

# Constraints on the origin of the ultrahigh energy cosmic rays using cosmic diffuse neutrino flux limits: An analytical approach

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Astrophysical neutrinos are expected to be produced in the interactions of ultrahigh energy cosmic rays with surrounding photons. The fluxes of the astrophysical neutrinos are highly dependent on the characteristics of the cosmic-ray sources, such as their cosmological distributions. We study possible constraints on the properties of cosmic-ray sources in a model-independent way using experimentally obtained diffuse neutrino flux above 100 PeV. The semianalytic formula is derived to estimate the cosmogenic neutrino fluxes as functions of source evolution parameter and source extension in redshift. The obtained formula converts the upper limits on the neutrino fluxes into the constraints on the cosmic-ray sources. It is found that the recently obtained upper limit on the cosmogenic neutrinos by IceCube constrains the scenarios with strongly evolving ultrahigh energy cosmic-ray sources, and the future limits from a 1 km<sup>3</sup> scale detector are able to further constrain the ultrahigh energy cosmic-rays sources with evolutions comparable to the cosmic star formation rate.

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## I. INTRODUCTION

The origin of the ultrahigh energy cosmic rays (UHECRs) has been a long-standing important question in astrophysics. While the observations by Auger [1,2] and HiRes [3] indicate that cosmic rays with energies above  $\sim 10^{18.5}$  eV are of extragalactic origin, identification of astronomical objects responsible for the UHECR emission has not been achieved. Neutrinos, secondarily produced by UHECR nucleons, are expected to provide direct information on the UHECR origin, since a neutrino penetrates over cosmological distance without being deflected by the cosmic magnetic field nor absorbed by the photon field. The ‘‘cosmogenic’’ neutrinos [4] are produced by the collisions of UHECR nucleons with the cosmic microwave background (CMB) photon via photo-produced  $\pi$  meson decay as  $\pi^\pm \rightarrow \mu^\pm \nu_\mu \rightarrow e^\pm \nu_e \nu_\mu$ , known as the Greisen-Zatsepin-Kuzmin (GZK) mechanism [5]. The intensity of cosmogenic neutrinos indicates redshift distributions of the parent UHECR sources [6,7] in the Universe. The source distributions derived from the cosmogenic neutrino intensities can then be compared with distributions of known classes of the astronomical objects possibly responsible for the UHECR emissions. Therefore reliable extractions of the UHECR source distribution function (SDF) is one of the key issues in cosmogenic diffusive neutrino searches. Constraints on the sources of UHECRs derived from the measurements or upper limits of the ultrahigh energy neutrino flux are highly complimentary to the constraints from the diffuse photon flux [8,9], because the former does not rely on uncertain estimation of extragalactic background light (EBL).

In this work, we develop a method to bound the UHECR source evolution and its redshift dependence in a comprehensive way without introducing specific astronomical models. We derive an analytical formula to calculate intensities of the neutrinos produced by the GZK mechanism in the range between 100 PeV and 10 EeV. Using the formula, we extract the relation among the neutrino intensity and the UHECR SDF parameters. The use of the analytical formula allows us to calculate neutrino intensities in the full phase space of the source evolution parameters without an intensive computational task. The analytical formula can also be used as a practical tool to approximately calculate cosmogenic GZK neutrino intensity with given UHECR SDF, for example, for the performance studies of the future detectors such as KM3NET [10]. Finally we present model-independent constraints on the UHECR sources using the obtained formula with the upper limit [11] and the future sensitivity [12] on the cosmogenic neutrino detection by the IceCube neutrino observatory [13].

The standard cosmology with  $H_0 \approx 73.5 \text{ km sec}^{-1} \text{ Mpc}^{-1}$ ,  $\Omega_M = 0.3$ , and  $\Omega_\Lambda = 0.7$  [14] is assumed throughout the paper.

## II. ANALYTICAL FORMULA FOR ESTIMATING COSMOGENIC $\nu$ INTENSITY

The neutrino flux per unit energy,  $dJ_\nu/dE_\nu$ , is generally written as

$$\frac{dJ_\nu}{dE_\nu} = n_0 c \int_0^{z_{\max}} \psi(z_s) \left| \frac{dt}{dz}(z_s) \right| dz_s \int_0^{z_s} \left| \frac{dt}{dz}(z_\nu) \right| dz_\nu \times \int_{E_\nu}^{\infty} \frac{dN_{p \rightarrow \nu}}{dE_p^g dt^g}(z_\nu, z_s) \delta(E_\nu^g - (1 + z_\nu)E_\nu) dE_p^g. \quad (1)$$

The first integral represents the total contribution of UHECR sources in the redshift up to  $z_{\max}$ , where  $z_{\max}$  is

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the maximum redshift of the UHECR source distribution or, in other words, the time of the first UHECR emission in the Universe.  $\psi(z_s)$  represents the cosmic evolution of the spectral emission rate per comoving volume and  $n_0$  is the number density of UHECR sources at the present Universe. The relation between time and redshift is given as

$$\left| \frac{dt}{dz}(z_s) \right| \equiv \left| \frac{dt_s}{dz_s} \right| = [H_0(1+z_s)\sqrt{\Omega_M(1+z_s)^3 + \Omega_\Lambda}]^{-1}.$$

The second integral calculates the total neutrino flux expected from a single UHECR source at redshift  $z_s$  generated via UHECR interactions at redshift  $z_\nu (\leq z_s)$ .  $dN_{p \rightarrow \nu}/dE_\nu^g dt^g$  is the yield of generated neutrinos with energy of  $E_\nu^g$  per unit time in the UHECR laboratory frame (the CMB rest frame). Suffixes  $s$  and  $g$  represent the quantities at the positions of UHECR sources and neutrino generation, respectively. The delta function indicates the neutrino energy loss due to the expansion of the Universe.

The neutrino yield,  $dN_{p \rightarrow \nu}/dE_\nu^g dt^g$ , at redshift  $z_\nu$  by the GZK mechanism is expressed by a convolution of the UHECR intensity from a source at  $z_s$ , the CMB photon density, and the photo-pion interaction kinematics as

$$\begin{aligned} \frac{dN_{p \rightarrow \nu}}{dE_\nu^g dt^g} &= \int dE_{\text{CR}} \frac{dN_{\text{CR}}}{dE_{\text{CR}}}(z_s, z_\nu) c \int ds \\ &\times \int dE_\pi \frac{d\sigma_{\gamma p}}{dE_\pi} \frac{d\rho_{\pi \rightarrow \nu}}{dE_\nu^g} \frac{dn_\gamma}{ds}, \end{aligned} \quad (2)$$

where  $dN_{\text{CR}}/dE_{\text{CR}}$  is the number of UHECRs per unit time and energy at the redshift  $z_\nu$ , originating from a source at  $z_s$ , and  $s$  is the Lorentz-invariant Mandelstam variable, the square of invariant mass of the cosmic-ray nucleon and the target CMB photon.  $\sigma_{\gamma p}$  is the photo-pion production cross section,  $d\rho_{\pi \rightarrow \nu}/dE_\nu^g$  is the energy distribution of neutrinos from the photo-produced pion, and  $dn_\gamma/ds$  is the CMB photon number density in the UHECR frame per unit  $s$ .

We introduce the following approximations to simplify the calculations: (1) the contribution of UHECR colliding with infrared and optical universal photon (IR/O) backgrounds is negligible and only the contribution of photo-pion production cross section from  $\Delta$  resonance is considered in collisions of UHECRs and CMB photons, and (2) the kinematics of the photo-pion production is represented by a single pion production. The first approximation allows the photon number density  $dn_\gamma/ds$  to be analytically obtained with the modification to the black-body distribution [6]. The contribution of neutrinos induced by UHECR interactions with IR/O becomes sizable only in the energy region below 100 PeV [15] while the effect is small in the higher energy region. Similarly the neutrinos from photo-produced pions outside the  $\Delta$  resonance are mostly visible only in the lower energy range below 100 PeV [16], and the single pion production is the most dominant channel in the  $\Delta$  resonance. The  $\Delta$ -resonance approximation simplifies the integral on  $s$  in Eq. (2) to a multiplication of the integrand at  $s = s_R (\simeq 1.5 \text{ GeV}^2)$ ,

where  $s_R$  is the Lorentz-invariant Mandelstam variable at the  $\Delta$  resonance, and  $\Delta s_R (\simeq 0.6 \text{ GeV}^2)$ , the width of the  $\Delta$  resonance. The second approximation then gives [17]

$$\frac{d\rho_{\pi \rightarrow \nu}}{dE_\nu^g} \simeq \frac{1}{E_\pi} \frac{3}{1 - r_\pi}, \quad (3)$$

where  $r_\pi = m_\mu^2/m_\pi^2 \simeq 0.57$  is the muon-to-pion mass squared ratio. The factor three arises from the fact that three neutrinos are produced from the  $\pi$  meson and  $\mu$  lepton decay chain. The allowed range of  $E_\nu^g$  due to the kinematics is given by

$$0 \leq \frac{E_\nu^g}{E_\pi} \leq 1 - r_\pi, \quad (4)$$

where neutrino mass is neglected. With a good approximation that a single pion is isotropically emitted in the center-of-momentum frame, one obtains

$$\frac{d\sigma_{\gamma p}}{dE_\pi} = \frac{1}{E_{\text{CR}}} \frac{d\sigma_{\gamma p}}{dx_\pi} \simeq \frac{1}{E_{\text{CR}}} \frac{\sigma_{\gamma p}}{x^+ - x^-}, \quad (5)$$

where  $x_\pi \equiv E_\pi/E_{\text{CR}}$  is the relative energy of emitted pion normalized by the parent proton energy  $E_{\text{CR}}$ .  $x^\pm$  are the maximal and minimal bound of  $x_\pi$  due to the kinematics and given by

$$x^\pm = \frac{s + m_\pi^2 - m_p^2}{2s} \pm \frac{\sqrt{(s + m_\pi^2 - m_p^2)^2 - 4sm_\pi^2}}{2s}, \quad (6)$$

where  $m_p$  is the proton mass.

Then we obtain the neutrino yield, Eq. (2), expressed as an analytical function with only a single energy integral,

$$\begin{aligned} \frac{dN_{p \rightarrow \nu}}{dE_\nu^g dt^g} &\simeq \frac{k_B T(1+z_\nu)}{8\pi^2 \hbar^3 c^2} (s_R - m_p^2) \sigma_{\gamma p}^R \\ &\times \frac{s_R \Delta s_R}{\sqrt{(s_R + m_\pi^2 - m_p^2)^2 - 4s_R m_\pi^2}} \frac{3}{1 - r_\pi} \\ &\times \int dE_{\text{CR}} \frac{1}{E_{\text{CR}}^3} \frac{dN_{\text{CR}}}{dE_{\text{CR}}} \\ &\times \ln\left(\frac{x_R^+}{\xi_R}\right) \{-\ln(1 - e^{-(E_\Delta)/(1+z_\nu)E_{\text{CR}}})\}. \end{aligned} \quad (7)$$

Here  $k_B$  is the Boltzmann constant,  $T$  is present temperature of the CMB.  $E_\Delta \equiv (s_R - m_p^2)/4k_B T$  corresponds to the energy of UHECR protons colliding via  $\Delta$  resonance at the present Universe. Suffix  $R$  denotes the values at the  $\Delta$  resonance in the photo-pion reaction. For example,  $\sigma_{\gamma p}^R = 2.1 \times 10^{-28} \text{ cm}^2$  represents the photo-pion production cross section of channel  $\gamma p \rightarrow n\pi^+$  at the  $\Delta$  resonance and  $x_R^\pm$  is given by Eq. (6) with  $s = s_R$ .

The parameter  $\xi_R$  reflects the kinematics bounds, Eqs. (4) and (6), and is defined by

$$\xi_R = \begin{cases} x_R^- & E_\nu^g \leq (1 - r_\pi)x_R^- E_{\text{CR}}, \\ \frac{E_\nu^g}{(1 - r_\pi)E_{\text{CR}}} & \text{otherwise.} \end{cases} \quad (8)$$

$dN_{\text{CR}}/dE_{\text{CR}}$  is calculated by the energy loss formula with the continuous energy loss approximation [18] represented by

$$-\frac{dE_{\text{CR}}}{cdt} = \frac{(1+z)^3}{\lambda_{\text{GZK}}(E_{\text{CR}}(1+z))} E_{\text{CR}}, \quad (9)$$

where  $\lambda_{\text{GZK}}$  is the energy attenuation length governed by the GZK mechanism, mainly due to the photo-pion production of UHECRs and the CMB. The factor  $(1+z)^3$  accounts for the increase of CMB photon number density with redshift  $z$ .

Here we introduce the final approximation that the energy attenuation length of UHECR by the GZK mechanism,  $\lambda_{\text{GZK}}$ , is constant with energies above  $E_{\text{GZK}} \equiv 10^{20}$  eV. While  $\lambda_{\text{GZK}}$  rapidly decreases with cosmic-ray energy increase, it becomes a slight increase or constant above  $\sim 3 \times 10^{20}$  eV for  $z_\nu \sim 0$ . Neutrinos from  $z_\nu \geq 1$  are the dominant contribution to the cosmogenic neutrino intensity at Earth and the turnover energy is shifted to lower energies  $\leq E_{\text{GZK}}$  due to the redshift effects for the Universe of  $z_\nu \geq 1$  [19]. Therefore this approximation reasonably describes the UHECR energy loss profile to calculate the neutrino yield. Assuming that the primary UHECR spectrum from a source at  $z_s$  follows the power law described by  $dN_{\text{CR}}(z_s, z_s)/dE_{\text{CR}} = \kappa_{\text{CR}}(E_{\text{CR}}/E_{\text{GZK}})^{-\alpha}$  up to  $E_{\text{max}}$ , the maximal injected energy from a source, then the  $dN_{\text{CR}}(z_s, z_\nu)/dE_{\text{CR}}$  is analytically obtained by

$$\frac{dN_{\text{CR}}}{dE_{\text{CR}}}(z_s, z_\nu) = \kappa_{\text{CR}} \left( \frac{E_{\text{CR}}}{E_{\text{GZK}}} \right)^{-\alpha} \times e^{-(\alpha-1)(c/\lambda_{\text{GZK}}H_0)(2/3\Omega_M)\{f(z_s)-f(z_\nu)\}}, \quad (10)$$

where  $f(z) \equiv \sqrt{\Omega_M(1+z)^3 + \Omega_\Lambda}$  and  $\kappa_{\text{CR}}$  is a normalization constant. With Eq. (10), the energy integral on  $E_{\text{CR}}$  in Eq. (7) becomes an addition of integrals in the forms of  $\int dy y^{-(\alpha+3)} \ln(1 - e^{-1/y})$  and  $\int dy y^{-(\alpha+3)} \ln(1 - e^{-1/y}) \ln y$ . An asymptotic approximation with numerical constants is found to provide approximate solutions of these integrals. The final formula of the neutrino yield is then obtained as

$$\begin{aligned} \frac{dN_{p \rightarrow \nu}}{dE_\nu^g dt^g} &= \kappa_{\text{CR}} \frac{k_B T}{8\pi^2 \hbar^3 c^2} \frac{(s_R - m_p^2)}{E_{\text{GZK}}^2} \left( \frac{E_\Delta}{E_{\text{GZK}}} \right)^{-(\alpha+2)} \sigma_{\gamma p}^R \\ &\times \frac{s_R \Delta s_R}{\sqrt{(s_R + m_\pi^2 - m_p^2)^2 - 4s_R m_\pi^2}} \frac{3}{1 - r_\pi} (1 + z_\nu)^{\alpha+3} \\ &\times e^{-(\alpha-1)(c/\lambda_{\text{GZK}}H_0)(2/3\Omega_M)\{f(z_s)-f(z_\nu)\}} \\ &\times \left\{ \epsilon_0 x_0^{-(\alpha+1)} e^{-2} \ln\left(\frac{x_R^+}{x_R^-}\right) + \epsilon_1 x_1^{-(\alpha+3)} e^{-1/x_1} \right. \\ &\left. \times e^{-2} \ln\left(x_1 \frac{E_\Delta}{E_\nu^g(1+z_\nu)} x_R^- (1 - r_\pi)\right) \right\}, \quad (11) \end{aligned}$$

where  $x_0 = 0.275$ , and  $x_1 = 0.16$  are the empirically determined numerical constants.  $\epsilon_0$  and  $\epsilon_1$  are either unity or null, depending on neutrino energy. These are consequences of the kinematics bound for pions and neutrinos, Eq. (8), and given by

$$\epsilon_0 = \begin{cases} 1 & E_\nu^g(1+z_\nu) \leq x_1 E_\Delta x_R^+ (1 - r_\pi), \\ 0 & \text{otherwise,} \end{cases} \quad (12)$$

and

$$\epsilon_1 = \begin{cases} 0 & E_\nu^g(1+z_\nu) \leq x_1 E_\Delta x_R^- (1 - r_\pi), \\ 1 & x_1 E_\Delta x_R^- (1 - r_\pi) \leq E_\nu^g(1+z_\nu) \leq x_1 E_\Delta x_R^+ (1 - r_\pi), \\ 0 & \text{otherwise.} \end{cases} \quad (13)$$

One can also find that  $x_1 E_\Delta x_R^\pm (1 - r_\pi)$  in these equations represents the effective energy of neutrinos from decay of the pions with kinematically allowed maximum ( $x_R^+$ ) and minimum ( $x_R^-$ ) energies from  $\Delta$  resonance in the  $\gamma p$  collision. The  $E_\nu^g(1+z_\nu) = E_\nu(1+z_\nu)^2$  factor reflects the redshift dependence of the CMB temperature and the redshift energy loss of neutrinos at  $z_\nu$ .

Equation (1) with the Eq. (11) finally give the cosmogenic neutrino flux with double integrals of redshift  $z_s$  and  $z_\nu$ . The  $z_\nu$  integral is analytically solvable neglecting  $O((\lambda_{\text{GZK}}H_0/c)^2)$  or higher order terms because the energy attenuation length is much shorter than the cosmological time dimension. Then the final form of the cosmogenic neutrino intensity is obtained as

$$\begin{aligned} \frac{dJ_\nu}{dE_\nu} &= (\alpha - 1) F_{\text{CR}} \frac{c}{H_0} \frac{k_B T}{8\pi^2 \hbar^3 c^3} \frac{(s_R - m_p^2)}{E_{\text{GZK}}^2} \left( \frac{E_\Delta}{E_{\text{GZK}}} \right)^{-(\alpha+2)} \sigma_{\gamma p}^R \\ &\times \frac{s_R \Delta s_R}{\sqrt{(s_R + m_\pi^2 - m_p^2)^2 - 4s_R m_\pi^2}} \frac{3}{1 - r_\pi} \zeta. \quad (14) \end{aligned}$$

Here  $F_{\text{CR}}$  represents the UHECR intensity above  $E_{\text{GZK}}$  and described by

$$\begin{aligned} F_{\text{CR}} &= \int_{E_{\text{GZK}}}^{E_{\text{max}}} dE_{\text{CR}} n_0 c \int_0^{z_{\text{max}}} \psi(z_s) \left| \frac{dt}{dz} \right| dz_s \frac{dN_{\text{CR}}}{dE_{\text{CR}}}(z_s, 0) \\ &\simeq n_0 \kappa_{\text{CR}} E_{\text{GZK}} \lambda_{\text{GZK}} / (\alpha - 1)^2 \quad (15) \end{aligned}$$

assuming that  $E_{\text{max}} \gg E_{\text{GZK}}$ .  $F_{\text{CR}}$  gives the normalization of the neutrino flux in the present formulation in Eq. (14). It can be estimated by the observational data for actual calculation.

$\zeta$  in Eq. (14) is the term which accounts for the redshift dependence and is given by

$$\begin{aligned} \zeta &= \int_0^{z_{\text{max}}} dz_s \frac{(1+z_s)^{(m+\alpha-1)}}{\sqrt{\Omega_M(1+z_s)^3 + \Omega_\Lambda}} \left\{ \epsilon_0 x_0^{-(\alpha+1)} e^{-2} \ln\left(\frac{x_R^+}{x_R^-}\right) \right. \\ &\left. + \epsilon_1 x_1^{-(\alpha+3)} e^{-1/x_1} e^{-2} \ln\left(x_1 \frac{E_\Delta}{E_\nu(1+z_s)^2} x_R^- (1 - r_\pi)\right) \right\}, \quad (16) \end{aligned}$$

where  $\epsilon_0$  and  $\epsilon_1$  are obtained by Eqs. (12) and (13), respectively, replacing  $z_\nu$  by  $z_s$ . The cosmic evolution

function  $\psi(z_s)$  is now parametrized as  $(1+z_s)^m$  such that the parameter  $m$  represents the ‘‘scale’’ of the cosmic evolution often used in the literature [20]. The integral on  $z_s$  in Eq. (16) is analytically solvable

$$\zeta = e^{-2\frac{1}{\gamma_m}\Omega_M^{-(\alpha+m)/(3)}} \left\{ (\Omega_M(1+z_{\text{up}})^3 + \Omega_\Lambda)^{\gamma_m/3} \left( x_0^{-(\alpha+1)} \ln\left(\frac{x_R^+}{x_R^-}\right) + x_1^{-(\alpha+3)} e^{-(1/x_1)} \left[ \ln\left(x_1 \frac{E_\Delta}{E_\nu(1+z_{\text{up}})^2} x_R^-(1-r_\pi)\right) + \frac{2}{\gamma_m} \right] \right) \right. \\ \left. - (\Omega_M + \Omega_\Lambda)^{\gamma_m/3} x_0^{-(\alpha+1)} \ln\left(\frac{x_R^+}{x_R^-}\right) - (\Omega_M(1+z_{\text{down}})^3 + \Omega_\Lambda)^{\gamma_m/3} x_1^{-(\alpha+3)} e^{-(1/x_1)} \left[ \ln\left(x_1 \frac{E_\Delta}{E_\nu(1+z_{\text{down}})^2} x_R^-(1-r_\pi)\right) + \frac{2}{\gamma_m} \right] \right\} \quad (17)$$

where  $\gamma_m \equiv (\alpha + m) - 3/2$ .  $z_{\text{up}}$  and  $z_{\text{down}}$  are the maximum and minimum bounds of the redshifts, respectively. These redshift bounds are associated with  $z_{\text{max}}$  in Eq. (1) but also depend on neutrino energies  $E_\nu$  due to kinematics of  $\pi$  decay and the redshift energy loss.  $z_{\text{up}}$  is given by

$$1 + z_{\text{up}} = \begin{cases} 1 & x_1 E_\Delta x_R^+(1-r_\pi) \leq E_\nu, \\ \left(\frac{x_1 E_\Delta}{E_\nu} x_R^+(1-r_\pi)\right)^{1/2} & \frac{x_1 E_\Delta x_R^+(1-r_\pi)}{(1+z_{\text{max}})^2} \leq E_\nu \leq x_1 E_\Delta x_R^+(1-r_\pi), \\ 1 + z_{\text{max}} & E_\nu \leq \frac{x_1 E_\Delta x_R^+(1-r_\pi)}{(1+z_{\text{max}})^2}. \end{cases} \quad (18)$$

$z_{\text{down}}$  is also given by Eq. (18) replacing  $x_R^+$  by  $x_R^-$ .

See Appendix B for the case of the astronomical objects of which cosmological evolution become constant above a certain redshift (see Ref. [21], for example).

### III. VALIDITY OF THE ANALYTICAL FORMULA

The analytical formula for estimating cosmogenic neutrino fluxes [see Eqs. (14), (17), and (18)] is derived under several assumptions. Here we demonstrate the applicability of the formula in estimating neutrino flux in  $100 \text{ PeV} \lesssim E_\nu \lesssim 10 \text{ EeV}$  which is the main energy range of several cosmogenic neutrino searches [11,22]. In Table I, the cosmogenic neutrino integral flux above 1 EeV obtained by the analytical formula with  $\alpha = 2.5$  is presented. We use the UHECR intensity  $F_{\text{CR}}(\geq E_{\text{GZK}}) = 2.96 \times 10^{-21} \text{ cm}^{-2} \text{ sec}^{-1} \text{ sr}^{-1}$  in the present study, which is obtained from the measurement of the HiRes experiment [3]. The fluxes obtained by the full numerical calculations with the same or comparable source evolution parameters are also listed for comparison. The values in each parameter subset show an agreement within a factor of 2 for a comparable evolution scenario in the wide range of parameter numbers.

Figure 1 presents the neutrino fluxes obtained with the present analytical estimation and the full-blown numerical calculations. The fluxes calculated with the different techniques show the best agreement at  $\sim 1 \text{ EeV}$ , the central energy in the cosmogenic neutrino search with IceCube [11]. The present formula provides a reasonable estimate of the neutrino flux from 100 PeV to 10 EeV with uncertainty of factor of  $\sim 2$ . Some deviations in the analytical formula from the full-blown numerical calculations arise mainly from the uncertainty in the intensity of the extragalactic UHECR component allowed by the observed UHECR spectrum, and the accuracy of the approximations used in derivation of the analytical formula. We discuss these issues in Sec. V.

when we use the fact that  $\sqrt{\Omega_M(1+z_s)^3 + \Omega_\Lambda} \gg \Omega_\Lambda$  in most of the integral range. Finally we obtain the final form of the redshift-dependent part of the analytical formula  $\zeta$  as

TABLE I. Cosmogenic neutrino fluxes predicted by the model-dependent full numerical calculations and those given by the present analytical formula with the corresponding parameters on source evolution. The numbers by the full calculations were converted to be the sum over all three neutrino flavors from the original when appropriate.

$\nu$ flux model	Integral flux $F$ ( $E_\nu \geq 1 \text{ EeV}$ ) [ $\text{cm}^{-2} \text{ sec}^{-1} \text{ sr}^{-1}$ ]
Yoshida and Teshima [6]	
$m = 2.0, z_{\text{max}} = 2.0$	$5.39 \times 10^{-18}$
Ahlers <i>et al.</i> [9]	
$m = 2.0, z_{\text{max}} = 2.0$	$1.85 \times 10^{-18}$
(‘‘the minimal case’’)	
The analytical formula	
$m = 2.0, z_{\text{max}} = 2.0$	$4.91 \times 10^{-18}$
Kotera <i>et al.</i> [15]	
SFR1	$1.07 \times 10^{-17}$
The analytical formula	
$m = 3.4 (z \leq 1.0)$	
const. ( $1 \leq z \leq 4$ )	$1.07 \times 10^{-17}$
Ahlers <i>et al.</i> [9]	
$m = 4.6, z_{\text{max}} = 2.0$	$3.39 \times 10^{-17}$
(‘‘the best fit’’)	
The analytical formula	
$m = 4.6, z_{\text{max}} = 2.0$	$4.09 \times 10^{-17}$
Kalashov <i>et al.</i> [23]	
$m = 5.0, z_{\text{max}} = 3.0$	$7.38 \times 10^{-17}$
The analytical formula	
$m = 5.0, z_{\text{max}} = 3.0$	$8.42 \times 10^{-17}$
Kotera <i>et al.</i> [15]	
Farano-Riley type II	$6.74 \times 10^{-17}$
The analytical formula	
$m = 5.02 (z \leq 1.5)$	
const. ( $1.5 \leq z \leq 2.5$ )	$5.21 \times 10^{-17}$

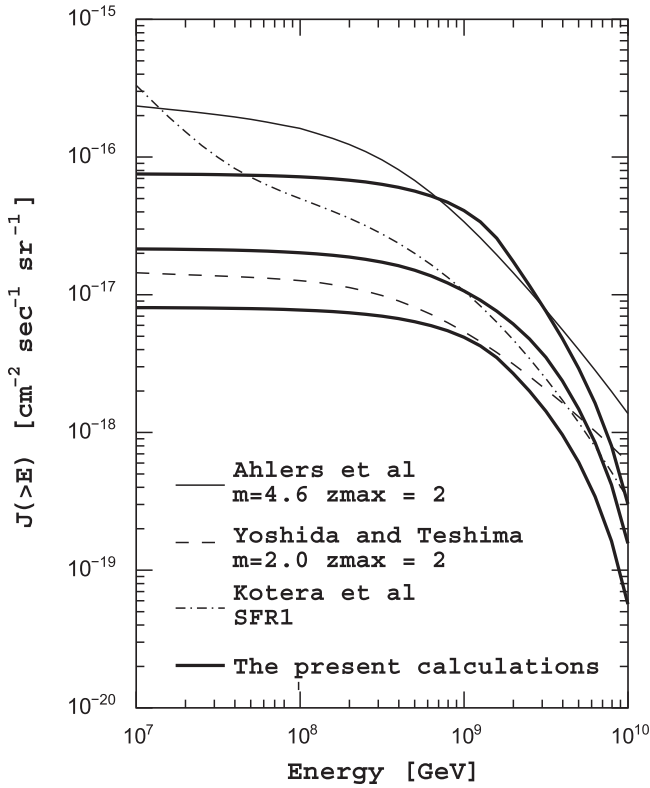


FIG. 1. Integral neutrino fluxes,  $J$  [ $\text{cm}^{-2} \text{sec}^{-1} \text{sr}^{-1}$ ], as a function of neutrino energy. Bold lines represent the present analytical estimates and thin lines represent corresponding predictions by the full numerical calculations [9] or the Monte-Carlo simulations [6,15].

## IV. RESULTS

### A. The relation between the $\nu$ flux and the cosmological evolution of the sources

Shown in Fig. 2 is the distribution of the cosmogenic neutrino integral fluxes with energies above 1 EeV in the parameter space of the evolution of UHECR sources ( $m, z_{\text{max}}$ ) calculated using the derived analytical formula. The fluxes vary by more than an order of magnitude with the evolution parameters. The distribution demonstrates that the neutrino intensity can indeed be an observable to imply the characteristics of the UHECR sources. The plot shows that cosmogenic neutrino flux around 1 EeV is mostly determined by source emissivity history up to redshift of  $z_s \sim 3$ . This is because the contributions of sources at  $z_s \gtrsim 3$  represent only a small fraction of the total flux due to the redshift dilution [15].

### B. Constraints on UHECR origin with the IceCube diffuse neutrino flux limit

Here we estimate the expected event rates with the IceCube neutrino observatory by using the derived analytical formula. The analytical function is valid in the IceCube cosmogenic neutrino detection energy range distributed

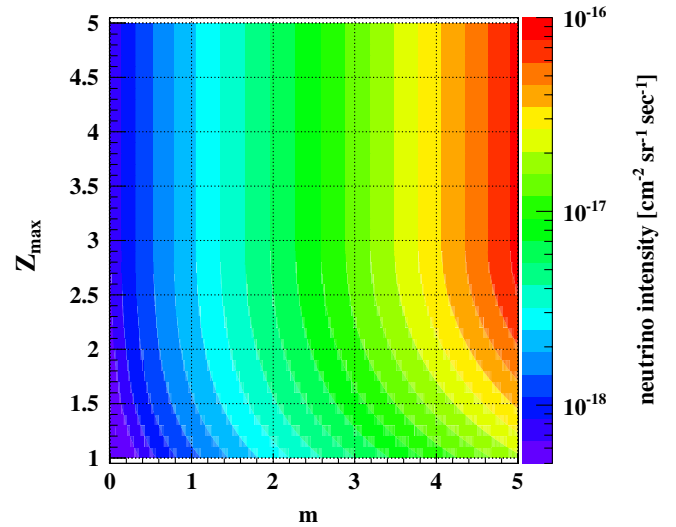


FIG. 2 (color online). Integral neutrino fluxes with energy above 1 EeV,  $J$  [ $\text{cm}^{-2} \text{sec}^{-1} \text{sr}^{-1}$ ], on the plane of the source evolution parameters,  $m$  and  $z_{\text{max}}$ .

around 1 EeV [11]. Convolution of Eq. (14) with the IceCube neutrino effective area [11,12] gives the event rate for the entire phase space of the evolution parameter  $m$  and the maximal redshift  $z_{\text{max}}$ . Full mixing in the standard neutrino oscillation scenario is assumed and the intensity of neutrinos of each of three neutrino flavors corresponds to one third of the estimated neutrino intensity by the analytical function. The Feldman-Cousins upper bound [24] then defines the excluded region on the  $m - z_{\text{max}}$  plane at a given confidence level. Figure 3 displays the resultant constraints. The shaded region represents the factor of 2 uncertainty in the analytical estimation discussed in the previous section. The upper limit with the IceCube 2008–2009 data [11] has already started to constrain hypotheses of UHECR sources with strong evolution of  $m \gtrsim 4.5$ . While this bound may be still weaker than that by the Fermi diffuse  $\gamma$ -ray flux measurement [9], nevertheless the limit by neutrinos is important because the neutrino estimate does not involve the uncertainties of the assumptions of  $E_{\text{max}}$  nor the EBL intensity. The full IceCube five-year observation would certainly probe the most interesting region of the source evolution phase space where the strong candidates for the UHECR sources of the powerful astronomical objects such as radio galaxies and gamma-ray bursts are included.

## V. DISCUSSION

The derived analytical formula to calculate intensities of the neutrinos produced by the GZK mechanism in the range between 100 PeV and 10 EeV is used to constrain the cosmological evolution of the UHECR sources.

The largest uncertainty in the present analytical formula at the lower energy range ( $E_\nu \ll 1$  EeV) is due to the omission of the IR contribution to the cosmogenic neutrino production. Photo-produced pions from the UHECR

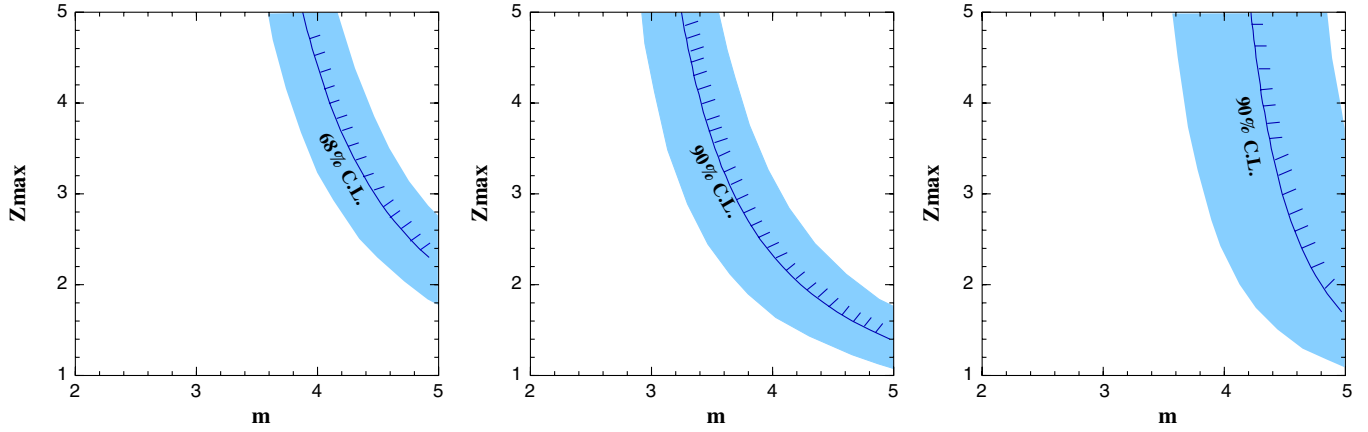


FIG. 3 (color online). Constraints on the UHECR source evolution  $m$  and  $z_{\max}$  with the IceCube 2008–2009 flux limit [11] (left) and with the full IceCube five-year sensitivity [12] (middle). The areas above the solid lines are excluded by null detection of  $\nu$  events. The shaded belts represent uncertainties in the present analytical estimation. The right panel shows the full IceCube five-year constraint when the emission rate per comoving volume becomes constant above  $z_s$  of 1.0. Excluded region at 68% confidence level (corresponding to  $\approx 1.1$  events assuming the same background rate of the IceCube 2008–2009 measurement [11]) and 90% confidence level ( $\approx 2.2$  events) is displayed.

interactions with the IR background are the major origin of neutrinos with energies below 10 PeV, however, the IR contribution is relatively minor in the higher energies where we mainly discuss the aspect of cosmogenic neutrino detection by IceCube. The amount of the IR contribution was studied, for example, in the calculations in Refs. [9,15]. The study in the latter reference exhibits much higher contribution of the IR background than in the former reference, where the effect is suppressed mainly due to the introduction of the minimal energy of extragalactic UHECR population. These differences can be seen in Fig. 1; the low energy component in Ref. [15] is substantially emphasized compared to the other calculations. These variations in the estimation of the IR contribution to the cosmogenic neutrino intensities are considered to be an additional uncertainty to the IR background yield itself, which is also not firmly understood [19,25]. Here we would like to emphasize that the omission of the IR background leads to a conservative constraint on the UHECR source evolution.

The second largest uncertainty is concerned with  $F_{\text{CR}}$ , the UHECR intensity above  $E_{\text{GZK}} \approx 10^{20}$  eV. The works in Refs. [9,26] allowed a sizable variation in the UHECR intensity within 99% confidence level of the statistical test against the observed data. This indicates that an extreme case of the UHECR intensity may lead to a large departure from the present estimate of the neutrino fluxes. For instance, the difference of their estimate for the scenario of  $(m, z_{\max}) = (2, 2)$  found in Table I arises from their assumption of very steep UHECR spectrum leading to a minimal  $F_{\text{CR}}$  estimation. This uncertainty is, however, expected to be reduced in the future when the statistical uncertainties in the observations of UHECRs and/or the systematic uncertainty in the energy estimation are improved.

We would like to also emphasize that the neutrino intensity below 10 EeV is not largely affected by the detailed behavior

of UHECR proton propagation in extragalactic space. This is because these neutrinos are mostly generated at cosmological distances which are substantially longer than the UHECR proton energy attenuation length in the CMB field. It is also suggested by no explicit dependence of  $\lambda_{\text{GZK}}$  in the final formula Eq. (14). This is related to the fact that the cosmogenic flux below 10 EeV is relatively insensitive to  $E_{\max}$  and  $\alpha$ , the maximal injection energy of UHECR protons from their sources and the spectral index of UHECR spectrum, respectively, while the flux above 10 EeV is sensitive to those parameters [6,19,23]. A scan of the parameter spaces of the cosmogenic neutrino sources for some known classes of astronomical objects with a numerical Monte-Carlo method was made in Refs. [15,19]. It was also shown that the intensity around 1 EeV is stable against  $E_{\max}$  variation and the transition models between the Galactic and extragalactic cosmic-ray components. These observations are consistent with the fact that the neutrino intensities around 1 EeV by the relatively old works [6,16] assuming harder UHECR spectrum of  $\alpha = 2.0$  and higher  $E_{\max}$ , and those by the recent works [9,15] with  $\alpha \sim 2.5$  show an agreement also within a factor of 2. The difference between the present analytical formula and the full-blown simulation above  $\sim 10$  EeV in Fig. 1 is attributed to responses to  $E_{\max}$ . The present analytical estimates of neutrino fluxes for  $100 \text{ PeV} \lesssim E_\nu \lesssim 10 \text{ EeV}$ , the main energy range by the IceCube cosmogenic neutrino search, is robust against these parameters. We should note however that we use  $F_{\text{CR}}$  for the normalization constant assuming  $E_{\max} \gg E_{\text{GZK}}$ . If  $E_{\max}$  is comparable or lower than  $E_{\text{GZK}}$ , the neutrino yield strongly depends on  $E_{\max}$  and the present simple treatment is not capable of providing reasonable estimates of the cosmogenic neutrino fluxes.

The present analysis indicates that a five-year observation by the IceCube observatory will scan the source evolution parameter space of the most interest where

many of the proposed UHECR astronomical sources are distributed. A null neutrino observation then would imply that either UHECR sources are only locally distributed ( $z_{\max} \lesssim 1$ ), very weakly evolved ( $m \lesssim 3$ ), or the mass composition of UHECRs is not dominated by proton primaries, but by heavier nuclei such as irons after all. The first two possibilities may lead to a speculation about the highest energy particle emission from an entirely different and probably dimmer class of objects than currently suggested. The last possibility has also been discussed with the measurement of the depth of maximum of air-showers by the Auger Collaboration [27]. A neutrino search in ultrahigh energies provides a complementary constraint on the proton fraction of UHECRs in this case [28].

## VI. SUMMARY

We have derived the analytical formula to estimate the cosmogenic neutrino fluxes for a wide range of cosmological evolution parameters of UHECR emission sources. The analytical formula provides a practical tool for estimating the neutrino intensity at around the EeV energy region with a limited accuracy within a factor of  $\sim 2$ . The obtained analytical estimates have indicated that the present IceCube neutrino limit in 100 PeV–10 EeV energies disfavors the scenarios with the strongly evolved UHECR sources. The future IceCube observation will be able to scan most of the interesting parameter space of UHECR source evolution. Furthermore, while the deep and highly energetic part of Universe is inaccessible with photons or cosmic rays due to the CMB field, the current study implies that the neutrinos can be used as a rare tool to probe the far Universe.

With the greater statistics of ultrahigh energy neutrino detections by the future neutrino telescopes of  $\sim 100$  km<sup>2</sup> areas, such as ARA [29] and ARIANNA [30], the analytical formula allows us to specify the astronomical classes of ultrahigh energy cosmic-ray sources. The pioneer prediction of the cosmogenic neutrinos in the 1960s [4] will finally lead to revealing the characteristics of an UHECR emission mechanism in the near future.

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## APPENDIX A: THE FLUX CALCULATION BASED ON ENERGETICS

Recently the possible upper limit on the cosmogenic neutrino flux has been discussed solely using the Fermi

measurement on extragalactic diffuse  $\gamma$ -ray background [31]. In this work, the neutrino flux was approximately estimated by the energetics argument, calculating energy channeling into secondary neutrinos from a UHECR proton during its propagation in the CMB field. The neutrino flux is then calculated by

$$E_\nu \frac{dJ_\nu}{dE_\nu} = n_0 c \int_0^{z_{\max}} dz_s \psi(z_s) \left| \frac{dt}{dz}(z_s) \right| \times \int dE_{\text{CR}} E_{\text{CR}} \frac{dN_{\text{CR}}}{dE_{\text{CR}}} R_\nu(E_{\text{CR}}) \frac{d\rho_\nu}{dE_\nu}. \quad (\text{A1})$$

Here  $dN_{\text{CR}}/dE_{\text{CR}}$  is the injected UHECR proton spectrum ( $\sim E_p^{-\alpha}$ ) at the source redshift  $z_s$ ,  $R_\nu$  is a fraction of UHECR proton injection energy carried by the secondary neutrinos, and  $d\rho_\nu/dE_\nu$  is a distribution of neutrino energy.  $R_\nu$  was calculated in the earlier work [16] represented by a numerically fitted function as

$$R_\nu = \frac{0.45}{1 + \left( \frac{2 \times 10^{11} \text{ GeV}}{(1+z_s)E_{\text{CR}}} \right)^2}. \quad (\text{A2})$$

While the original work [31] represented  $d\rho_\nu/dE_\nu$  as  $\sim \delta(E_\nu - E_{\text{CR}}/(20(1+z_s)))$  approximating each secondary neutrino receiving 1/20 of the UHECR proton energy, we found that the single pion kinematics approximation would give a better agreement in the neutrino spectral shape with those obtained by the full-blown simulation. It is then written as

$$\frac{d\rho_\nu}{dE_\nu} \simeq (1+z_s) [E_{\text{CR}}(x_R^+ - x_R^-)(1-r_\pi)]^{-1} \ln \left( \frac{x_R^+}{\xi_R} \right), \quad (\text{A3})$$

where  $\xi_R$  is given by Eq. (8), replacing  $E_\nu^g$  with  $E_\nu(1+z_s)$ .

This approach has an advantage in that it does not rely on the  $\Delta$ -resonance approximation. Although we are not able to find out a complete analytical solution of the integrals in Eq. (A1), the numerical calculations indeed confirmed that Eq. (A1) reasonably reproduces the full simulation/numerical calculation results. It gives a better agreement than our formula at around 100 PeV, owing to inclusion of direct pion production yielding a pair of  $\pi^+ \pi^-$  by Eq. (A2) [16]. However, this energetics-based formulation significantly overestimates neutrino intensities with energy above 1 EeV. We suspect this is due to neglecting of the energy loss of UHECR protons. Energy of an UHECR proton is in many cases largely lower than its injected energy when it yields neutrinos, because of energy loss by the photo-pion production during the UHECR propagation. Without accounting for this effect, higher energy neutrino production is overweighted in the formulation. The overproduced high-energy neutrinos are then redshifted and accumulated even in the PeV regime when UHECR sources are strongly evolved. As a consequence, the estimated intensity departs from the calculation with the full-blown simulation in case of the strong evolution scenario. Since the GZK neutrino search by the IceCube

detector is sensitive to the EeV range and to emission from strongly evolved sources, we concluded that the energetics-based formulation is not appropriate for our purpose.

## APPENDIX B: THE ANALYTICAL FORMULA FOR THE PARTIALLY CONSTANT SOURCE EVOLUTION

Some classes of astronomical objects, like galaxy star formation, seem to exhibit evolution nearly constant above a certain redshift. For these cases, cosmological evolution

of sources is written as  $\psi(z_s) \sim (1 + z_s)^m$  for  $0 \leq z_s \leq \tilde{z}$  and  $\psi(z_s) \sim (1 + \tilde{z})^m$  for  $\tilde{z} \leq z_s \leq z_{\max}$ . The redshift-dependence term  $\zeta$  (Eq. (16)) is then obtained with minor modifications on Eq. (17) and given as

$$\zeta = \zeta_{\tilde{z}} + \tilde{\zeta}, \quad (\text{B1})$$

where  $\zeta_{\tilde{z}}$  is given by Eq. (17) by replacing  $z_{\max}$  with  $\tilde{z}$ , accounting for the evolution up to  $z_s \leq \tilde{z}$ .

The additional term  $\tilde{\zeta}$  is obtained in functions similar to Eq. (17) as

$$\begin{aligned} \tilde{\zeta} = e^{-2} \frac{1}{\gamma_\alpha} \Omega_M^{-(\alpha/3)} (1 + \tilde{z})^m & \left\{ (\Omega_M (1 + \tilde{z}_{\text{up}})^3 + \Omega_\Lambda)^{\gamma_\alpha/3} \left( x_0^{-(\alpha+1)} \ln \left( \frac{x_R^+}{x_R^-} \right) \right. \right. \\ & + x_1^{-(\alpha+3)} e^{-(1/x_1)} \left[ \ln \left( x_1 \frac{E_\Delta}{E_\nu (1 + \tilde{z}_{\text{up}})^2} x_R^- (1 - r_\pi) \right) + \frac{2}{\gamma_\alpha} \right] - (\Omega_M (1 + \tilde{z}_{\text{down}})^3 + \Omega_\Lambda)^{\gamma_\alpha/3} x_0^{-(\alpha+1)} \ln \left( \frac{x_R^+}{x_R^-} \right) \\ & \left. \left. - (\Omega_M (1 + \tilde{z}_{\text{down}})^3 + \Omega_\Lambda)^{\gamma_\alpha/3} x_1^{-(\alpha+3)} e^{-(1/x_1)} \left[ \ln \left( x_1 \frac{E_\Delta}{E_\nu (1 + \tilde{z}_{\text{down}})^2} x_R^- (1 - r_\pi) \right) + \frac{2}{\gamma_\alpha} \right] \right\}, \end{aligned} \quad (\text{B2})$$

where  $\gamma_\alpha \equiv \alpha - \frac{3}{2}$  and the redshift bound  $\tilde{z}_{\text{up}}$  is given by

$$1 + \tilde{z}_{\text{up}} = \begin{cases} 1 + \tilde{z} & \frac{x_1 E_\Delta x_R^+ (1 - r_\pi)}{(1 + \tilde{z})^2} \leq E_\nu, \\ \left( \frac{x_1 E_\Delta x_R^+ (1 - r_\pi)}{E_\nu} \right)^{1/2} & \frac{x_1 E_\Delta x_R^+ (1 - r_\pi)}{(1 + z_{\max})^2} \leq E_\nu \leq \frac{x_1 E_\Delta x_R^+ (1 - r_\pi)}{(1 + \tilde{z})^2}, \\ 1 + z_{\max} & E_\nu \leq \frac{x_1 E_\Delta x_R^+ (1 - r_\pi)}{(1 + z_{\max})^2}. \end{cases} \quad (\text{B3})$$

$\tilde{z}_{\text{down}}$  is written as the same equation (B3) by replacing  $x_R^+$  with  $x_R^-$ .

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