particles, k_i is energy of the secondary neutrinos of type i, $d\sigma/dE_{rec}$ is the differential cross section of the interaction in the cascade, θ is the collision angle between UHE neutrinos and ν_{bb}, dn/dk_i is the energy distribution of the secondary neutrinos which are produced by the recoil particle from the collision with E_{rec}. In the case of elastic collisions, $dn/dk_i = \delta(k_i - E_{rec})$. Since the energies of UHE neutrinos are much higher than those of ν_{bb}, the energy distribution is scaled approximately:

$$\frac{dN_{j \to i}}{dLdk_i}(k_j, k_i, z) = \frac{1}{k_j} \frac{dN_{j \to i}}{dLd\eta}(k_j, k_j\eta, z) \simeq \frac{1}{k_j} \frac{dN_{j \to i}}{dLd\eta}(k_j, \eta, z), \tag{8}$$

where $\eta = k_i/k_j$.

Fig.3 shows the energy distribution of neutrinos in the cascade after propagation through 1 Gpc. The input spectra are electron neutrinos with energies of 10^{16} GeV and 3×10^{17} GeV respectively. The pions in the jets initiated by the quarks produced in the cascade are the main supplier of the bulk of the neutrinos in the lower energy region. The amount of secondary muon neutrinos is about twice that of electron neutrinos, as expected when the decay processes of charged pions are considered. The tail of the distribution of the secondary neutrinos are mainly determined by the hadronic fragmentation formula dn_h/dx expressed in Eq.(2).

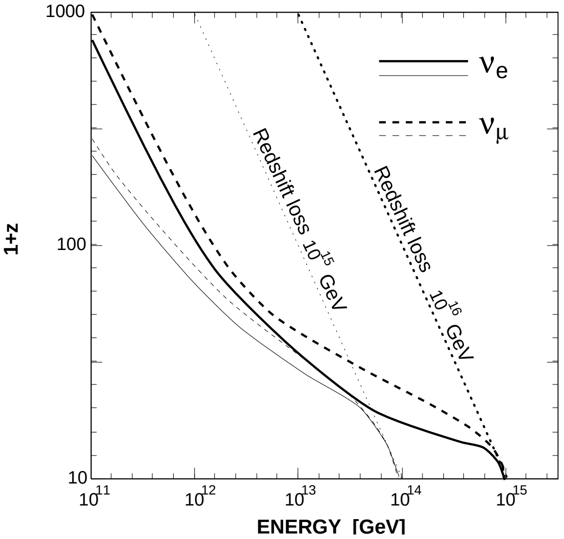

Figure 4: The horizon of the Universe for UHE neutrinos as a function of present-day neutrino energies. The primary energies are 10^{15} GeV (thin lines) and 10^{16} GeV (thick lines). The solid lines show electron-neutrinos and the dashed lines show muon-neutrinos. The dotted lines show the upper bounds of the horizon taking into account the energy loss due to redshift.

To illustrate the effect of the neutrino cascade on the propagation of UHE neutrinos, let us determine the maximum redshift up to which neutrinos are not attenuated in their propagation (referred to as the "horizon of the Universe for neutrinos" hereafter). For a primary energy spectrum at a given epoch z_a with a monochromatic energy k_0 which is represented by $dN_i/dk_i(z_a) = N_0\delta(k_i - k_0)$, the present-day energy distribution of neutrinos after propagation, $dN_i/dk_i(0)$, is calculated by Eq.(5). The effective cutoff energy k is defined from

$$\int_k^{k_0} dk_i \frac{dN_i}{dk_i}(0) = N_0 \exp(-1). \tag{9}$$

We consider the redshift z_a as the horizon of the Universe for neutrinos with present-day energy k for a primary energy k_0. Fig.4 shows the curves of the horizon as a function of the present-day neutrino energies. The primary energies are 10^{15} GeV and 10^{16} GeV respectively. The upper bounds of the horizon determined from the redshift energy loss are also shown for comparison. It is found that the energy loss effect due to the interactions with the black-body neutrinos contracts the horizon. This means that the distant sources at very high redshift epoch do not contribute to the bulk of UHE neutrinos. These UHE neutrinos are *absorbed*. However, the bulk of the secondary neutrinos moderate the absorption effect for the propagation from a high redshift epoch when the neutrino cascade develops significantly. As seen in Fig.4, the cascading effect expands the horizon for neutrinos below 10^{12} GeV compared with that obtained by extrapolation from the higher energy region in which the cascade does not develop significantly during propagation. Twice as many muon neutrinos are produced as electron neutrinos by the pion decay processes, and the cascade expands the horizon for muon neutrinos further. The absorption effect due to the energy loss and the cascade development may provide us with a "signature" of the black-body neutrinos in studying the bulk of the UHE neutrinos.

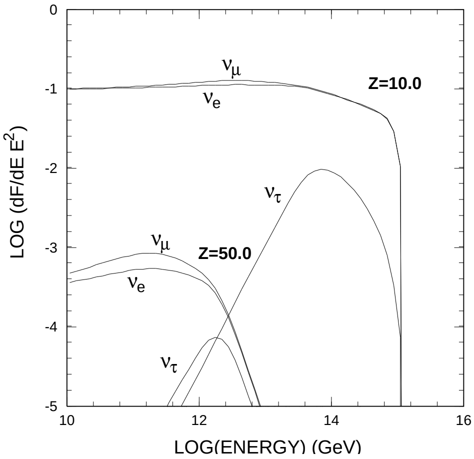

Figure 5: The energy spectra of UHE neutrinos from a single object where the neutrinos are emitted with power spectrum of $\sim E^{-2}$. The flux is in arbitrary units. The primary absolute flux is assumed to be the same for $z = 10$ and $z = 50$.

3. ENERGY SPECTRUM OF UHE NEUTRINOS

The interactions between the UHE neutrinos and the black-body neutrinos described in the previous section would modify the energy spectrum of the UHE neutrinos during their propagation for sources of primary neutrinos at high redshift epochs ($z \gg 10$), which contribute to the bulk of UHE neutrinos. To verify this, a "benchmark" calculation was performed under the simple assumptions that the primary energy spectra of neutrinos are power laws of E^{-2} which consist equally of ν_e and ν_μ and that a single source is responsible for all UHE neutrinos. The resulting spectra calculated by Eq.(5) for this simple case are given in Fig.5. The maximum energy of primary neutrinos is assumed to be 10^{16} GeV. The case of $z = 10$ gives only slight modifications while the case of $z = 50$ gives a cut-off and a small pile-up in the spectrum.

When TDs are destroyed the contained energy is released in the form of supermassive gauge bosons and Higgs bosons (so called "X-particles") with masses given by the typical GUT scale, $\sim 10^{15}$ GeV [4]. The subsequent decays of X-particles produce leptons and quarks. The quarks will hadronize producing jets of hadrons. The decay of the charged pions in the jets can give rise to UHE neutrinos ranging up to an energy on the order of the mass of the X-particles. In our calculation we assume that half the mass of the X-particles is the total energy of the jet (assuming that each X-particle decays into a quark and a lepton of equal energy, and that each quark produces one hadronic jet). The primary energy spectrum of UHE neutrinos resulting from the decay of the pions and muons which are hadronized in the jet can be written as

$$f_{\nu_\mu}(E_\nu) = \frac{2}{m_X}\int_{\frac{2E_\nu}{(1-r_\pi)m_X}}^{1} \frac{dx}{x}\frac{1}{1-r_\pi}\frac{dN_h}{dx} + \frac{2}{m_X}\int_{\frac{2E_\nu}{m_X}}^{1}\frac{dy}{y}\int_y^{\frac{y}{r_\pi}}\frac{dx}{x}\frac{dN_{\nu_\mu}}{dy}(x,y)\frac{1}{1-r_\pi}\frac{dN_h}{dx},$$

$$f_{\nu_e}(E_\nu) = \frac{2}{m_X}\int_{\frac{2E_\nu}{m_X}}^{1}\frac{dy}{y}\int_y^{\frac{y}{r_\pi}}\frac{dx}{x}\frac{dN_{\nu_e}}{dy}(x,y)\frac{1}{1-r_\pi}\frac{dN_h}{dx}, \qquad (10)$$

where $r_\pi = m_\mu^2/M_\pi^2$, m_μ is the muon mass, M_π is the pion mass, m_X is mass of the X-particles, N_{ν_μ}, N_{ν_e} are the numbers of neutrinos from the muons decay processes, and $y =$

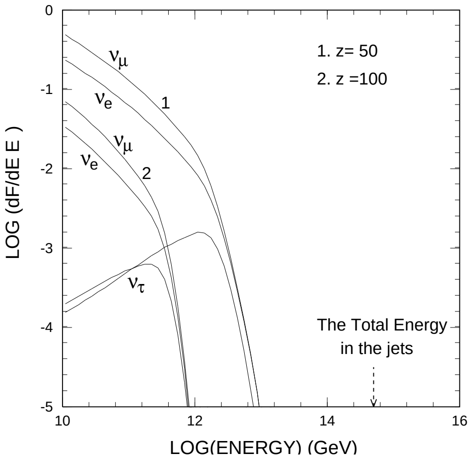

Figure 6: The energy spectra of UHE neutrinos from a single topological defect at z=50 and z=100.The flux is in arbitrary units. The primary absolute flux is assumed to be the same between $z = 50$ and $z = 100$. The total energy in the jet produced by the decay of X-particles is also shown. This energy is the maximum primary energy of UHE neutrinos.

E_ν/E_μ represents the produced neutrino energy normalized by the parent muon's energy. The distribution of the hadronic fragmentation, dN_h/dx, is given by Eq.(2). The energy distribution of neutrinos produced by the muon decay, dN_ν/dy , is derived from the matrix elements of muon decay (see e.g. Ref. [5]). The expected spectra of UHE neutrinos from a single TD after propagation in the ν_{bb} field which are calculated by Eq.(5) are shown in Fig.6. We assume that $m_X = 10^{15}\ GeV$. The cut-off energy of neutrinos depends on the propagation length. All ν_τ are the products of the neutrino cascades during propagation because the primary spectrum consists of only ν_e and ν_μ in this model.

Now we consider the case of the emission of UHE neutrinos from TDs throughout the Universe. X-particles are released throughout the history of Universe and the spectrum is a superposition of those from the early epochs to the present. The rate of production of X-particles is given by (Bhattacharjee, Hill, and Schramm [2])

$$\frac{dn_X}{dz} = \kappa(1+z)^{\frac{7-3p}{2}}, \qquad (11)$$

where κ and p are the constants whose values depend on the process involving specific kinds of TDs. The case of p=1 corresponds to those processes involving collapsing cosmic string loops or monopole-antimonopole annihilation [1] while p=0 corresponds to processes involving saturated superconducting cosmic string loops [3]. The energy spectrum of UHE neutrinos from TDs is obtained by

$$J(E_0)dE_0 = \frac{1}{H_0}\int dz_r(1+z_r)^{-3}\frac{dn_x}{dz_r}\int_{E_0} dE_{z_r} G(E_{z_r}, E_0, z_r) f_\nu(E_{z_r}) \qquad (12)$$

where z_r is the epoch when the X-particle is released, E_{z_r} is the UHE neutrinos energy at z_r, $f_\nu(E_{z_r})$ is the primary energy spectrum given by Eq.(10), E_0 is the present-day energy of the neutrinos, and $G(E_{z_r}, E_0, z_r)$ gives the energy distribution at the present epoch for UHE neutrinos which were input at time z_r with energy of E_{z_r}, resulting from the neutrino cascades in the ν_{bb} field and adiabatic loss. This quantity is calculated by Eq.(5).

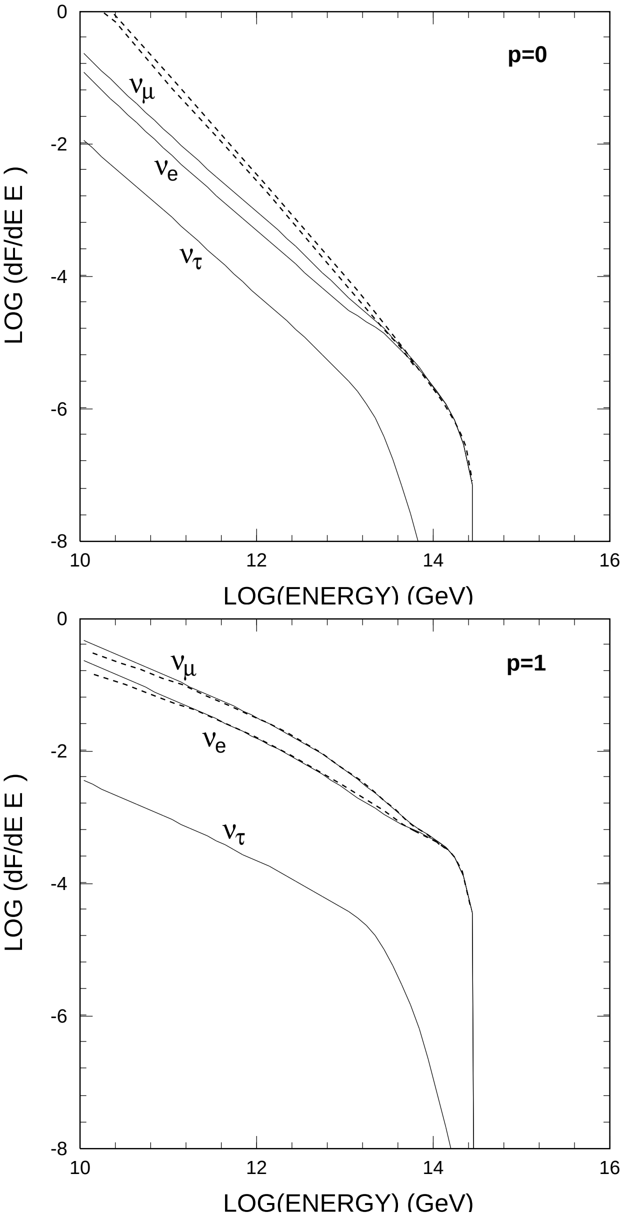

Figure 7: The energy spectra of UHE neutrinos from TDs for (a) $p = 0$ (saturated superconducting cosmic string loops) and (b) $p = 1$ (collapsing cosmic string loops and/or annihilation of monopoles). m_X is assumed to be 10^{15} GeV. The flux is in arbitrary units. The dashed curves show the results obtained if only the energy loss due to expansion of the Universe is taken into account.

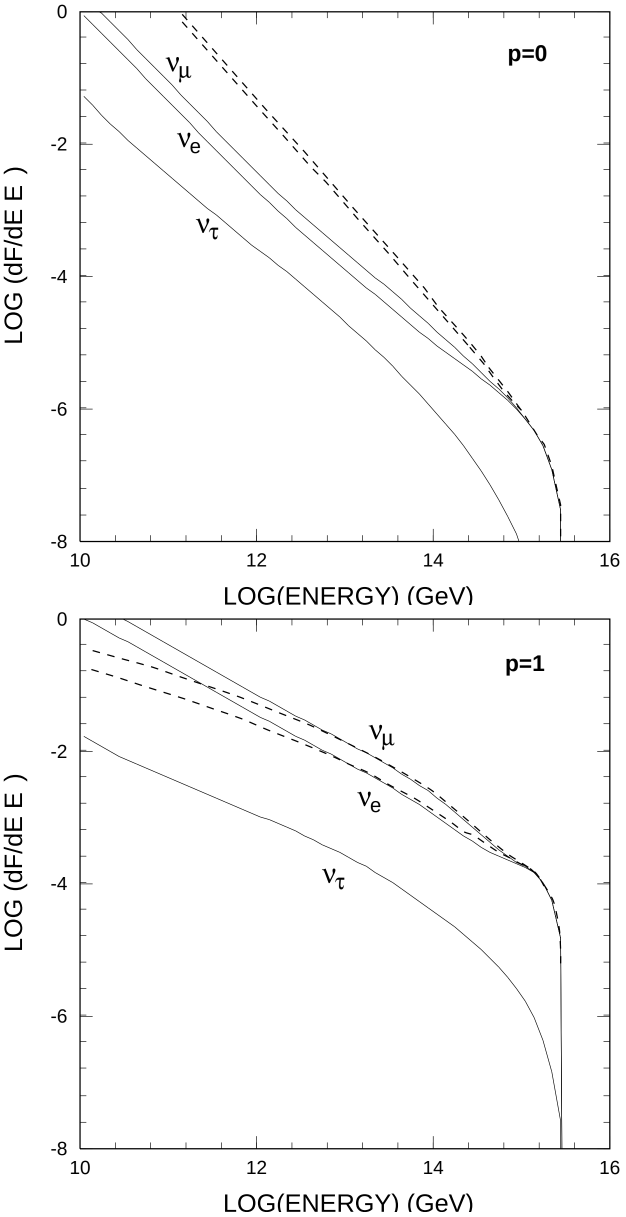

Figure 8: The energy spectra of UHE neutrinos for $m_X = 10^{16}\ GeV$ and all other parameters as in Fig.7. The dashed curves show the results obtained if only the energy loss due to expansion of the Universe is taken into account.

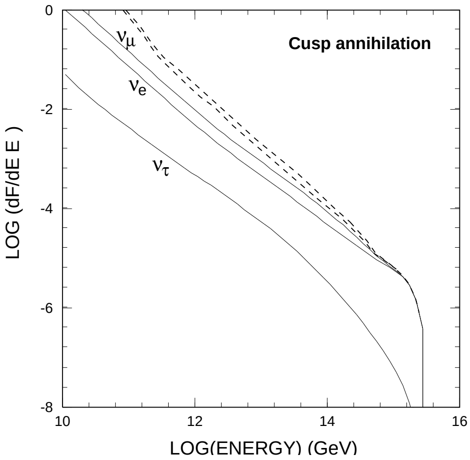

Figure 9: The energy spectra of UHE neutrinos for the "cusp annihilation" model with $\gamma G\mu = 10^{-5}$. The total energy of jets emitted from the cusps is assumed to be 5×10^{15} GeV. The dashed curves show the results obtained if only the energy loss due to expansion of the Universe is taken into account.

The calculated spectra of the UHE neutrinos of TD origin are presented in Fig.7 for $m_X = 10^{15}$ GeV and in Fig.8 for $m_X = 10^{16}$ GeV. The dashed curves correspond to the spectra taking into account only energy loss due to expansion of the Universe. The energy loss process due to the interactions with the ν_{bb} field flattens the energy spectrum for $p = 0$ because the UHE neutrinos from early epochs are absorbed by the ν_{bb} field and do not contribute to the present day flux while the release rate of the X-particles favors early epochs. The horizon of the Universe for UHE neutrinos above 10^{19} eV in this case is $z \sim 700$. If ν_{bb} are not taken into account, however, the horizon is decided by the time of equal matter and radiation in the Universe which is given as $z_{eq} = 3.9 \times 10^4 (H_0/100\ km\,\mathrm{sec}^{-1}\,Mpc^{-1})^2$. In case of $p = 1$, the weak evolution does not favor the contribution of TDs at early epochs and the absorption effect of the ν_{bb} field is not responsible for the spectral shape. Instead, the secondary neutrinos produced in the cascading of UHE neutrinos modify the spectrum for $p = 1$ to increase the flux below $\sim 10^{20}$ eV. In any case, the slope of the spectrum depends on the parameter p and the mass of the X-particle. In case of $p = 0$, $dN/dE \sim E^{-2.25}$ for $m_X = 10^{15}$ GeV and $\sim E^{-2.35}$ for $m_X = 10^{16}$ GeV respectively at around 10^{19} eV, while $dN/dE \sim E^{-2.5}$ without any black-body neutrinos. In case of $p = 1$, $dN/dE \sim E^{-1.6}$ for $m_X = 10^{15}$ GeV and $\sim E^{-1.7}$ for $m_X = 10^{16}$ GeV, while $dN/dE \sim E^{-1.45}$ when the black-body neutrinos are not taken into account.

There is another proposed mechanism by which cosmic strings can emit UHE neutrinos, so called "cusp annihilation" [12,13]. In this model, cosmic strings can continue to release their entire energy as high-energy particles in the region of cusps. The initial particles emitted from cusp fragments to produce jets. UHE neutrinos are produced as decay products of pions in the jets [14]. In this model, it is not necessary that the initial energy of the jets should reach the GUT energy scale. However, an appropriate choice of parameters such as mass per unit length of the string can give rise to jets with the GUT energy scale though the predicted flux of neutrinos is quite low [14]. In this case, interactions with the black-body neutrinos are significant. The rate of emission of UHE particles is given approximately as follows:

$$\frac{dn_X}{dz} \simeq \begin{cases} \alpha(1+z)^{\frac{5}{2}} & (z < z_{eq}(\gamma G\mu)^{\frac{2}{3}}), \\ \frac{8}{11}\frac{1}{\sqrt{\gamma G\mu}}\alpha(1+z_{eq})^{-\frac{3}{4}}(1+z)^{\frac{13}{4}} & (z_{eq}(\gamma G\mu)^{\frac{2}{3}} \ll z \ll z_{eq}), \\ \frac{32}{33}\frac{1}{\sqrt{\gamma G\mu}}\alpha(1+z_{eq})^{-\frac{7}{6}}(1+z)^{\frac{11}{3}} & (z_{eq} < z) \end{cases} \tag{13}$$

Here, γ is a constant of order 10^2, μ is the mass of the cosmic string per unit length, z_{eq} is the redshift when radiation in the Universe is equal to matter ($\sim 2 \times 10^4$ for $H_0 = 75 km\,\mathrm{sec}^{-1}\,Mpc^{-1}$), and α is the constant whose value is determined from the number density of cosmic strings. The first case in the right hand side of Eq.(13) gives the rate when all particles are emitted in the matter dominated era and all loops contributing to the emission were laid in the matter dominated era. The second case describes the emission rate when all particles are emitted in the matter dominated era while the contributing loops were produced in the radiation dominated era. In the third case all of the particle emission is in the radiation dominated epoch. We need to consider the emission in the matter dominated epoch only, because in the radiation dominated era the neutrinos lose almost all their energy during propagation and do not survive as UHE particles with energies above 10^{19} eV. In Fig.9 we show the energy spectra of UHE neutrinos in the cusp annihilation model for $\gamma G\mu = 10^{-5}$ (this value is used in the cosmic string model of galaxy formation [15]). The total energy of jets from the cusps is assumed to be 5×10^{15} GeV. In the cusp annihilation model, the strong source evolution modifies the spectrum through the absorption effect of the ν_{bb} field.

4. DISCUSSION

It is certain that the spectra of UHE neutrinos propagating from distant sources at $z \gg 10$ have the pile-up and cut-off as shown in Fig.5 and Fig.6. However, these modifications are too moderate to form any distinct structures in the spectrum, to which many sources distributed universally from early epochs to the present contribute. The model of TDs corresponds to this case since the X-particles have been released during almost whole history of the Universe as represented by Eq.(11). None of features, such as pile-up or dip, which would appear in the UHE cosmic ray spectrum (see refs.[7-9]) is created in the TDs model. The main modification on the spectral shape of the black-body neutrinos is to change the slope as shown in Figs.7 and 8. In the case of the strong evolution model of TDs, for example with $p = 0$, the absorption effect, *i.e.* the horizon of the Universe for UHE neutrinos, is main contributor for the spectral shape. In case of the weak evolution model of TDs, with $p = 1$, the bulk of the secondary neutrinos produced in the cascade of neutrinos modify the spectral shape and increase the flux of UHE neutrinos below $\sim 10^{20}$ eV. The slope depends strongly on the parameter p and, thus, we must know the value of p, the time dependence of formation of the TDs, to verify the existence of the black-body neutrino in the Universe from the shape of the UHE neutrino spectrum. In other words, the effect of the presence of black-body neutrinos on the shape of the energy spectrum can be reproduced by changing the time dependence of formation of the TDs (the parameter p) without considering the black-body neutrinos at all. In the context of the cosmic string theory, it is unlikely that the value of the parameter p is not in the range between 0 and 1 [6]. Certain values of p (in particular 0 and 1) are favored because of the special way in which TDs evolve cosmologically (so called "scale-free" evolution). If this is real, then the black-body neutrinos may be searched for by studying the spectral shape of the UHE neutrinos from TDs. In this case, it is possible to estimate the end-point energy of the UHE neutrinos, because the slope of the spectrum is a function of not only the parameter p but the end-point energy of the primary neutrinos.

In the present work, we considered hadronic jets produced by TDs as the probable sources of UHE neutrinos ranging up to the GUT scale. It should be noted that we cannot exclude the other possibilities [18,19]. There might exist several different channels for the X-particles decay processes which depends on the properties of the X-particles. Generally UHE neutrinos can be emitted as direct products of the X-particles decay, $X \rightarrow \nu + all$. In this case, these UHE neutrinos constitute approximately a flat energy distribution at injection. However, if the decay process of X-particles produce the hadronic jets, the contribution from fragmentation in the jets dominates over that from the direct products of the X-particle decay [3] and constitutes approximately a power law energy distribution ($\sim E^{-1.3}$) described by Eq.(2). Therefore, the uncertainties in the branching ratios of different channels for their decay processes would make it difficult to draw reliable inferences of the presence of the black-body neutrino background from observation of the spectral shape of the UHE neutrinos. For any cosmological sources to produce UHE neutrinos, the interactions with the black-body neutrinos would change the spectral slope but create no special structures such as the pile-up, in case of the sum over cosmological UHE neutrino emitters with different redshift, and it is essential to know the primary energy distribution at the sources and the evolution of the release rate of primary UHE neutrinos (corresponding to the Eqs.(10) and (11) in the TDs model).

A bulk of UHE neutrinos above $\sim 10^{19}\ eV$ can also be produced by photopion production of UHE cosmic rays in the cosmic background photon field during the propagation [7,9]. If extragalactic astrophysical sources are the main supplier of the highest energy cosmic rays above $10^{19}\ eV$, the secondary neutrino flux is roughly $\sim 10^{-33}\ m^{-2}\ \sec^{-1}\ sr^{-1} eV^{-1}$ at $10^{19}\ eV$ [9]. In addition, the direct neutrino production as a result of particle acceleration in AGNs may lead to a significant neutrino intensity above $10^{19}\ eV$ [22] though the predicted flux and maximum energy have a lot of ambiguity [23]. It should be noted that these sources may complicate the neutrino spectrum resulting from the cascade.

If neutrinos are massive ($m_\nu \simeq 10 \sim 100\ eV$), the interactions with black-body neutrinos become more sizable for $E \geq 10^{19}\ eV$. Roulet studied the absorption of UHE neutrinos when the dark matter in the Universe consists of massive neutrinos [17]. In this hypothesis, the neutrino cascading may lead to a significant dip and a pile-up in the UHE spectrum at $E \geq 10^{19}\ eV$. A study of the neutrino cascade in the massive cosmological neutrino field is in progress and will be presented elsewhere.

UHE neutrinos will interact with not only the cosmic background neutrinos but other particles in the Universe. The most important process is the scattering off background nucleons [14]. However, the cross section is approximately $\sigma_{\nu N} \propto E_\nu^{0.35}$ for $10^{12} \ll E_\nu \ll 10^{19}\ eV$ [20,21] while the neutrino-neutrino cross section is $\sigma_{\nu\nu} \propto s \propto E_\nu$ at $s \ll M_Z^2$, which leads to the contribution of neutrino-nucleon scattering being negligible at higher energies. For $E_\nu \geq 10^{19}\ eV$, the interactions with the background neutrinos become dominant [19] and the present results are not affected.

We should also mention the uncertainty in the fragmentation formula of jets. This formula determines the shape of the primary neutrino spectra from TDs in our calculation and plays an important role in the neutrino cascading. Equation 2 reproduces well the particle multiplicity growth as seen in GeV-TeV jets in colliders [10]. The jets produced by the decay of X-particles and those produced in collider experiments should be reasonably similar in characteristics, at least to a first approximation. Nothing is better than extrapolation of the results in the collider experiments to the GUT energy scales, and this uncertainty should be borne in mind in discussions about UHE neutrinos from TDs and the resulting "cascades".

In this paper no attempt has been made to calculate the absolute flux of the UHE neutrinos. However, we can set an upper limit on the flux of UHE neutrinos though the limit has a lot of ambiguity [16]. If cosmic strings are the main supplier of UHE cosmic rays at around $10^{20}\ eV$ [1], the expected flux of UHE neutrinos can be estimated from the observed flux of cosmic rays. Bhattacharjee *et al.* [2] obtained $\sim 3 \times 10^{-22}\ GeV^{-1} m^{-2} sec^{-1} sr^{-1}$ at $10^{19}\ eV$ for $p = 1$ and $m_X = 10^{15}\ GeV$. This value would reach the sensitivity of some currently planned experiments (see e.g. Ref. [11]). Our results show that the existence of the blackbody neutrinos increases the flux of UHE neutrinos for $p = 1$ (collapsing cosmic string loops or annihilation of monopoles), by about factor of 6 at 10^{19} eV in case of $m_X = 10^{16}$ GeV for example. This effect is encouraging for the detection of UHE neutrinos above 10^{19} eV.

ACKNOWLEDGEMENTS

The author would like to thank P.Bhattacharjee and M.Honda for useful discussions, encouragement, and support. I also wish to thank P.G.Edwards for his careful reading of the manuscript. I am grateful to an anonymous referee for many valuable remarks and bringing Ref.[17-19] to my attention.

REFERENCES

[1] P.Bhattacharjee, in Astrophysical Aspects of the Most Energetic Cosmic Rays, edited by M. Nagano and F. Takahara *p.382* (World Scientific 1991, Singapore)

[2] P.Bhattacharjee, C.T.Hill, and D.N.Schramm, Phys.Rev.Lett. **69** (1992) 567

[3] C.T.Hill, D.N.Schramm, and T.P.Walker, Phys.Rev.D **36** (1987) 1007

[4] P.Bhattacharjee and N.C.Rana, Phys.Lett. **B 246** (1990) 365

[5] T.K.Gaisser, in Cosmic Rays and Particle Physics (Cambridge University Press 1990) *p.89*

[6] P.Bhattacharjee, (*Private Communications* 1993)

[7] C.T.Hill and D.N.Schramm, Phys.Rev.**D31** (1985) 564

[8] V.S.Berezinsky and S.I.Grigor'eva, Astron.Astrophys. **199** (1988) 1

[9] S.Yoshida and M.Teshima, Prog.Theor.Phys. **89** (1993) 833

[10] C.T.Hill, Nucl.Phys **B224** (1983) 469

[11] M.Teshima et al., Nucl.Phys.B (Proc.Suppl.), **28B** (1992) 169

[12] R.H.Brandenberger, Nucl.Phys. **B293** (1987) 812

[13] R.H.Brandenberger and A.Matheson, Mod.Phys.Lett. **A2** (1987) 461

[14] J.H.MacGibbon and R.H.Brandenberger, Nucl.Phys. **B331** (1990) 153

[15] A.Vilenkin, Phys.Rep. **121** (1985) 263

[16] X.Chi *et al.* Astropart.Phys. **1** (1992) 129

[17] E.Roulet, Phys.Rev. **D47** (1993) 5247

[18] V.S.Berezinsky, Nucl.Phys. **B380** (1992) 478

[19] P.Gondolo, G.Gelmini and S.Sarkar, Nucl.Phys. **B392** (1993) 111

[20] D.W.Mackey and J.P.Ralston, Physics Lett. **167B** (1986) 103

[21] C.Quigg, M.H.Reno, and T.P.Walker, Phys.Rev.Lett. **57** (1986) 774

[22] P.L.Biermann, in High Energy Neutrino Astrophysics, edited by V.J.Stenger *et al.*, *p.86* (World Scientific 1992, Singapore)

[23] A.P.Szabo and R.J.Protheroe, Proc. 23rd ICRC at Calgary **4** (1993) 438